



## Physics for Scientists & Engineers 2

Spring Semester 2005  
Lecture 25

## Induction



- Last week we learned that a current-carrying loop in a magnetic field experiences a torque
- If we start with a loop with no current in a magnetic field, and force the loop to rotate, we find that a current is induced in the loop
- Further, if we start with a loop with no current and turn on a magnetic field without moving the coil, again a current is induced in the loop
- These effects are described by Faraday's Law of Induction and are the basis of electric motors and electric power generation from mechanical motion

## Faraday's Experiments

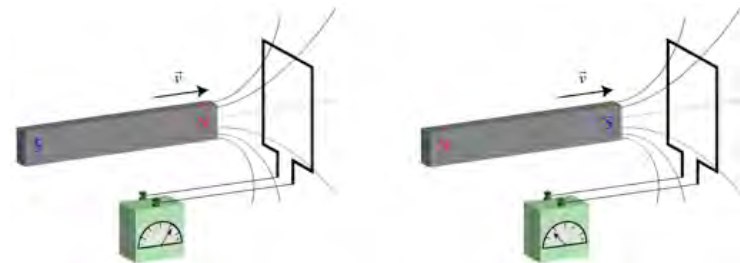


- Consider the situation in which we have a wire loop connected to an ammeter so that we can measure current flowing in the loop
- We hold a bar magnet some distance from the loop, pointing the north pole of the magnet toward the loop
- While the magnet is stationary, there is no current flowing in the loop
- What happens if we move the magnet?

## Faraday's Experiments (2)



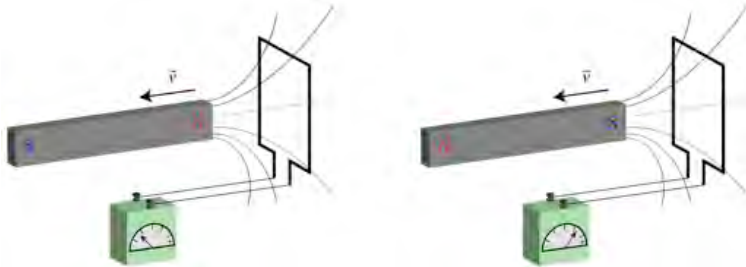
- When we move the magnet toward the loop, we induce a positive current in the loop
- Now we turn the magnet around so that the south pole points toward the loop
- When we move the magnet toward the loop, we induce a negative current in the loop



## Faraday's Experiments (3)



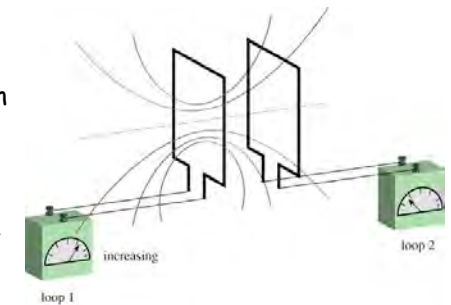
- Now let's point the north pole toward the loop but move away from the loop
  - We get a negative current
- We turn the magnet around so that the south pole points toward the loop and move away from the loop
  - We get a positive current



## Faraday's Experiments (4)



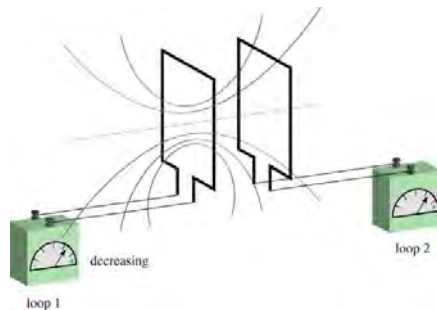
- We can create similar effects by placing a second loop near the first loop but with a more quantitative result as shown below
- If a constant current is flowing through loop 1, no current will be induced in loop 2
- If we increase the current in the loop 1, we observe that a current is induced in the loop 2 in the opposite direction
- Thus not only does the changing current in the first loop induce a current in the first loop, the induced current is in the opposite direction



## Faraday's Experiments (5)



- Now if we have the current flowing in loop 1 in the same direction as before, and decrease the current as shown below, we induce a current flowing loop 2 in the same direction as the current in loop 1



## Law of Induction



- From these observations we see that a changing magnetic field induces a current in a loop
- We can visualize the change in magnetic field as a change in the number of magnetic field lines passing through the loop
- Faraday's Law of Induction** states that:
  - An emf is induced in a loop when the number of magnetic field lines passing through the loop changes with time
- The rate of change of magnetic field lines determines the induced emf

## Magnetic Flux



- To quantify the amount of magnetic field lines we define the magnetic flux in analogy to the electric flux
- When we introduced Gauss' Law for the electric field, we defined the electric flux as

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

- For the magnetic field, we can define **magnetic flux** in analogy as

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

- The unit of magnetic flux is the weber (Wb)

$$1 \text{ Wb} = 1 \text{ Tm}^2$$

## Magnetic Flux - Special Case



- Consider the special case of a flat loop of area  $A$  in a constant magnetic field  $B$
- In this case we can re-write the magnetic flux as
  - $\Phi_B = BA \cos \theta$
  - $\theta$  is the angle between the surface normal vector of the plane of the loop and the magnetic field
- If the magnetic field is perpendicular to the plane of the loop
  - $\theta = 0^\circ, \Phi_B = BA$
- If the magnetic field is parallel to the plane of the loop
  - $\theta = 90^\circ, \Phi_B = 0$

## Faraday's Law of Induction



- We can then recast Faraday's Law of Induction in terms of the magnetic flux as
  - The magnitude of the  $V_{emf}$  induced in a conducting loop is equal to the time rate of change of the magnetic flux from the loop. This induced emf tends to oppose the flux change.
- Faraday's Law of Induction is thus contained in the equation

$$V_{emf} = -\frac{d\Phi_B}{dt}$$

- We can change the magnetic flux in several ways including changing the magnitude of the magnetic field, changing the area of the loop, or by changing the angle the loop with respect to the magnetic field

## Induction in a Flat Loop



- Let's explore Faraday's Law for the special case of a flat loop inside a magnetic field that is constant in space, but that can vary in time
- The magnetic flux for this case is

$$\Phi_B = BA \cos \theta$$

- The induced emf is then

$$V_{emf} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA \cos \theta)$$

- Carrying out the derivative we get

$$V_{emf} = -A \cos \theta \frac{dB}{dt} - B \cos \theta \frac{dA}{dt} + AB \sin \theta \frac{d\theta}{dt}$$

- Taking  $d\theta/dt = \omega$  we get

$$V_{emf} = -A \cos \theta \frac{dB}{dt} - B \cos \theta \frac{dA}{dt} + \omega AB \sin \theta$$

## Induction in a Flat Loop - Special Cases



- If we leave two of the three variables ( $A, B, \theta$ ) constant, then we can have the following three special cases
  - We leave the area of the loop and its orientation relative to the magnetic field constant, but vary the magnetic field in time
 
$$A, \theta \text{ constant: } V_{emf} = -A \cos \theta \frac{dB}{dt}$$
  - We leave the magnetic field as well as the orientation of the loop relative to the magnetic field constant, but change the area of the loop that is exposed to the magnetic field
 
$$B, \theta \text{ constant: } V_{emf} = -B \cos \theta \frac{dA}{dt}$$
  - We leave the magnetic field constant and keep the area of the loop fixed as well, but allow the angle between the two to change as a function of time
 
$$A, B \text{ constant: } V_{emf} = \omega AB \sin \theta$$

## Example: Changing Magnetic Field



- A direct current of 600 mA is delivered to an ideal solenoid, resulting in a magnetic field of 0.025 T
- Then the current is increased according to
 
$$i(t) = i_0(1 + 2.4s^{-2}t^2)$$
- **Question:**
  - If a circular loop of radius 3.4 cm with 200 windings is located inside the solenoid and perpendicular to the magnetic field, what is the induced voltage at  $t = 2.0$  s in this loop?

## Example: Changing Magnetic Field (2)



- **Answer:**
  - First, the area of the loop is computed (the number of winding acts as a simple multiplier):
 
$$A = N\pi R^2 = 200\pi(0.034 \text{ m})^2 = 0.73 \text{ m}^2$$
  - The magnetic field inside an ideal solenoid is
 
$$B = \mu_0 in$$
  - Because the magnetic field is linearly proportional to the current, we obtain the time dependence of the magnetic field in this case
 
$$B(t) = B_0(1 + 2.4s^{-2}t^2)$$

## Example: Changing Magnetic Field (3)



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$$B(t) = B_0(1 + 2.4s^{-2}t^2), B_0 = 0.025 \text{ T}$$

## Example: Changing Magnetic Field (4)



- The area of the loop and the angle are kept constant so

$$A, \theta \text{ constant: } V_{emf} = -A \cos \theta \frac{dB}{dt}$$

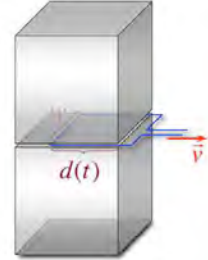
- For the induced voltage we then find

$$\begin{aligned} V_{emf} &= -A \cos \theta \frac{dB}{dt} \\ &= -A \cos \theta \frac{d}{dt} (B_0 (1 + 2.4s^{-2}t^2)) \\ &= -AB_0 \cos \theta (2 \cdot 2.4s^{-2}t) \\ &= -(0.73 \text{ m}^2)(0.025 \text{ T})(\cos 90^\circ)(4.8s^{-2})t \\ \text{at } t = 2.0 \text{ s, } V_{emf} &= -0.17 \text{ V} \end{aligned}$$

## Example: Motion Voltage



- A rectangular loop of width 3.1 cm and depth 4.8 cm is pulled out of the gap between two permanent magnets, with a field of 0.073 T throughout the gap.
- Question:**
  - If the loop is removed with a constant velocity of 1.6 cm/s, what is the induced voltage in the loop as a function of time?

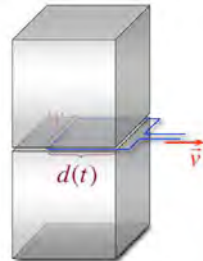


- Answer:**
  - The magnetic field as well as the orientation of the loop relative to the field remains constant
  - What changes is the area of the loop that is exposed to the magnetic field

## Example: Motion Voltage (2)



- With the narrow gap, there will be very little field outside the gap
- The effective area of the loop exposed to the field is  $A(t) = w \cdot d(t)$
- While the entire loop is still inside the gap, no voltage is produced
- We select the time of arrival of the right edge of the loop at the right edge of the gap as  $t = 0$



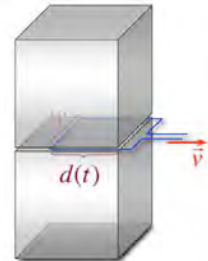
- Then we find
 
$$A(t) = w \cdot d(t) = w \cdot (d - vt)$$
- This formula holds until the left edge of the loop reaches the edge of the gap, after which the area of the loop is zero
- The left edge arrives at
 
$$t_f = d / v = 4.8 \text{ cm} / 1.6 \text{ cm/s} = 3.0 \text{ s}$$

## Example: Motion Voltage (3)



- Now we can calculate the induced voltage

$$\begin{aligned} V_{emf} &= -B \cos \theta \frac{dA}{dt} \\ &= -B \cos \theta \frac{d}{dt} (w \cdot (d - vt)) \\ &= wvB \cos \theta \\ &= (0.031 \text{ m})(0.016 \text{ m/s})(0.073 \text{ T}) \\ &= 3.6 \cdot 10^{-5} \text{ V} \end{aligned}$$



- So our result is that during the time interval between 0 and 3 s, a constant voltage of 36  $\mu\text{V}$  is induced, and no voltage is produced outside this time interval