Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 26

Review

- Faraday's Law of Induction in words is
  - *The magnitude of the V_{emf} induced in a conducting loop is equal to the time rate of change of the magnetic flux from the loop. This induced emf tends to oppose the flux change.*
- Faraday's Law of Induction in equation form is

$$\boxed{V_{emf} = -\frac{d\Phi_B}{dt}}$$

  - V_{emf} is the induced voltage
  - $d\Phi_B/dt$ is time rate change of the magnetic flux
  - The negative sign means that the induced voltage opposes the change in flux

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Review (2)

- If we have a flat loop, we can keep two of the three variables (A,B,θ) constant, and vary the third, then we can have the following three special cases
  - We leave the area of the loop and its orientation relative to the magnetic field constant, but vary the magnetic field in time

$$A,\theta \text{ constant:} \quad V_{emf} = -A\cos\theta\frac{dB}{dt}$$

  - We leave the magnetic field as well as the orientation of the loop relative to the magnetic field constant, but change the area of the loop that is exposed to the magnetic field

$$B,\theta \text{ constant:} \quad V_{emf} = -B\cos\theta\frac{dA}{dt}$$

  - We leave the magnetic field constant and keep the area of the loop fixed as well, but allow the angle between the two to change as a function of time

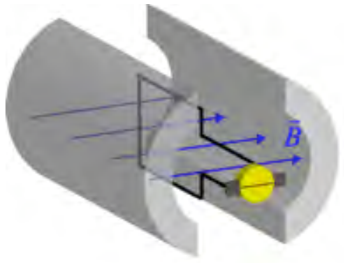
$$A,B \text{ constant:} \quad V_{emf} = \omega AB\sin\theta$$

Generators and Motors

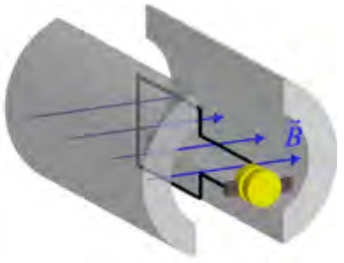
- The third special case of our simple induction processes is the technologically by far the most interesting
- It is the variation of the angle between the loop and the magnetic field with time, while keeping the area of the loop as well as the magnetic field strength constant in time
- In this way, Faraday's Law of Induction can be applied to the generation and use of electric current
- A device that produces electric current from mechanical motion is called a generator
- A device that produces mechanical motion from electric current is called a motor

Generators and Motors (2)

- A simple generator consists of a loop forced to rotate in a fixed magnetic field
- The driving force that causes the loop to rotate can be supplied by hot steam running over a turbine
- Or the loop can be made to rotate by water or wind in a completely pollution-free way of generating electrical power



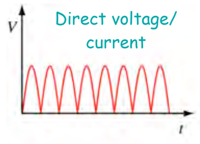
Direct current generator
split ring commutator

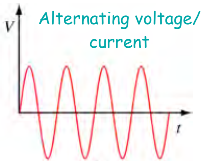


Alternating current generator
slip ring commutator

Generators and Motors (3)

- In a direct current generator the rotating coil is connected to an external circuit using a split commutator ring
- As the coil turns, the connection is reversed such that the induced voltage always has the same sign
- In alternating current generator, each end of the loop is connected to the external circuit through a slip ring
  - Thus this generator produces an induced voltage that varies from positive to negative and back, and is called an alternator
- The voltages and currents produced by these generators are illustrated to the right





Back EMF

- Another fact-of-life concerning electric motors is back emf
- An electric motor is essentially the same as an electric generator
- Thus as the motor runs faster and faster, it begins to generate a voltage opposing the voltage that is being supplied to create current in the motor
- You might have noticed that when a large electric motor such as an air conditioner compressor starts up, the lights dim, because the motor is drawing a large amount of current
- As the motor speeds up, the dimming effect disappears
- When the motor is running at normal speed, it draws less current
- If an electric motor is overloaded and stops, it will draw large amounts of current that could produce enough heat to damage the motor

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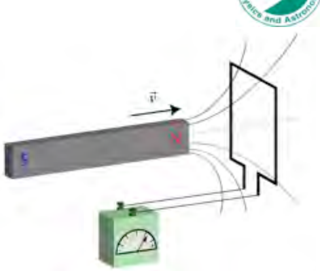
Lenz's Law

- Lenz's Law defines a rule for determining the direction of an induced current in a loop
- An induced current will have a direction such that the magnetic field due to the induced current opposes the change in the magnetic flux that induces the current
- The direction of the induced current corresponds to the direction of the induced emf
- We can apply Lenz's Law to the situations described in yesterday's lecture

Lenz's Law (2)



- The physical situation shown here involves moving a magnet toward a loop with the north pole pointed toward the loop
- The magnetic field lines point toward the north pole of the magnet
- As the magnet moves toward the loop, the magnitude of the field increases in the direction pointing toward the north pole
- Lenz's law states that a current is induced in the loop that tends to oppose the change in magnetic flux
- This induced magnetic field then points in the opposite direction as the field from the magnet

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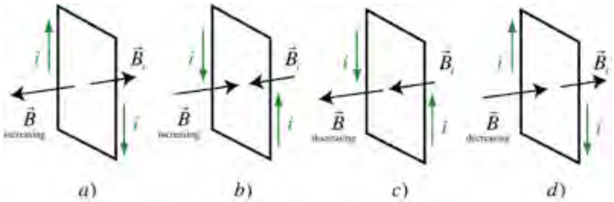
Lenz's Law - Four Cases



- *a*) An increasing magnetic field pointing to the left induces a current that creates a magnetic field to the right
- *b*) An increasing magnetic field pointing to the right induces a current that creates a magnetic field to the left
- *c*) A decreasing magnetic field pointing to the left induces a current that creates a magnetic field to the left
- *d*) A decreasing magnetic field pointing to the right induces a current that creates a magnetic field to the right

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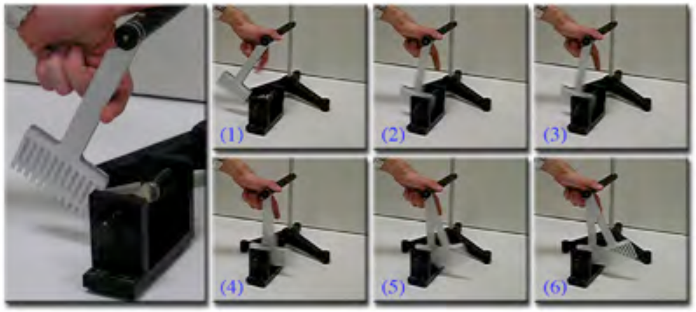
Eddy Currents

- Let's consider two pendulums, each with a non-magnetic conducting metal plate at the end that is designed to pass through the gap of a strong permanent magnet
- One metal plate is solid and the other has slots cut in it



- We pull back both pendulums and release them

Eddy Currents (2)

- We observe that the pendulum with the solid metal sheet stops in the gap of the magnet while the grooved sheet passes through the magnetic field, only slowing slightly
- This demonstration illustrates the very important concept of induced eddy currents
- As the pendulum with the solid plate enters the magnetic field, Lenz's law tells us that the changing magnetic flux will induce currents that tend to oppose the change in flux
- These currents interact with the magnetic field to stop the pendulum
- For the slotted plate, the induced eddy currents are blocked and the slotted plate passes through the magnetic field, only slowing

Eddy Currents (3)

- Eddy currents are not like the current induced in the loop in Example 28.2, but swirl like eddy currents we see in turbulent flowing water
- Eddy currents are often undesirable and steps are taken to minimize them by segmenting or laminating an electrical device that must operate in an environment of changing magnetic fields
  - Pulsed magnets
  - Transformers
- Induced eddy currents can also be employed in practical situations such as braking railroad cars

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Induced Electric Fields

- A changing magnetic field produces an electric field, as we have shown from Faraday's Law of Induction
- Let's explore the consequences of this statement
- Consider a test charge q moving in a circular path with radius r
- The work done is equal to the integral of the force times the distance
- Considering one revolution of the test charge we obtain the work done on the test charge to be

$$\int \vec{F} \bullet d\vec{s} = (qE)(2\pi r)$$

Induced Electric Fields (2)

- Remembering that the work done by an electric field is $V_{emf}q$ we get

$$V_{emf} = 2\pi r E$$

- We can generalize this result by considering the work done a test particle with charge q moving along an arbitrary closed path as

$$W = \oint \vec{F} \bullet d\vec{s} = q\oint \vec{E} \bullet d\vec{s}$$

- Again substituting $V_{emf}q$ for the work we obtain

$$V_{emf} = \oint \vec{E} \bullet d\vec{s}$$

Induced Electric Fields (3)

- Now we can express the induced emf as

$$\boxed{\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}}$$

- Which states that a changing magnetic field induces an electric field
- This equation can be applied to any closed path drawn in a changing magnetic field
- We will encounter this idea in electromagnetic waves

Inductance

- In studying capacitors, we found that independent of the geometry of two conductors, the charge on the plates *q* is always proportional to the electric potential I between the plates and the proportionality constant is called the capacitance *C* such that

$$q = CV$$

- Consider a long solenoid with turns carrying a current *i*
- This current creates a magnetic field in the center of the solenoid resulting in a magnetic flux of Φ_B
- For this case we find that the quantity $N\Phi_B$, called the flux linkage, is always proportional to the current with a proportionality constant called the inductance *L*

$$N\Phi_B = Li$$

Inductance (2)

- Thus the inductance is a measure of the flux linkage produced by the inductor per unit current
- For an inductor to behave in this manner, it must not have any magnetic materials in its core
- The unit of inductance is the henry (H) given by

$$[L] \;=\; \frac{[\Phi_B]}{[i]} \Rightarrow 1\ \mathrm{H} = \frac{1\ \mathrm{Tm}^2}{1\ \mathrm{A}}$$

- Which allows us to write the magnetic permeability of free space as

$$\mu_0 = 4\pi \cdot 10^{-7}\ \mathrm{H/m}$$

Inductance of a Solenoid

- Consider a solenoid with cross sectional area *A* and length *l*
- The flux linkage is

$$N\Phi_B = (nl)(BA)$$

  - *n* is the number turns per unit length and $B = \mu_0 in$
- The inductance of a solenoid is then

$$L = \frac{N\Phi_B}{i} = \frac{(nl)(\mu_0 in)(A)}{i} = \mu_0 n^2 lA$$

- You can see that the inductance of a solenoid depends only on its geometry