Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 34

Review

- The speed of an electromagnetic wave can be expressed in terms of two fundamental constants related to electric fields and magnetic fields, the magnetic permeability and the electric permittivity of the vacuum

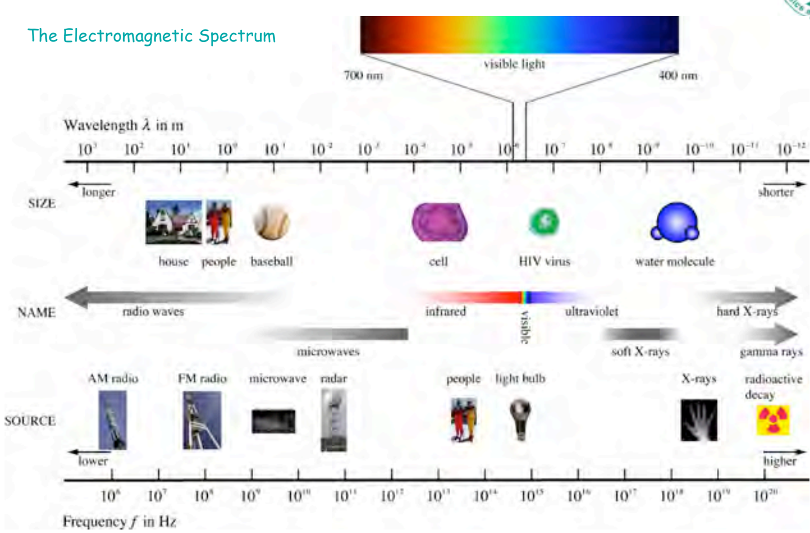
$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

- The speed of light is constant in any reference frame
  - Relativity
- For an electromagnetic wave, the wavelength and frequency of the wave are related to the speed of light

$$c = \lambda f$$

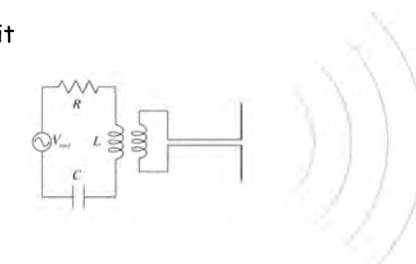
Review (2)

The Electromagnetic Spectrum



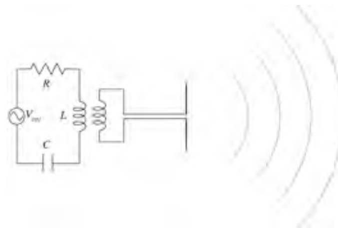
Traveling Electromagnetic Waves

- Sub-atomic processes can produce electromagnetic waves such as gamma rays, X-rays, and light
- Electromagnetic waves can also be produced by an oscillator connected to an antenna
- The connection between the circuit and the circuit on the right is accomplished using a transformer
- A dipole antenna is used to approximate an electric dipole
- The voltage and current in the antenna vary sinusoidally with time and cause charge in the antenna to oscillate with frequency ω of the circuit
- The electromagnetic waves created by moving charges travel from the antenna with speed c and frequency $f = \omega/(2\pi)$



Traveling Electromagnetic Waves (2)

- We can think of these traveling electromagnetic waves as wave fronts spreading out spherically from the antenna



- However, at a large distance from the antenna, the wave fronts will appear to be almost flat, or planar
- So we can think of the electromagnetic wave in terms of our assumed form

$$E(\vec{r},t) = E_{\max} \sin(kx - \omega t)$$
$$B(\vec{r},t) = B_{\max} \sin(kx - \omega t)$$

Traveling Electromagnetic Waves (3)

- If we now place in the path of these electromagnetic waves, a second *RLC* circuit tuned to the same frequency ω_0 as the emitting circuit, voltage and current will be induced in this second circuit
- These induced oscillations are the basic idea of radio transmission and reception
- If the second circuit has

$$\omega_{0,2} = 1/\sqrt{LC} \neq \omega_0$$

- smaller voltages and currents will be induced, providing selective tuning for different frequencies
- This principle of transmission of electromagnetic waves was discovered by German physicist Heinrich Hertz (1857-1894) in 1888, and then used by Italian physicist Guglielmo Marconi (1874-1937) to transmit wireless signals

Energy Transport

- When we walk out into the sunlight, we feel warmth
- If we stay out too long in the bright sunshine, we will get sunburn
- These phenomena are related to electromagnetic waves emitted from the Sun
- These electromagnetic waves carry energy generated in the nuclear reactions of the Sun to our skin
- The rate of energy transported by an electromagnetic wave is usually defined as

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

- This quantity is called the Poynting vector after British physicist John Poynting (1852-1914) who first discussed its properties

Energy Transport (2)

- The magnitude of the Poynting vector is related to the instantaneous rate at which energy transported by an electromagnetic wave over a given area
  - more simply, the instantaneous power per unit area

$$S = |\vec{S}| = \left(\frac{\text{power}}{\text{area}} \right)_{\text{instantaneous}}$$

- The units of the Poynting vector are W/m^2
- For an electromagnetic wave, in which B is perpendicular to E, we can write

$$S = \frac{1}{\mu_0} EB$$

Energy Transport (3)

- We know that the magnitude of the electric field and the magnetic field are directly related via $E/B = c$
- We can express the instantaneous power per unit area of an electromagnetic wave in terms of the magnitude the electric field or the magnetic field
- However, usually it is easier to measure an electric field than a magnetic field so we express the instantaneous power per unit area as

$$S = \frac{1}{c\mu_0} E^2$$

- We can now substitute a sinusoidal form for the electric field and obtain an expression for the transmitted power per unit area

$$E = E_{\max} \sin(kx - \omega t)$$

Energy Transport (4)

- The usual method of describing the power per unit area in an electromagnetic wave is the intensity I of the wave given by

$$I = S_{ave} = \left(\frac{\text{power}}{\text{area}}\right)_{ave} = \frac{1}{c\mu_0}\left[E_{\max}^2 \sin^2(kx - \omega t)\right]_{ave}$$

- The units of intensity are the same as the units of the Poynting vector, W/m^2
- The average of $\sin^2(kx-\omega t)$ over time is 1/2
- So we can express the intensity as

$$I = \frac{1}{c\mu_0} E_{rms}^2 \qquad \left(E_{rms} = E_{\max} / \sqrt{2}\right)$$

Energy Transport (5)

- Because the magnitude of the electric and magnetic fields of the electromagnetic wave are related by $E = cB$, and c is such a large number, one might conclude that the energy transported by the electric field is much larger than the energy transmitted by the magnetic field
- Actually the energy transported by the electric field is the same as the energy transported by the magnetic field
- We can understand this fact by remembering that the energy density of an electric field is given by

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

- And the energy density of a magnetic field is given by

$$u_B = \frac{1}{2\mu_0} B^2$$

Energy Transport (6)

- If we substitute

$$E = cB \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

- We get

$$u_E = \frac{1}{2}\varepsilon_0 (cB)^2 = \frac{1}{2}\varepsilon_0 \left(\frac{B}{\sqrt{\mu_0\varepsilon_0}}\right)^2 = \frac{1}{2\mu_0} B^2 = u_B$$

- We obtain the result that the energy density of electric field is the same as the energy density of the magnetic field everywhere in the electromagnetic wave

Radiation Pressure

- When you walk out into the sunlight, you feel warmth, but you do not feel any force from the sunlight
- Sunlight is exerting a pressure on you, but that pressure is small enough that you do not notice it
- Because these electromagnetic waves are radiated from the Sun and travel to you on the Earth, we call these electromagnetic waves radiation
- This type of radiation is not the same as radioactive radiation resulting from the decay of unstable nuclei
- Let's calculate the magnitude of the pressure exerted by these radiated electromagnetic waves

Radiation Pressure (2)

- Electromagnetic waves have energy
- Electromagnetic waves also have linear momentum
- Let's assume that an electromagnetic wave is incident on an object and that this object totally absorbs the wave over some time interval Δt
- The object will then gain energy ΔU over that time interval from the radiated electromagnetic waves and we can relate the change in momentum of the object Δp to the change in energy by

$$\Delta p = \frac{\Delta U}{c}$$

- The change in momentum of the object will be in the same direction as the incident electromagnetic wave

Radiation Pressure (3)

- If instead, the object reflects the electromagnetic wave, the object will gain twice the momentum of the incident wave, because of momentum conservation, in the direction of the incident wave

$$\Delta p = \frac{2\Delta U}{c}$$

- Newton's Second Law tells us that

$$F = \frac{\Delta p}{\Delta t}$$

- To get an expression for the pressure exerted by the electromagnetic wave in terms of the intensity of the wave, we remember that
  - the intensity is power per unit area
  - power is energy per unit time

Radiation Pressure (4)

- Therefore we can relate the intensity of the wave to the momentum transferred to the object when the electromagnetic wave is absorbed

$$I = \frac{\Delta U / \Delta t}{A} = \frac{c\Delta p}{A\Delta t}$$

- which gives us an expression for the force exerted by the radiation on the object

$$F = \frac{\Delta p}{\Delta t} = \frac{IA}{c}$$

- The pressure is defined as force per unit area so we can write the radiation pressure due to a totally absorbed electromagnetic wave p_r as

$$p_r = \frac{I}{c}$$

Radiation Pressure (5)

- If the electromagnetic wave is reflected the radiation pressure is

$$p_r = \frac{2I}{c}$$

- The radiation pressure from sunlight is small
- The intensity of sunlight is at most 1400 W/m^2
- Thus maximum the radiation pressure for sunlight that is totally absorbed is

$$p_r = \frac{I}{c} = \frac{1400 \text{ W/m}^2}{3 \cdot 10^8 \text{ m/s}} = 4.67 \cdot 10^{-6} \text{ N/m}^2$$

- However, physicists using very intense lasers focused to a small area can exert pressure on small objects such as DNA molecules. These devices are called "laser tweezers"
- Using laser tweezers, DNA molecules can be stretched or molecules can be inserted in the strands