Physics for Scientists & Engineers 2

Spring Semester 2005

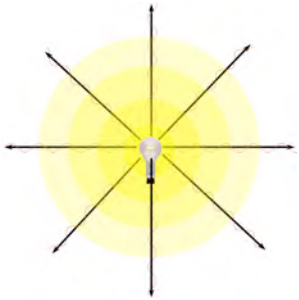
Lecture 36

Geometric Optics

- The study of light divides itself into three fields
  - geometric optics
  - wave optics
  - quantum optics
- In the previous chapter, we learned that light is an electromagnetic wave
- In the next chapter, we will deal with the wave properties of light
- In this chapter we will deal with geometric optics in which we will treat light that travels in straight lines called light rays
- Quantum optics makes use of the fact that light is quantized

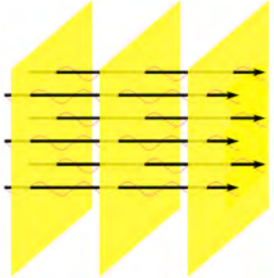
Spherical Waves

- Electromagnetic waves spread spherically from a source
- The concentric yellow spheres shown below represent the spreading spherical wave fronts of the light emitted from the light bulb
- The black arrows are the light rays, which are perpendicular to the wave fronts at every point in space.



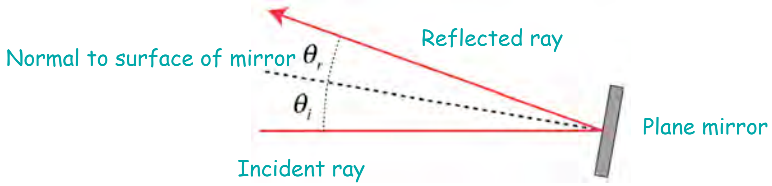
Plane Waves

- We can treat light waves from far away sources as plane waves whose wave front is traveling in a straight line
- We can further abstract these traveling planes by vectors or arrows perpendicular to the surface of these planes
- These planes can then be represented by a series of parallel rays or just one ray
- In this chapter we will treat light as a ray traveling in a straight line
- We can solve many problems geometrically and by various constructions
- Thus for the remainder of this chapter we will ignore our knowledge of the structure of light and attack a broad range of practical problems



Reflection and Plane Mirrors

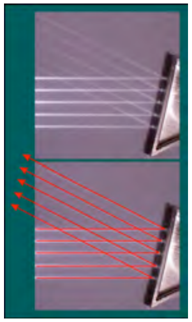
- A mirror is a surface that reflects light
- A plane mirror is a flat mirror
- For reflection from plane mirrors we have a simple rule for light rays incident on the surface of the mirror
- This rule states that the angle of incidence, θ_i, is equal to the angle of reflection, θ_r
- These angles are always measured from the normal, which is defined to be a line perpendicular to the surface of the plane



Reflection and Plane Mirrors (2)

- Parallel rays incident on a plane mirror will be reflected such that the reflected rays are also parallel, because every normal to the surface is also parallel
- The law of reflection is given by

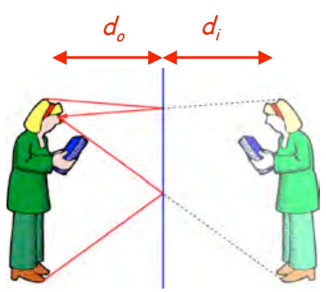
$$\theta_r = \theta_i$$



- Images can be formed by light reflected from plane mirrors
- For example, when you stands in front of a mirror, you see your image in the mirror
- This image appears to be behind the mirror
- This type of image is referred to as a virtual image because it cannot be projected on a screen

Mirror Image

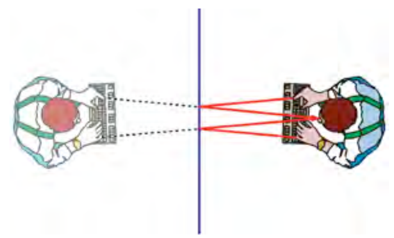
- Images formed by plane mirrors appear to be reversed because the light rays incident on the surface of the mirror are reflected back on the other side of the normal
- A mirror image looks correct vertically
- The image seen by the person can be constructed with two light rays as shown
  - of course light rays are coming from every visible point of the person
- The image is
  - upright (meaning not "upside-down")
  - virtual (implying that the image is formed behind the surface of the mirror



- The distance the person is standing from the mirror is called the object distance, d_o, and the distance the image appears to be behind the mirror is called the image distance, d_i, and for a plane mirror, $d_o = d_i$

Mirror Image (2)

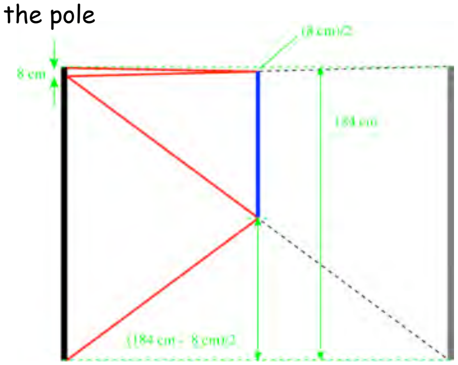
- Now let's discuss the left-right question for the mirror image
- Again we construct the virtual image with two rays
  - all rays behave the same way



- One can see that the real live person has his watch on his left hand and he sees that his virtual self has his watch on his right hand
- Thus when one looks in a mirror one sees an image that is upright but flipped left and right forming a "mirror image"

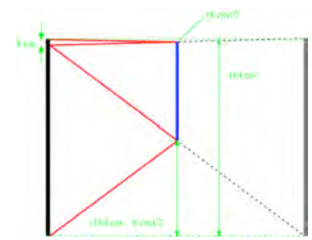
Example: Full-length Mirror

- *Question:*
- A 184 cm (6 ft 1/2 inch) tall person wants to buy a mirror so that he can see himself full length. His ey⊠s are 8 cm from the top of his head
- What is the minimum height of the mirror?
- Let us abstract the person as a 184 cm tall pole with eyes 8 cm from the top of the pole



Example: Full-length Mirror

- The distance from the floor to the bottom of the mirror is
  - (184 cm – 8cm)/2 = 88 cm
  - the angle of incidence equals the angle of reflection so the point on the mirror where the person can see the bottom of his feet will be half way between his eyes and the bottom of his feet
- Similarly the difference between the top of the mirror and 184 cm is
  - (8 cm)/2 = 4 cm
- So the minimum length of the mirror is
  - 184 cm – 88 cm – 4 cm = 92 cm
- The required length of the "Full-length mirror" is just half the height of the person wanting to see himself full-length

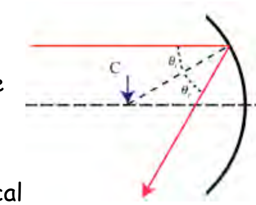


Curved Mirrors

- When light is reflected from the surface of a curved mirror, the light rays follow the law of reflection at each point on the surface
- However, unlike the plane mirror, the surface of a curved mirror is not flat
- Thus light rays that are parallel before they strike the mirror are reflected in different directions depending on the part of the mirror that they strike
- Depending of the shape of the mirror, the light rays can be focused or made to diverge
- Suppose we have a spherical mirror where the reflecting surface is on the inside of the sphere
- Thus we have a concave reflecting surface and the reflected rays will converge

Concave Spherical Mirrors

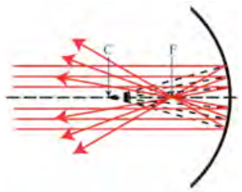
- We can abstract this sphere to a two dimensional semicircle
- The optical axis of the mirror is a line through the center of the sphere, represented in this drawing by a horizontal dashed line
- Imagine that a horizontal light ray above the optical axis is incident on the surface of the mirror
- At the point the light ray strikes the mirror, the law of reflection applies
  - $\theta_i = \theta_r$
- The normal to the surface is a radius line that points to the center of the sphere marked as *C*

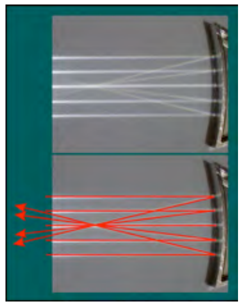


Concave Spherical Mirror (2)

- Now let us suppose that we have many horizontal light rays incident on this spherical mirror as shown
- Each light ray obeys the law of reflection at each point
- All the reflected rays cross the optical axis at the same point called the focal point F
- Point F is half way between point *C* and the surface of the mirror
- *C* is located at the center of the sphere so the distance of *C* from the surface of the mirror is just the radius of the mirror, *R*
- Thus the focal length, *f*, of a spherical mirror is

$$f = \frac{R}{2}$$





Images with Concave Mirror

- Now let us talk about forming actual images with a concave mirror
- For this explanation we choose the case where an object with height h_o is placed a distance d_o from the mirror where $d_o > f$
- As is traditional, we will represent the object with an arrow, which tells us the height and direction of the object
- We orient the object so that the tail of the arrow is on the optical axis
- We use three light rays to determine where the image is formed
  - The first light ray emanates from the bottom of the arrow along the optical axis
    - This ray tells us that the bottom of the image will be located on the optical axis
  - The second light ray starts from the top of the arrow parallel to the optical axis and is reflected through the focal point of the mirror
  - The third ray begins with the top of the arrow, passes through the center of the sphere, and is reflected back on itself
  - The last two rays intersect as the point where the image of the top of the object will be located on the optical axis

Forming an Image with a Concave Mirror

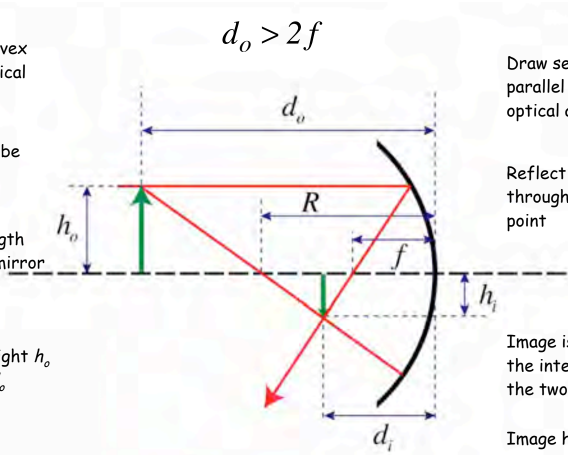
Start with convex mirror and optical axis

Add object to be imaged

Draw focal length and radius of mirror

Object has height h_o and distance d_o

Draw first ray through center of circle, reflecting on itself



Draw second ray parallel to the optical axis

Reflect the ray through the focal point

Image is formed at the intersection of the two rays

Image has height h_i

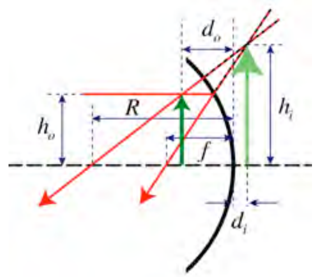
Image has distance d_i

Image with Concave Mirror

- The reconstruction of this special case produces a real image
  - defined by the fact that the image is on the same side of the mirror as the object
  - not behind the mirror
- The image has a height h_i located a distance d_i from the surface of the mirror on the same side of the mirror as the object with height h_o and distance d_o from the mirror
- The image is inverted and reduced in size relative to the object that produced the image
- The image is called real when you are able to place a screen at the image location and obtain a sharp projection of the image on the screen at that poin
- For a virtual image, it is impossible to place a screen at the location of the image

Concave Mirror, $d_o < f$

- Now let's reconstruct another case for a convex mirror where $d_o < f$
- Again we place the object standing on the optical axis
- We again use three light rays
  - The first again establishes that the tail of the image lies on the optical axis
  - The second ray starts from the top of the object parallel to the optical axis and is reflected through the focal point
  - The third ray leaves the top of the object along a radius and reflected back on itself through the center of the sphere
- The reflected rays are clearly diverging
- To determine the location of the image, we must extrapolate the reflected rays to the other side of the mirror
- These two rays intersect a distance d_i from the surface of the mirror producing an image with height h_i



Concave Mirror, $d_o < f$ (2)

- In this case, we have the image formed on the opposite side of the mirror from the object, a virtual image
- To an observer, the image appears to be behind the mirror
- The image is upright and larger than the object. These results for are very different than those for $d_o > f$
- Clearly to treat all possible cases for convex mirrors, we must define some conventions for distances and heights
- We define all distances on the same side of the mirror as the object to be positive and all distances on the far side of the mirror from the object to be negative
- Thus f and d_o are positive for concave mirrors
- In the case of real images, d_i is positive
- In cases where the image is virtual, we define d_i to be negative
- If the image is upright, then h_i is positive and if the image is inverted, h_i is negative

Mirror Equation

- Using these sign conventions we can express the mirror equation in terms of the object distance, d_o, and the image distance, d_i, and the focal length f of the mirror

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

- The magnification m of the mirror is defined to be

$$m = -\frac{d_i}{d_o} = -\frac{h_i}{h_o}$$

- We can summarize the image characteristics of concave mirrors

Case	Type	Direction	Magnification
$d_o < f$	Virtual	Upright	Enlarged
$d = f$	Real	Upright	Image at infinity
$f < d_o < 2f$	Real	Inverted	Enlarged
$d_o = 2f$	Real	Inverted	Same size
$d_o > 2f$	Real	Inverted	Reduced