# Physics for Scientists \& Engineers ? 

Spring Semester 2005
Lecture 37

## March 26, 2005

## Review (2)

- We can express the mirror equation in terms of the object distance, $d_{0}$, and the image distance, $d_{i}$, and the focal length $f$ of the mirror

$$
\frac{1}{d_{o}}+\frac{1}{d_{i}}=\frac{1}{f}
$$

- The magnification $m$ of the mirror is

$$
m=-\frac{d_{i}}{d_{o}}=-\frac{h_{i}}{h_{o}}
$$



- We can summarize the image characteristics of concave mirrors

| Case | Type | Direction | Magnification |
| :--- | :--- | :--- | :--- |
| $d_{o}<f$ | Virtual | Upright | Enlarged |
| $d=f$ | Real | Upright | Image at infinity |
| $f<d_{o}<2 f$ | Real | Inverted | Enlarged |
| $d_{o}=2 f$ | Real | Inverted | Same size |
| $d_{o}>2 f$ | Real | Inverted | Reduced |

## Reflection and Plane Mirrors (2)

- The law of reflection is given by

$$
\theta_{r}=\theta_{i}
$$



- the focal length, $f$, of a spherical mirror with radius $R$ is

$$
f=\frac{R}{2}
$$



$$
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$$

## Convex Spherical Mirrors

- Suppose we have a spherical mirror where the reflecting surface is on the outside of the sphere
- Thus we have a convex reflecting surface and the reflected rays will diverge
- The optical axis of the mirror is a line through the center of the sphere, represented in this drawing by a horizontal dashed line
- Imagine that a horizontal light ray above the
 optical axis is incident on the surface of the mirror
- At the point the light ray strikes the mirror, the law of reflection applies, $\theta_{i}=\theta_{r}$
- The normal to the surface is a radius line that points to the center of the sphere marked as $C$


## Convex Spherical Mirrors (2)

## Convex Spherical Mirrors

- In contrast to the concave mirror, the normal points away from the center of the sphere
- When we extrapolate the normal through the surface of the sphere, it intersects the optical axis of the sphere at the center of the sphere marked $C$
- When we observe the reflected ray, it seems to be coming from inside the sphere



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## Images from Convex Mirrors

- Now let us discuss images formed by convex mirrors starting with the case of the $d_{0}>f$
- Again we use three rays
- The first ray establishes that the tail of the arrow lies on the optical axis
- The second ray starts from the top of the object traveling parallel to the optical axis and is reflected from the surface of the mirror such that its extrapolation crosses
 the optical axis a distance from the surface of the mirror equal to the focal length of the mirror
- The third ray begins at the top of the object and is directed so that its extrapolation would intersect the center of the sphere
- This ray is reflected back on itself
- Now let us suppose that we have many horizontal light rays incident on this spherical mirror as shown
- Each light ray obeys the law of reflection at each point
- You can see that the rays diverge and do not seem to form any kind of image
- However, if we extrapolate the reflected rays through the surface of the mirror they all intersect the optical axis at one point
- This point called the focal point of this convex spherical mirror



## Images from Convex Mirrors (2)

- We can see that we form an upright, reduced image on the side of the mirror opposite the object
- This image is virtual because it cannot be projected
- These characteristics are valid for all cases for a convex mirror
- In the case of a convex mirror, we define the focal length $f$ to be negative because the focal point of the mirror is on the opposite side of the object
- We always take the object distance $d_{0}$ to be positive
- We recall the mirror equation

$$
\frac{1}{d_{o}}+\frac{1}{d_{i}}=\frac{1}{f}
$$

## Images from Convex Mirrors (3)

- We can rearrange the mirror equation to get

$$
d_{i}=\frac{d_{o} f}{d_{o}-f}
$$

- If $d_{o}$ is always positive and $f$ is always negative, we can see that $d_{i}$ will always be negative
- Applying the equation for the magnification we find that is always positive
- Looking at our diagram of the ray construction for the convex mirror will also convince you that the image will always be reduced in size
- Thus, for a convex mirror, we always will obtain a virtual, upright, and reduced image
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## Spherical Aberration

- The equations we have derived for spherical mirrors apply only to light rays that are close to the optical axis
- If the light rays are far from the optical axis, they will not be focused through the focal point of the mirror
- Thus we will see a distorted image
- In the drawing several light rays are incident on a spherical concave mirror
- You can see that the rays far from the optical axis are reflected such that they cross the optical axis closer to the mirror
 that rays that are incident closer to the optical axis
- As the rays approach the optical axis, they are reflected through points closer and closer to the focal point



## Parabolic Mirror

- Parabolic mirrors have a surface that reflects light to the focal point from anywhere on the mirror
- Thus the full size of the mirror can be used to collect light and form images
- In the drawing to the right, horizontal light rays are incident on a parabolic mirror
- All rays are reflected through the focal point of the mirror
- Parabolic mirrors are more difficult to produce than spherical mirrors and are accordingly more expensive

- Most large telescopes use parabolic mirrors


## Refraction

Refraction (2)

- Light travels at different speeds in different optically transparent materials
- The ratio of the speed of light in a material divided by the speed of light in vacuum is called the index of refraction of the material
- The index of refraction, $n$, is given by

$$
n=\frac{c}{v}
$$

- where $c$ is the speed of light in a vacuum and $v$ is the speed of light in the medium
- Thus the index of refraction of a material is always greater than or equal to one, and by definition the

| Material | Index of Refraction |
| :--- | :--- |
| Air | 1.00029 |
| Water | 1.333 |
| Ice | 1.310 |
| Ethyl alcohol | 1.362 |
| Quartz glass | 1.459 |
| Linseed oil | 1.486 |
| Typical glass | 1.5 |
| Typical oil | 1.5 |
| Diamond | 2.417 | index of refraction of vacuum is one

- When light crosses the boundary between two transparent materials, it is refracted
- Refraction means that the light rays do not travel in a straight line across the boundary
- When light crosses a boundary from a medium with a lower index of refraction, $n_{1}$, to a medium with a higher index of refraction, $n_{2}$, the light rays change their direction toward the normal
- Changing direction toward the normal means that $\theta_{2}<\theta_{1}$



## Refraction (3)

- When light crosses a boundary from a medium with a higher index of refraction, $n_{1}$, to a medium with a lower index of refraction, $n_{2}$, the light rays are bent away from the normal
- The light rays are refracted away from the normal
- Changing direction toward the normal means that $\theta_{2}>\theta_{1}$



## Law of Refraction

- The Law of Refraction can be expressed as

$$
n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2}
$$

- where $n_{1}$ and $\theta_{1}$ are the index of refraction and angle from the normal in the first medium and $n_{1}$ and $\theta_{1}$ are the index of refraction and angle from the normal in the second medium
- The law of refraction is also called Snell's Law
- We will always assume the index of refraction of air is one
- The situation of light incident on various media from air is common so we will write the formulas for refraction of light incident on a surface with index of refraction

$$
n_{\text {medium }}=\frac{\sin \theta_{\text {air }}}{\sin \theta_{\text {medium }}}
$$

- Note that $\theta_{\text {air }}$ is always greater than $\theta_{\text {medium }}$


## Total Internal Reflection

- Now let us consider the situation where light is travels across the boundary between two media where the first medium has an index of refraction less than the second medium, $n_{1}<n_{2}$
- In this case the light is bent away from the normal
- As one increases the angle of incidence from medium $1, \theta_{1}$, one can see that the angle of the transmitted light, $\theta_{2}$, can approach $90^{\circ}$
- When $\theta_{2}$ reaches $90^{\circ}$, total internal reflection takes place instead of refraction


## Optical Fibers

- If the second medium is air, then we can take $n_{2}$ equal to 1 and we obtain an expression for the critical angle for total internal reflection for light leaving a medium with index of refraction $n$ and entering air

$$
\sin \theta_{c}=\frac{1}{n}
$$

- An important application of total internal reflection is the optical fiber
- Light injected into an optical fiber so that the reflection angle at the surface of the fiber is greater than the critical angle for total internal reflection will be transported the length of the fiber

Total Internal Reflection (2)

- The critical angle, $\theta_{c}$, at which total internal reflection takes place is given by

$$
\frac{n_{2}}{n_{1}}=\frac{\sin \theta_{1}}{\sin \theta_{2}}=\frac{\sin \theta_{c}}{\sin 90^{\circ}}
$$

- Which we can rewrite as

$$
\sin \theta_{c}=\frac{n_{2}}{n_{1}} \quad\left(n_{2} \leq n_{1}\right)
$$

- One can see from this equation that total internal reflection can only occur for light traveling from a medium with a higher index of refraction to one with a lower index of refraction because the sine of an angle cannot be greater than 1
- At angles less than $\theta_{c}$, some light is reflected and some is transmitted
- At angles greater than $\theta_{c}$, all the light is reflected and none is transmitted


## Optical Fibers (2)

- An important application of total internal reflection is the optical fiber
- Optical fibers can be used to transport light from a source to a destination
- In this picture, a bundle of optical fibers is shown with the end of the fibers open to the camera
- The other end of the fiber is optically
 coupled to a light source
- Note that optical fibers can transport light in directions other that a straight line as long as the radius of curvature of the fiber is not small enough to allow the light traveling in the optical fiber to exceed when reflecting off the surface of the fiber


## Optical Fibers (3)

- One type optical fiber used for digital communications consists of a glass core surrounded by cladding composed of glass with lower index refraction than the core
- The cladding is then coated to prevent damage

- For a typical commercial optical fiber, the core material is $\mathrm{SiO}_{2}$ doped with Ge to increase its index of refraction
- The typical commercial fiber can transmit light 500 m with small losses
- The light is generated with long wavelength light emitting diodes as digital pulses


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