Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 42

Review

- If light waves are traveling from some point, then the phase difference Δx can be related to the path difference between the two waves
- The criterion for constructive interference is given by a path difference Δx given by

$$\Delta x = m\lambda \quad (m = 0, m = \pm 1, m = \pm 2, ...)$$

- Destructive interference will take place if the path difference Δx is a half wavelength plus an integer times the wavelength

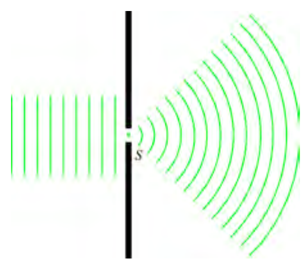
$$\Delta x = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, m = \pm 1, m = \pm 2, ...)$$

Double Slit Interference

- Our first example of the interference of light is Young's double slit experiment
- For this situation we assume that we have coherent light
  - light with the same wavelength and phase
- This light is incident on a pair of slits
- Each slit is smaller than the wavelength of light
- The slits are separated by a distance *d*
- For each slit we will use a Huygens' construction and assume all the light observed passing through each slit is due to wavelets emitted from a single point at the center of that slit

Double Slit Interference (2)

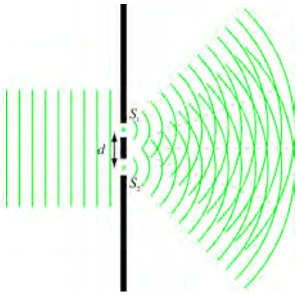
- Below we see that spherical wavelets emitted from a point in the center of the slit



- We assume that the slit is much smaller than the wavelength of light so that we can represent the source of the wavelets with one point

Double Slit Interference (3)

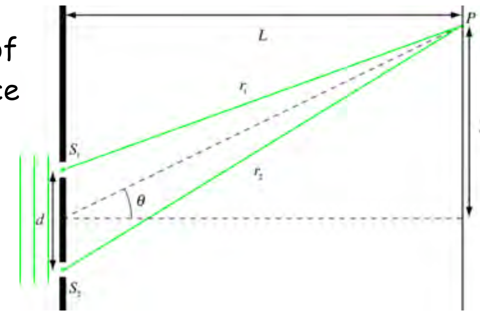
- Now let's look at two slits like the one on the previous page



- We have coherent light incident from the left and a source of spherical wavelets at the center of each slit
- We see that the gray dashed lines represent lines along which there is constructive interference

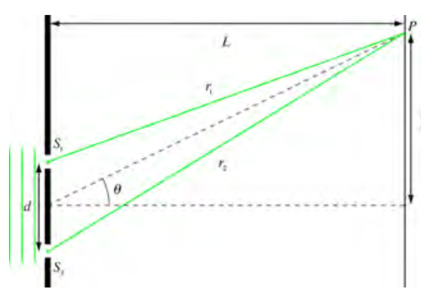
Double Slit Interference (4)

- If we place a screen to the right of the slits we will observe an alternating pattern of bright lines and dark lines corresponding to constructive and destructive interference between the light waves emitted from the two slits
- To quantify these lines of constructive interference we expand and simplify the plot on the previous page



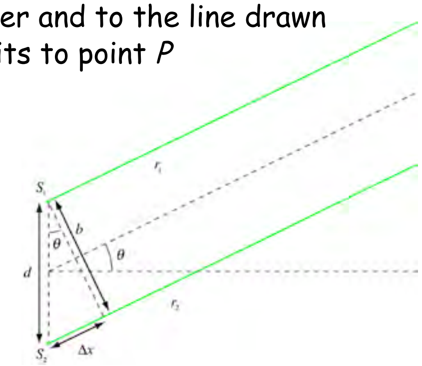
Double Slit Interference (5)

- The two lines r_1 and r_2 represent the distance from the center of slit S_1 and slit S_2 respectively to a point P on a screen that is placed a distance L away from the slits
- A line drawn from a point midway between the two slits to the same point on the screen makes an angle θ with respect to a line drawn from the slits perpendicular to the screen
- The point P on the screen is a distance y above the centerline



Double Slit Interference (6)

- To further quantify the two slit geometry we expand and simplify the figure on the previous slide
- In this figure we assume that we have placed the screen a large distance L away from the slits such that the lines r_1 and r_2 are parallel to each other and to the line drawn from the center of the two slits to point P
- We draw a line from S_1 perpendicular to r_1 and r_2 making a triangle with sides d, b, and Δx
- The quantity Δx represents the path length difference between r_1 and r_2



Double Slit Interference (7)

- This path length difference will produce different phases for light originating from the two slits and illuminating the screen at point *P*
- The path length difference can be expressed in terms of the distance between the slits and the angle at which the light is observed

$$\sin\theta = \frac{\Delta x}{d} \quad \text{or} \quad \Delta x = d\sin\theta$$

- For constructive interference this path length difference must be a multiple of the wavelength of the incident light

$$\Delta x = d\sin\theta = m\lambda \quad (m = 0, m = \pm1, m = \pm2, ...)$$

Double Slit Interference (8)

- For destructive interference the path length difference must be an integer plus a half times the wavelength

$$\Delta x = d\sin\theta = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, m = \pm1, m = \pm2, ...)$$

- A bright fringe on the screen signals constructive interference
- A dark fringe on the screen signals destructive interference
- Note that for constructive interference and $m = 0$, we obtain $\theta = 0$, which means that $\Delta x = 0$ and we have a bright fringe at zero degrees
- This bright fringe is called the central maximum

Order of Interference Fringes

- The integer *m* is called the order of the fringe
- The order has a different meaning for bright fringes and for dark fringes
- For constructive interference
  - $m = 1$ would give us the angle of the first order bright fringe
  - $m = 2$ would give us the second order fringe, etc.
- For destructive interference
  - $m = 0$ would give us the angle of the first order dark fringe
  - $m = 1$ would give us the second order fringe, etc.
- For both bright and dark fringes, the first order fringe is the one closest to the central maximum

Fringes on a Distance Screen

- If the screen is placed a sufficiently large distance from the slits, the angle θ will be small and we can make the approximation

$$\sin\theta \approx \tan\theta = y / L$$

- We can express constructive interference as

$$d\sin\theta = d\frac{y}{L} = m\lambda \quad (m = 0, m = \pm1, m = \pm2, ...)$$

- Rearranging this equation gives us the distance of the bright fringes from the central maximum along the screen

$$y = \frac{m\lambda L}{d} \quad (m = 0, m = \pm1, m = \pm2, ...)$$

Fringes on a Distance Screen (2)

- Similarly we can express the distance of the dark fringes from the central maximum along the screen as

$$y = \frac{\left(m + \frac{1}{2}\right)\lambda L}{d} \qquad (m = 0, m = \pm 1, m = \pm 2, \ldots)$$

- These formulas allow us to locate the positions of the bright and dark fringes
- However, we can also calculate the intensity of the light at any point on the screen

Double Slit Intensity on a Distant Screen

- We start our calculation of the intensity of light from a double slit by assuming that the light emitted at each slit is in phase
- The electric field of the light waves can be described by

$$E = E_{\max} \sin \omega t$$

- where E_{max} is the amplitude of the wave and ω is the angular frequency
- When the light waves arrive at the screen from the two slits, they have traveled difference distances, and so can have difference phases that depend on the angle of observation

Double Slit Intensity on a Distance Screen (2)

- Let's express the electric field of the light arriving at a given point on the screen from S_1 as

$$E_1 = E_{\max} \sin(\omega t)$$

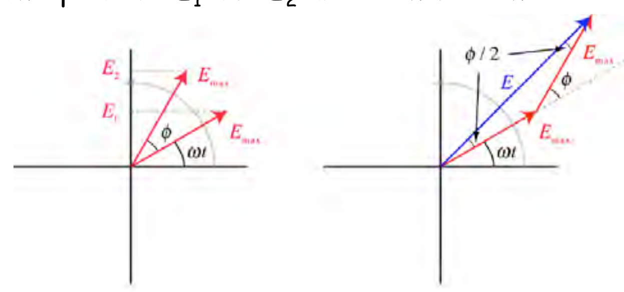
- and the electric field of the light arriving at the same point from S_2 as

$$E_2 = E_{\max} \sin(\omega t + \phi)$$

- where ϕ is the phase constant of E_2 with respect to E_1

Double Slit Intensity on a Distant Screen (3)

- The two phasors E_1 and E_2 are shown below



a) b)

- The magnitude of the sum of the two phasors is

$$E = 2E_{\max} \cos(\phi / 2)$$

Double Slit Intensity on a Distant Screen (4)

- We know that the intensity of an electromagnetic wave is proportional to the square of the electric field so we can write

$$\frac{I}{I_{max}} = \frac{E^2}{E_{max}^2}$$

- Which gives us the relative intensity of the summed wave as a function of the phase difference between the two light waves

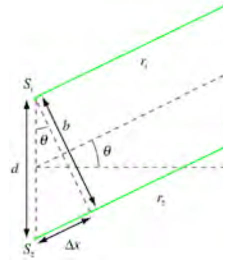
$$I = 4I_{max}\cos^2(\phi/2)$$

- Now we need to relate the phase difference to the path length difference

Double Slit Intensity on a Distant Screen (5)

- Looking at our previous figure we can see that the path length difference Δx will cause a phase shift given by

$$\phi = \frac{\Delta x}{\lambda}(2\pi)$$



- We remember that

$$\Delta x = d\sin\theta$$

- So we can write the phase constant as

$$\phi = \frac{2\pi d}{\lambda}\sin\theta$$

Double Slit Intensity on a Distant Screen (6)

- Thus we can write an equation for the intensity of the light produced by the interference from two slits as

$$I = 4I_{max}\cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)$$

- Because we are assuming that we are looking at the intensity pattern in a distant screen we can make the approximation

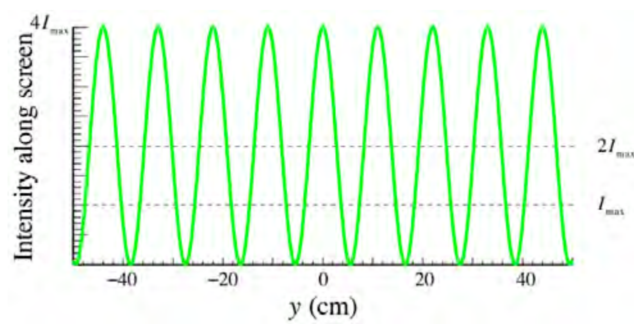
$$\sin\theta \approx \tan\theta = y/L$$

- And get an expression for the intensity

$$I = 4I_{max}\cos^2\left(\frac{\pi d y}{\lambda L}\right)$$

Double Slit Intensity on a Distant Screen (7)

- The graph below represents the intensity a distance L away from the slits, where the slits are separated by $d = 10^{-5}$ m and the wavelength of the incident light is $\lambda = 550$ nm



Double Slit Intensity on a Distant Screen (8)

- We can see that the intensity varies from 0 to $4I_{max}$
- Covering one slit, we get a constant intensity of I_{max}
- If we illuminate both slits with light that has random phases, we would observe a constant intensity of $2I_{max}$
- Only when we illuminate both slits with coherent light do we observe the oscillatory pattern characteristic of two-slit interference