Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 43

Review

- Coherent light with wavelength λ incident on two narrow slits separated by a distance d produces an interference pattern on a screen located a large distance L from the slits
- The position of the bright fringes from the center line is given by

$$y = \frac{m\lambda L}{d} \quad (m = 0, m = \pm 1, m = \pm 2, ...)$$

- The position of the dark fringes from the center line is given by

$$y = \frac{\left(m + \frac{1}{2}\right)\lambda L}{d} \quad (m = 0, m = \pm 1, m = \pm 2, ...)$$

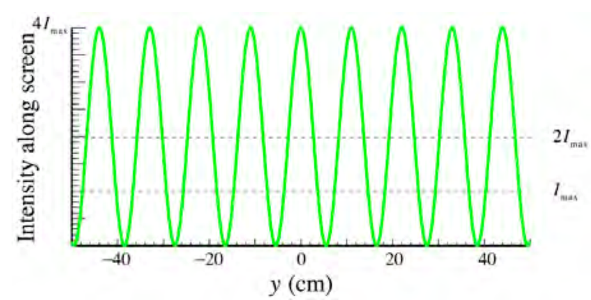
Review (2)

- The integer *m* is called the order of the fringe
- The order has a different meaning for bright fringes and for dark fringes
- For constructive interference
  - *m* = 1 would give us the angle of the first order bright fringe
  - *m* = 2 would give us the second order fringe, etc.
- For destructive interference
  - *m* = 0 would give us the angle of the first order dark fringe
  - *m* = 1 would give us the second order fringe, etc.
- For both bright and dark fringes, the first order fringe is the one closest to the central maximum

Review (3)

- Coherent light with wavelength λ incident on two narrow slits separated by a distance d produces an interference pattern on a screen located a large distance L from the slits with intensity given by

$$I = 4I_{max}\cos^2\left(\frac{\pi d y}{\lambda L}\right)$$

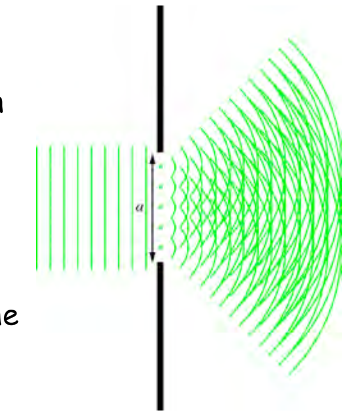


Diffraction

- Any wave passing through an opening that is about the same size as the wavelength of the wave will experience diffraction
- The same applies to light waves
- Diffraction means that the wave will spread out on the other side of the opening rather than having the opening cast a sharp shadow
- In addition, if light passes through a narrow slit, it will produce an interference pattern called a diffraction pattern
- Light passing a sharp edge will also exhibit a diffraction pattern

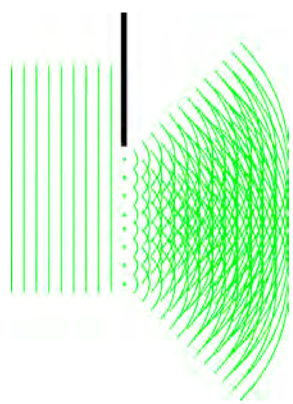
Diffraction (2)

- Huygens' principle can describe this spreading out and we can use a Huygens' construction to quantify the diffraction phenomenon
- For example, to the right we show coherent light incident on an opening with dimension comparable to the wavelength of the light
- Rather than casting a sharp shadow, light spreads out on the other side of the opening



Diffraction (3)

- We can describe this spreading out by using a Huygens' construction and assuming that spherical wavelets are emitted at several points inside the opening
- The resulting light waves on the right side of the opening can undergo interference and produce a characteristic diffraction pattern
- Light waves can also encounter barriers such as the one shown



Diffraction (4)

- In this case, the light far from the edge of the barrier continues to travel in a straight line
- The light near the edge of the barrier seems to bend around the barrier is described by the sources of wavelets near the edge
- Diffraction phenomena cannot be described by geometric optics. Often the resolution of an optical instrument can be limited by diffraction effects rather than geometric effects

Single Slit Diffraction

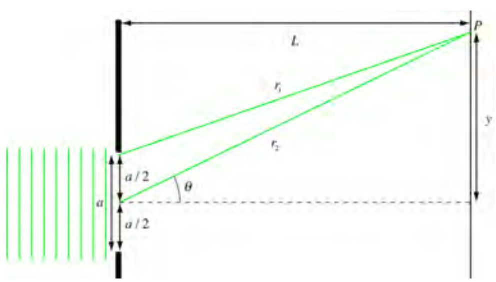
- We start our quantitative description of diffraction by studying the diffraction of light through a single slit of width *a* that is comparable to the wavelength of light that is passing through the slit
- We will approach the calculation using a Huygens' construction
- We assume that the light passing through the single slit is described by spherical wavelets emitted from a distribution of points located in the slit
- The light emitted from these points will superimpose and interfere based on the path length difference for each wavelet at each position

Single Slit Diffraction (2)

- At a distant screen we will observe an intensity pattern characteristic of diffraction
- This intensity pattern will consist of bright and dark fringes
- For the case of two-slit interference, we were able to work out the equations for the bright fringes based on constructive interference
- For diffraction we will analyze the dark fringes as destructive interference because constructive interference is beyond the scope of this book

Single Slit Diffraction (3)

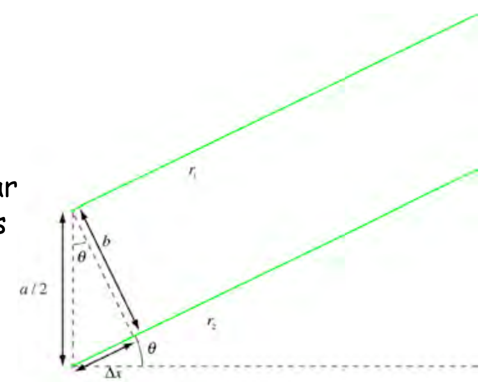
- To study the interference let's expand and simplify our previous figure for single slit diffraction
- We assume coherent light with wavelength λ incident on a slit with width *a* that produces an interference pattern on a screen a distance *L* away



- We employ a simplifying method of analyzing pairs of light waves emitted from points in the slit

Single Slit Diffraction (4)

- We start with light emitted from the top edge of the slit and from the center of the slit as shown
- To analyze the path difference we show an expanded version of our figure to the right
- Here we assume that the point P on the screen is far enough away that the rays r_1 and r_2 are parallel and make an angle θ with the central axis



Single Slit Diffraction (5)

- Therefore the path length difference for these two rays is

$$\sin\theta = \frac{\Delta x}{a/2} \quad \text{or} \quad \Delta x = \frac{a\sin\theta}{2}$$

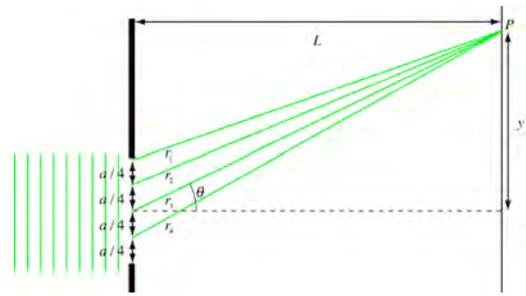
- The criterion for the first dark fringe is

$$\Delta x = \frac{a\sin\theta}{2} = \frac{\lambda}{2}$$

- Although we chose one ray originating from the top edge of the slit and one from the middle of the slit to locate the first dark fringe, we could have used any two rays that originated $a/2$ apart inside the slit

Single Slit Diffraction (6)

- Now let's consider four rays instead of two



- Here we choose a ray from the top edge of the slit and three more rays originating from points spaced $a/4$ apart

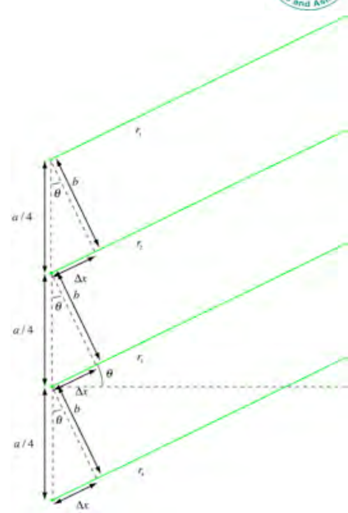
Single Slit Diffraction (7)

- We can expand the previous drawing to represent the case of the screen being far away as shown to the right
- The path length difference between the pairs of rays (r_1,r_2), (r_2,r_3), (r_3,r_4) is given by

$$\sin\theta = \frac{\Delta x}{a/4} \quad \text{or} \quad \Delta x = \frac{a\sin\theta}{4}$$

- The dark fringe is given by

$$\frac{a\sin\theta}{4} = \frac{\lambda}{2} \quad \text{or} \quad a\sin\theta = 2\lambda$$



Single Slit Diffraction (8)

- This equation describes the second dark fringe
- At this point we could take six pairs and eight pairs and describe the third and fourth dark fringes, etc.
- The result is that the dark fringes from single slit diffraction can be described by

$$a\sin\theta = m\lambda \quad (m = 1,2,3,\ldots)$$

- If the screen is placed a sufficiently large distance from the slits, the angle θ will be small and we can make the approximation

$$\sin\theta \approx \tan\theta = y/L$$

Single Slit Diffraction (9)

- We can express the position of the dark fringes as

$$\frac{ay}{L} = m\lambda \quad \text{or} \quad y = \frac{m\lambda L}{a} \qquad (m = 1, 2, 3, \ldots)$$

- The intensity of light passing through a single slit can be calculated but we will not present the derivation here
- The intensity I relative to I_{max} that we would get if there were no slit is

$$I = I_{\max}\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{where } \alpha = \frac{\pi a}{\lambda}\sin\theta$$

- We can see that this expression for the intensity will be zero for sin(α) = 0, which means $\alpha = m\pi$ for m = 1, 2, 3, …
- Note that α cannot be zero or sin(α)/ α would diverge

Single Slit Diffraction (10)

- We can write

$$m\pi = \frac{\pi a}{\lambda}\sin\theta \quad \text{or} \quad a\sin\theta = m\lambda \qquad (m = 1, 2, 3, \ldots)$$

- which gives the same result for the diffraction minima as our Huygen's construction
- If the screen is placed a sufficiently large distance from the slits we can make the small angle approximation and write

$$\alpha = \frac{\pi a y}{\lambda L}$$

Single Slit Diffraction (11)

- If we take L = 2.0 m, a = 5.0·10^{-6} m, and λ = 550 nm, we get the following intensity distribution