Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 44

Review

- If coherent light with wavelength λ is incident on a single slit of width a, a single-slit diffraction pattern will be produced
- The angles of the minima are given by

$$a\sin\theta = m\lambda \quad (m = 1, 2, 3, ...)$$

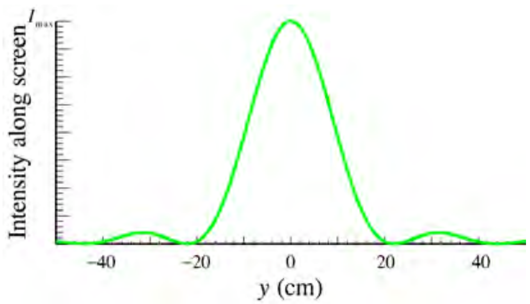
  - m is the order of each minimum
- If we project this diffraction pattern on a screen a large distance L away from the slit, the position on the screen, measured from the center line, is given by

$$y = \frac{m\lambda L}{a} \quad (m = 1, 2, 3, ...)$$

Review (2)

- The intensity of light passing through a single slit I relative to I_{max} that we would get if there were no slit is

$$I = I_{\max}\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{where } \alpha = \frac{\pi a}{\lambda}\sin\theta$$



Thin Film Interference

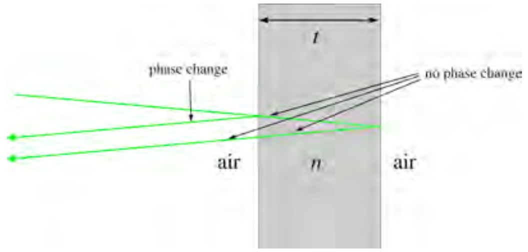
- Another way of producing interference phenomena is partial reflection of light from the front and back layers of thin films
- A thin film is an optically clear material with thickness on the order few wavelengths of light
- Examples of thin films include the walls of soap bubbles and thin layers of oil floating on water
- When the light reflected off the front surface constructively interferes with the light reflected off the back surface of the thin film, we see the color corresponding to the wavelength of light that is interfering constructively

Thin Film Interference (2)

- When light travels from an optical medium with an index of refraction n_1 into a second optical medium with index of refraction n_2, several things can happen
- The light can be transmitted through the boundary
  - In this case the phase of the light is not changed
- A second process that can occur is that the light can be reflected
  - In this case, the phase of the light can be changed depending on the index of refraction of the two optical media
    - If $n_1 < n_2$, the phase of the reflected wave will be changed by half a wavelength
    - If $n_1 > n_2$ then there will be no phase change

Thin Film Interference (3)

- Let's begin our analysis of thin films by studying a thin film with index of refraction n in air as shown below



- We will assume that light is incident perpendicular to the surface of the thin film
- An angle of incidence is shown for the light waves in the figure for clarity

Thin Film Interference (4)

- When the light wave reaches the boundary between air and the film, part of the wave is reflected and part of the wave is transmitted
- The reflected wave undergoes a phase shift of half a wavelength when it is reflected because $n_{air} < n$
- The light that is transmitted has no phase shift and continues to the back surface of the film
- At the back surface, again part of the wave is transmitted and part of the wave is reflected
- The transmitted light passes through the film completely
- The reflected light has no phase shift because $n > n_{air}$ and travels back to the front surface of the film

Thin Film Interference (5)

- At the front surface, some of the light is transmitted and some is reflected
- The reflected light will not be considered
- The transmitted light has no phase shift and emerges from the film and interferes with the light that was reflected when the light originally entered the film
- The transmitted light has traveled a longer distance than the originally reflected light and has a phase shift given by the path length difference that is twice the thickness t of the film

Thin Film Interference (4)

- The fact that the originally reflected light has undergone a phase shift and the transmitted light has not means that the criterion for constructive interference is given by

$$\Delta x = \left(m + \frac{1}{2}\right)\lambda = 2t \quad (m = 0, m = \pm 1, m = \pm 2, \ldots)$$

- The wavelength λ refers to the wavelength the light traveling in the thin film, which has index of refraction *n*
- The wavelength of the light traveling in air is related to the wavelength of the light traveling in the film by

$$\lambda = \frac{\lambda_{air}}{n}$$

Thin Film Interference (5)

- We can then write

$$\left(m + \frac{1}{2}\right)\frac{\lambda_{air}}{n} = 2t \quad (m = 0, m = \pm 1, m = \pm 2, \ldots)$$

- The minimum thickness t_{min} that will produce constructive interference corresponds to

$$t_{\min} = \frac{\lambda_{air}}{4n}$$

- Note that this result applies only to the case where of a material with index of refraction *n* and air on both sides, like a soap bubble

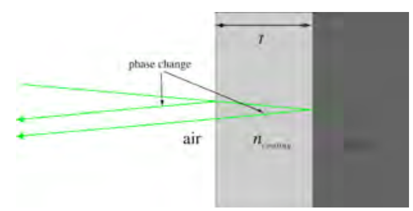
Example: Lens Coating

- Many high quality lenses are coated to prevent reflections
- This coating is designed to set up destructive interference for light that is reflected from the surface of the lens
- Assume that the coating is MgF_2, which has $n_{coating}$ = 1.38 and the lens is glass with n_{lens} = 1.51

- Question:
- What is the minimum thickness of the coating that will produce destructive interference for light with a wavelength of 550 nm?

Example: Lens Coating (2)

- We assume that the light is incident perpendicularly on the surface of the coated lens as shown below



- Light reflected at the surface of the coating will undergo a phase change of half a wavelength because $n_{air} < n_{coating}$
- The light transmitted through the coating has no phase change

Example: Lens Coating (3)

- Light reflected at the boundary between the coating and the lens will undergo a phase change of half a wavelength because $n_{coating} < n_{lens}$
- This reflected light will travel back through the coating and exit with no phase change
- Thus both the light reflected from the coating and from the lens will have suffered a phase change of half a wavelength
- Thus the criterion for destructive interference is

$$\left(m+\frac{1}{2}\right)\frac{\lambda_{air}}{n_{coating}} = 2t \quad (m=0, m=\pm 1, m=\pm 2, \ldots)$$

Example: Lens Coating (4)

- Thus the minimum thickness for the lens coating to provide destructive interference corresponds to *m* = 0

$$t_{\min} = \frac{\lambda_{air}}{4n} = \frac{550 \cdot 10^{-9} \text{ m}}{4 \cdot 1.38} = 9.96 \cdot 10^{-8} \text{ m} = 99.6 \text{ nm}$$

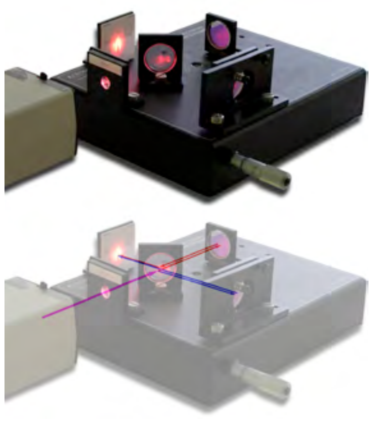
- Note that this formula is the same as the one we found for constructive interference in a film with air on both sides
- To analyze thin film interference, one must always take into account the phase changes at the boundary
  - *An even number of phase changes is the same as no phase changes*
  - *An odd number of phase changes is the same as one phase change*

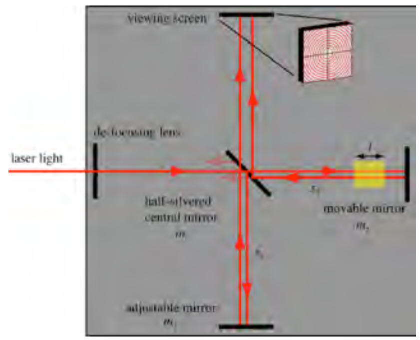
Interferometer

- An interferometer is a device designed to measure lengths or changes in length using interference of light
- An interferometer can measure lengths or changes in lengths to a fraction of the wavelength of light using interference fringes
- Here we will describe an interferometer similar to one constructed by Albert Michelson and Edward Morley at the Case Institute in Cleveland, Ohio in 1887
- The interferometer we will describe is simpler than the one used by Michelson and Morley but is based on the same physical principles

Interferometer (2)

- A photograph and drawing of a commercial interferometer used in physics labs are shown below





Interferometer (3)

- The interferometer consists of a light source in the form of a laser emitting coherent light with wavelength λ = 632.8 nm
- The light passes through a de-focusing lens to spread out the normally very narrowly focused laser beam
- The light then passes through a half-silvered mirror m_1
- Part of the light is reflected toward the adjustable mirror m_3 and part of the light is transmitted to the movable mirror m_2
- The distance between m_1 and m_2 is x_2
- The distance between m_1 and m_3 is x_3
- The transmitted light is totally reflected from m_2 back toward m_1
- The reflected light is totally reflected from m_3 back toward m_1
- Part of the light from m_2 is reflected by toward the viewing screen and part of the light is transmitted and not considered
- Part of the light from m_3 is transmitted through m_1 and part is reflected and not considered

Interferometer (4)

- The light from mirrors m_2 and m_3 will interfere based on the path length difference between the two ways of arriving at the viewing screen
- Both paths undergo two reflections, each resulting in a phase change of half a wavelength so the condition for constructive interference is

$$\Delta x = m\lambda \quad (m = 0, m = \pm 1, m = \pm 2, ...)$$

- The two different paths will have a path length difference of

$$\Delta x = 2x_2 - 2x_3 = 2(x_2 - x_3)$$

- The viewing screen will display concentric circles or vertical fringes corresponding to constructive and destructive interference depending on the type of de-focusing lens

Interferometer (4)

- If the movable mirror m_2 is moved a distance of $\lambda/2$, the fringes will shift by one fringe
- Thus this type of interferometer can be used to measure changes in distance on the order of a fraction of a wavelength of light, depending on how well one can measure the shift of the interference fringes
- Another type of measurement can be made with this interferometer by placing a material with index of refraction n and thickness t in the path of the light traveling to the movable mirror m_2

Interferometer (4)

- The path length difference in terms of the number of wavelengths will change because the wavelength of light in the material λ_n in this material is related to the wavelength of light in air λ by

$$\lambda_n = \frac{\lambda}{n}$$

- The number of wavelengths in the material is

$$N_{material} = \frac{2t}{\lambda_n} = \frac{2tn}{\lambda}$$

- The number of wavelength that would have been there if the light traveled through only air is

$$N_{air} = \frac{2t}{\lambda}$$

Interferometer (5)

- The difference in the number of wavelengths is

$$N_{material} - N_{air} = \frac{2tn}{\lambda} - \frac{2t}{\lambda} = \frac{2t}{\lambda}(n-1)$$

- When the material is placed between m_1 and m_2, an observer will see a shift of one fringe for every wavelength shift in the path length difference
- Thus we can substitute the number of fringes shifted for in $N_{material}$ - N_{air} and obtain the thickness of the material knowing the index of refraction
- One could also insert material with a well-known thickness and determine the index of refraction