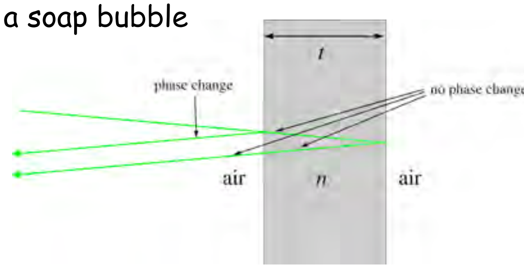
Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 45

Review

- Imagine light incident on a thin, optically clear medium in air such as a soap bubble



- Transmitted light has no phase change
- For reflected light
  - If $n_1 < n_2$, the phase of the reflected wave will be changed by half a wavelength
  - If $n_1 > n_2$ then there will be no phase change

Review (2)

- The criterion for constructive interference of light incident on a thin, optically clear medium in air such as a soap bubble is

$$\left(m+\frac{1}{2}\right)\frac{\lambda_{air}}{n}=2t \quad (m=0, m=\pm1, m=\pm2,...)$$

- The minimum thickness t_{min} that will produce constructive interference corresponds to

$$t_{\min}=\frac{\lambda_{air}}{4n}$$

- We get the same answer for the destructive interference of light passing from air to two clear optical media such that $n_{air} < n_1 < n_2$ such as the coating on a camera lens

Review (3)

- An interferometer is a device designed to measure lengths or changes in length using interference of light
- An interferometer can measure lengths or changes in lengths to a fraction of the wavelength of light using interference fringes
- An interferometer can be used to measure the thickness of a material or the index of refraction of a material by placing in one of the paths of the interfering light and counting the change in the number of fringes

$$N_{material}-N_{air}=\frac{2tn}{\lambda}-\frac{2t}{\lambda}=\frac{2t}{\lambda}(n-1)$$

Diffraction by a Circular Opening

- We have considered interference through two slits and diffraction through a single slit
- Now we consider diffraction of light through a circular opening
- Diffraction through a circular opening relates to observing objects with telescopes with circular mirrors and cameras that have circular lens
- The resolution of a telescope or camera is limited by diffraction phenomena
- The first diffraction minimum from light with wavelength λ passing through a circular opening with diameter d is

$$\sin\theta = 1.22\frac{\lambda}{d}$$

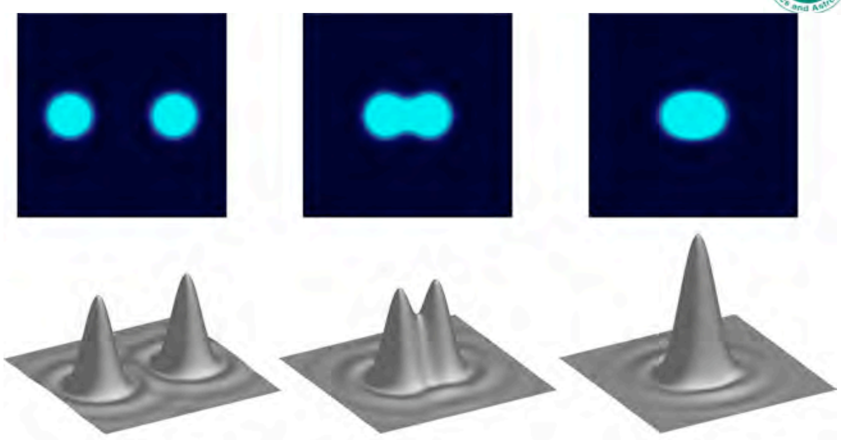
Diffraction by a Circular Opening (2)

- This result is similar to the result from a single slit except for the factor of 1.22
- If one is using a circular lens to observe two distant points objects, such as two stars, whose angular separation is small, diffraction limits the ability of the lens to distinguish these two objects
- The criterion for being able to separate two point objects is based on the idea that if the first image is centered on the first diffraction minimum of the second object, the objects are just resolved
- This criterion is called Rayleigh's Criterion and is expressed as

$$\theta_R = \sin^{-1}\left(\frac{1.22\lambda}{d}\right)$$

- Where θ_R is the minimum observable angular separation, λ is the wavelength, and d is diameter of the lens

Examples of Resolution



Resolved Barely Resolved Not Resolved

Hubble Space Telescope

- The diameter of the Hubble Space Telescope is 2.4 m. What is the minimum angular resolution of the Hubble Space Telescope?
- Using Rayleigh's Criterion with green light of wavelength 550 nm we get

$$\theta_R = 1.22\frac{550\cdot 10^{-9}\text{ m}}{2.4\text{ m}} = 2.8\cdot 10^{-7}$$

- which corresponds to the angle subtended by a dime located 64 km away
- When the Hubble Space Telescope was first launched, flaws were discovered in the main mirror that limited its ability to resolve images
- A repair mission fixed the mirror so that it now functions at the diffraction limit

Double Slit Diffraction

- We have discussed the interference pattern produced by two slits
- For that analysis we assumed that the slits themselves were very narrow compared with the wavelength of light, $a \ll \lambda$
- For these narrow slits, the diffraction maxima are very wide and we saw peaks in the intensity that were the same intensity at all angles
- For many sets of real-world double slits, the condition $a \ll \lambda$ is not met and we observe that not all the interference fringes have the same intensity

Double Slit Diffraction (2)

- With diffraction effects the intensity of the interference pattern from double slits is given by

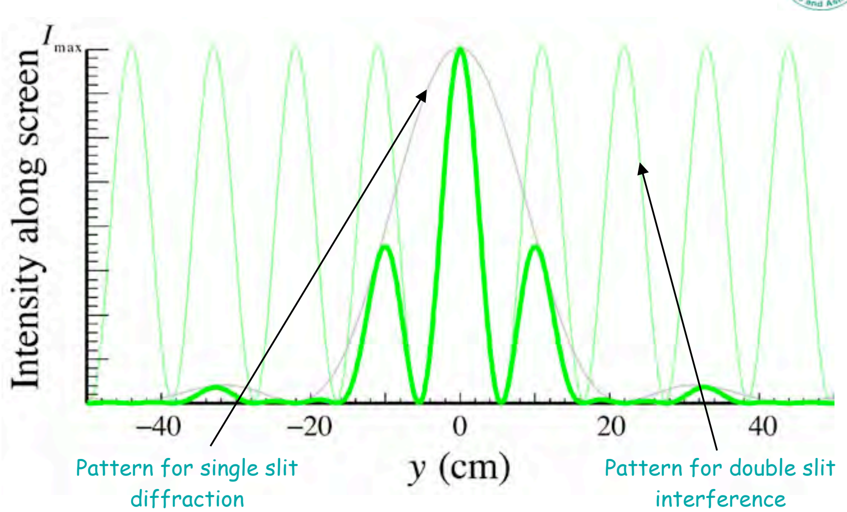
$$I = I_{\max} \cos^2 \beta \left(\frac{\sin \alpha}{\alpha} \right)^2 \quad \alpha = \frac{\pi a}{\lambda} \sin \theta \quad \beta = \frac{\pi d}{\lambda} \sin \theta$$

- If the screen is placed a sufficiently large distance from the slits then we can write

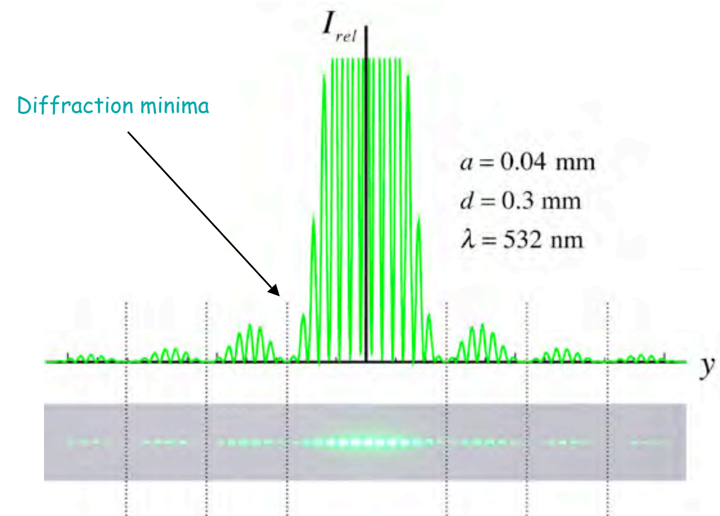
$$\alpha = \frac{\pi a y}{\lambda L} \quad \text{and} \quad \beta = \frac{\pi d y}{\lambda L}$$

- On the next slide we calculate the intensity pattern for a double slit including interference and diffraction assuming L = 2.0 m, $a = 5.0 \cdot 10^{-6}$ m, $d = 1.0 \cdot 10^{-5}$ m, and λ = 550 nm

Double Slit Diffraction (3)



Real Life Two Slit Diffraction Pattern

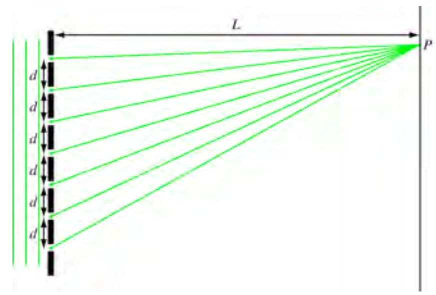


Diffraction Gratings

- We have discussed diffraction and interference for a single slit and for two slits
- Now we will discuss the application of diffraction and interference to a system of many slits
- Putting many slits together forms a device called a diffraction grating
- A diffraction grating has a large number of slits, or rulings, placed very close together
- A diffraction grating can also be constructed using an opaque material with grooves rather than actual slits
- A diffraction grating produces an intensity pattern that consists of narrow bright fringes separated by wide dark areas
- This characteristic pattern results from the use of many slits that produce destructive interference away from the maxima

Diffraction Gratings

- A portion of a diffraction grating is shown below
- In this drawing we see coherent light with wavelength incident on a series of narrow slits each separated by a distance *d*



- A diffraction pattern is produced on a screen a long distance *L* away

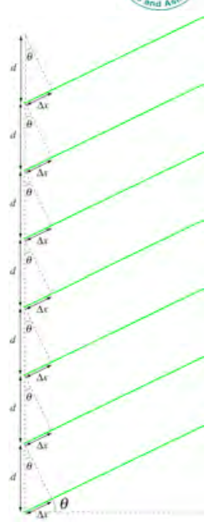
Diffraction Gratings (2)

- We can expand our drawing as we did for the single slit and double slit to enable our analysis of the path length difference for the light from each of the slits to the screen
- The distance *d* is called the grating spacing
- If the grating is *W* wide, the number *N* of slits or gratings will be

$$N = \frac{W}{d}$$

- Diffraction gratings are often specified in terms the number of slits or rulings per unit length, *n*
- We can obtain *d* from the specified *n* using

$$d = \frac{1}{n}$$



Diffraction Gratings (3)

- We can calculate the path length differences for the paths shown on the previous slide
- Using an adjacent pair of rays, the path length difference is

$$\Delta x = d \sin\theta$$

- To produce bright lines or constructive interference this path length difference must be an integer multiple of the wavelength so

$$d \sin\theta = m\lambda \quad (m = 0, 1, 2, ...)$$

- The values of *m* correspond to different bright lines
  - For $m = 0$ we have the central maximum at $\theta = 0$
  - For $m = 1$ we have the first order maximum
  - For $m = 2$ we have the second order maximum, etc.

Diffraction Gratings (4)

- Typically diffraction gratings are designed to produce large angular separations between the maxima so we do not make the small angle approximation when discussing diffraction gratings
- Because diffraction gratings produce widely spaced narrow maxima, they can be used to determine the wavelength of monochromatic light by rearranging

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) \qquad (m = 0, 1, 2, ...)$$

- Monochromatic light incident on a diffraction grating will produce lines on a screen at widely separated angles

Diffraction Gratings (5)

- In addition, diffraction gratings can be used to separate out different wavelength light from a spectrum of wavelengths
- Sunlight will be dispersed into multiple sets of rainbow-like colors as a function of θ
- If the light is composed of several discrete wavelengths, the light will be separated into sets of a few lines corresponding to those wavelengths
- The quality of a diffraction grating can be quantified in terms of its dispersion
- The dispersion describes the ability of a diffraction grating to spread apart the various orders
- Dispersion is defined by

$$D = \frac{\Delta\theta}{\Delta\lambda}$$

- $\Delta\theta$ is the angular separation between two lines with wavelength difference $\Delta\lambda$

Diffraction Gratings (6)

- We can get an expression for the dispersion by differentiating with respect to λ

$$\frac{d\theta}{d\lambda} = \frac{d}{d\lambda}\left(\sin^{-1}\left(\frac{m\lambda}{d}\right)\right) = \frac{1}{\sqrt{1-\left(\frac{m\lambda}{d}\right)^2}}\frac{m}{d} = \frac{m}{\sqrt{d^2-(m\lambda)^2}}$$

- We can use the relation $d\sin\theta = m\lambda$ to get

$$\frac{d\theta}{d\lambda} = \frac{1}{\sqrt{1-\sin^2\theta}}\frac{m}{d} = \frac{m}{d\cos\theta}$$

- Taking interval of θ and λ that are not too large we get

$$D = \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta} \qquad (m = 1, 2, 3, ...)$$