Physics for Scientists & Engineers 2

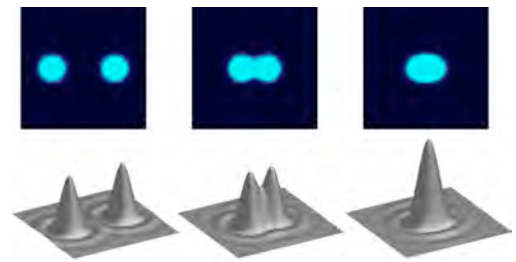
Spring Semester 2005

Lecture 46

Review

- The criterion for being able to separate two point objects is called Rayleigh's Criterion and is expressed as

$$\theta_R = \sin^{-1}\left(\frac{1.22\lambda}{d}\right)$$



Resolved Barely Resolved Not Resolved

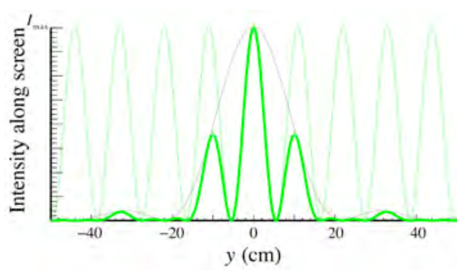
Review (2)

- With diffraction effects the intensity of the interference pattern from double slits is given by

$$I = I_{\max}\cos^2\beta\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \alpha = \frac{\pi a}{\lambda}\sin\theta \quad \beta = \frac{\pi d}{\lambda}\sin\theta$$

- If the screen is placed a sufficiently large distance from the slits then we can write

$$\alpha = \frac{\pi a y}{\lambda L} \quad \text{and} \quad \beta = \frac{\pi d y}{\lambda L}$$



Review (3)

- A diffraction grating has a large number of slits, or rulings, placed very close together
- To produce bright lines or constructive interference this path length difference must be an integer multiple of the wavelength so

$$d\sin\theta = m\lambda \quad (m = 0, 1, 2, \ldots)$$

- The values of *m* correspond to different bright lines
- The dispersion describes the ability of a diffraction grating to spread apart the various orders

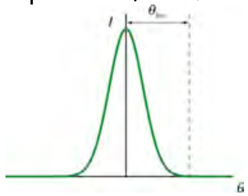
$$D = \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta} \quad (m = 1, 2, 3, \ldots)$$

Resolving Power of a Grating

- The resolving power R of a diffraction grating describes the ability of the diffraction grating to resolve closely spaced maxima, which depends on the width of each maximum
- We define the power of a diffraction grating to resolve two wavelengths, λ_1 and λ_2, as

$$R = \frac{\lambda_{ave}}{\Delta\lambda} \quad \lambda_{ave} = (\lambda_1 + \lambda_2)/2 \quad \Delta\lambda = |\lambda_2 - \lambda_1|$$

- Thus to discuss the resolving power, we need an expression for the width of each maximum
- The width of each maximum is defined by the position of the first minimum on each side of the maximum
- We can then define the half-width θ_{hw} of the maximum as the angle between the maximum and the first minimum



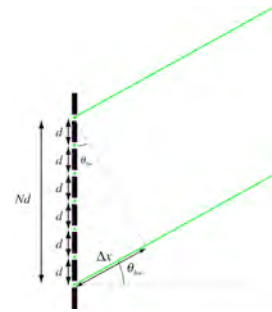
Resolving Power of a Grating (2)

- We base our argument our analysis of single slit diffraction using the whole grating as the single slit as shown
- The angle of the first minimum for single slit diffraction can be obtained where we substitute Nd for the slit width a

$$Nd \sin\theta_{hw} = \lambda$$

- Because θ_{hw} is small, we can write

$$\theta_{hw} = \frac{\lambda}{Nd}$$



- One can show that the width of the maxima for other orders is

$$\theta_{hw} = \frac{\lambda}{Nd\cos\theta}$$

  - θ is the angle corresponding to the maximum intensity for that order

Resolving Power of a Grating (3)

- We can substitute θ_{hw} for $\Delta\theta$

$$\frac{\Delta\theta}{\Delta\lambda} = \frac{\lambda}{Nd\cos\theta\Delta\lambda} = \frac{m}{d\cos\theta}$$

- Which gives us

$$R = \frac{\lambda}{\Delta\lambda} = Nm \qquad \lambda \approx (\lambda + (\lambda + \Delta\lambda))/2$$

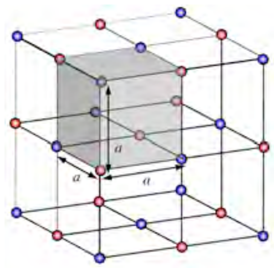
- Note that the resolving power of a diffraction grating depends on the total number of rulings and the order

X-Ray Diffraction

- Wilhelm Röntgen discovered x-rays in the late 1800's
- These experiments suggested that x-rays were electromagnetic waves with a wavelength of about 10^{-10} m
- At about the same time, the study of crystalline solids suggested that the atoms of those solids were arranged in a regular repeating pattern with a spacing of about 10^{-10} m between the atoms
- Putting these two ideas together, Max von Laue proposed in the early 1900's that a crystal could serve as a three dimensional diffraction grating for x-rays
- Von Laue and Friederich Knipping did the first x-ray diffraction experiment that showed diffraction of x-rays by a crystal in 1912
- Soon after Sir William Bragg and his son William Bragg derived Bragg's law and carried out a series of experiments involving x-ray diffraction from crystals

X-Ray Diffraction (2)

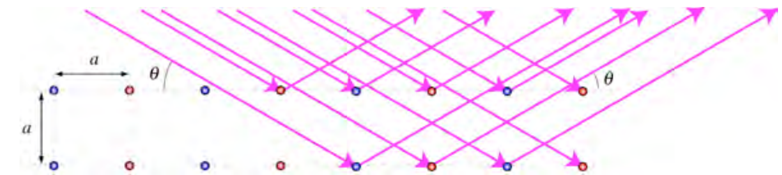
- Let's assume that we have a cubic crystal as shown



- Each atom in the lattice is a distance *a* away from the next atom in all three directions
- We can imagine various planes of atoms in this crystal

X-Ray Diffraction (3)

- For example, the horizontal planes are composed of atoms spaced a distance *a* apart with the planes themselves being spaced a distance *a* from each other
- We can imagine x-rays incident on these planes and that the rows of atoms in the crystalline lattice can act like a diffraction grating
- The x-rays can be thought of as scattering from the atoms



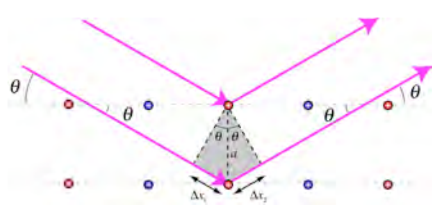
X-Ray Diffraction (4)

- Interference effects are caused by path length differences
- If we look at x-rays scattering off one plane, all the waves remain in phase
- However, if we consider adjacent planes, we can see below that the path length difference for the scattered x-rays from the two planes is

$$\Delta x = \Delta x_1 + \Delta x_2 = 2a\sin\theta$$

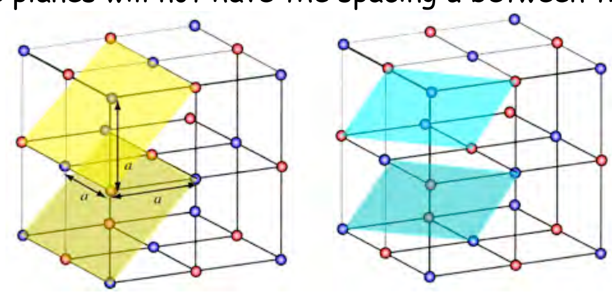
- The criterion for constructive interference is

$$2a\sin\theta = m\lambda \quad (m = 0, 1, 2, ...)$$



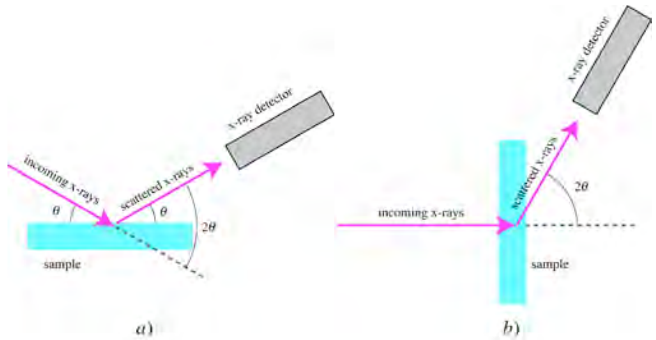
X-Ray Diffraction (5)

- Of course when x-rays are incident on a crystal, there can be several different planes that can function as diffraction gratings
- Some examples are illustrated below
- These planes will not have the spacing *a* between the planes



X-Ray Diffraction (6)

- To study the atomic structure of a substance using x-ray diffraction one can scatter x-rays parallel to the surface of a sample as shown below in a) or one can transmit the x-rays through the sample and detect the x-rays on the opposite side of the sample and shown in b)



X-Ray Diffraction (7)

- For the parallel scattering method, the angle of incidence θ should equal the angle of observation
- For the transmission method, the observed angle is twice the Bragg angle θ
- By measuring the intensity of the x-rays as a function of θ one can determine details of the structure of the material being studied
- Modern particle accelerators such as the National Synchrotron Light Source at Brookhaven National Laboratory are used to produce high quality, intense beams of x-rays to carry out material science research