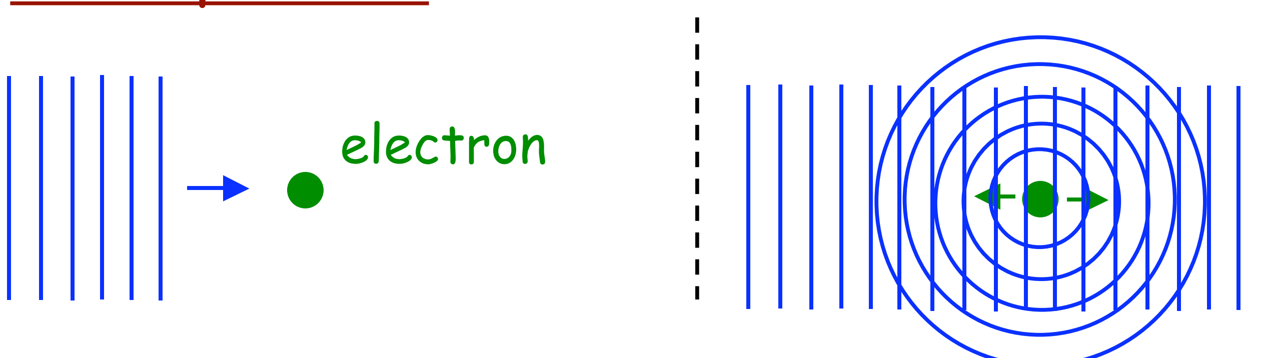
Compton Scattering

• Photoelectric effect, X-ray spectra suggest that photons, as well as being electromagnetic waves, also act like particles.

• This effect is even more pronounced in scattering of X-rays off electrons.

Wave picture:



Particle picture:

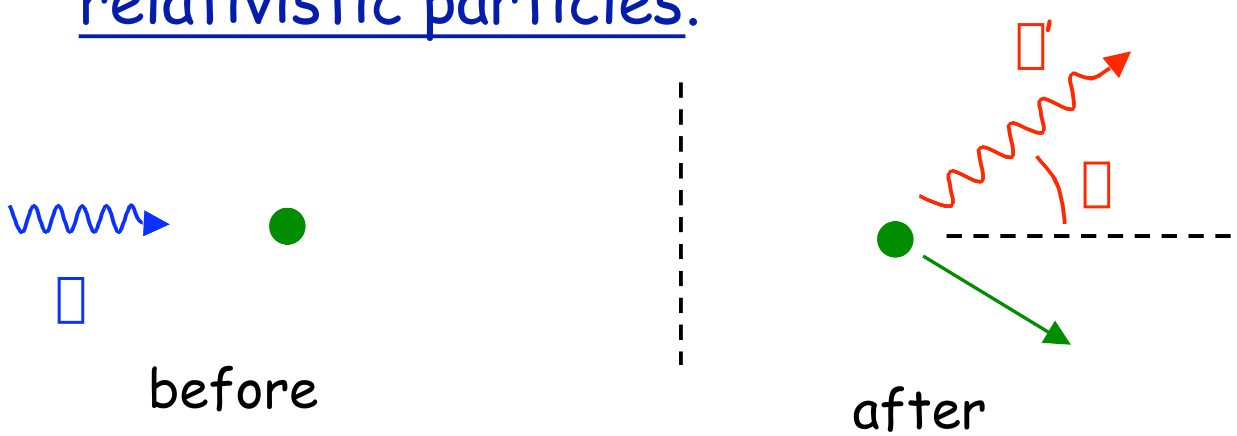
- 1923 -
  Experiments by Arthur Compton.
  He observed:

  Some component of the scattered wave (especially at backward-scattering angles) has a longer wavelength than the incoming wave.

  $$\lambda' > \lambda$$

- This could not be understood purely in terms of light as a wave.

- Compton showed that it was understandable as a scattering of relativistic particles.



The formula for the shift in wavelength is

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

with

λ' the scattered wavelength

λ the incoming wavelength

θ the scattering angle

m the electron mass

The combination

$$h/(mc) = 2.43 \times 10^{-12}\ \mathrm{m} = 2.43 \times 10^{-3}\ \mathrm{nm}$$

is called the Compton wavelength of the electron.

Atomic Structure

Thornton and Rex, Ch. 4

Models of the Atom

Required features:

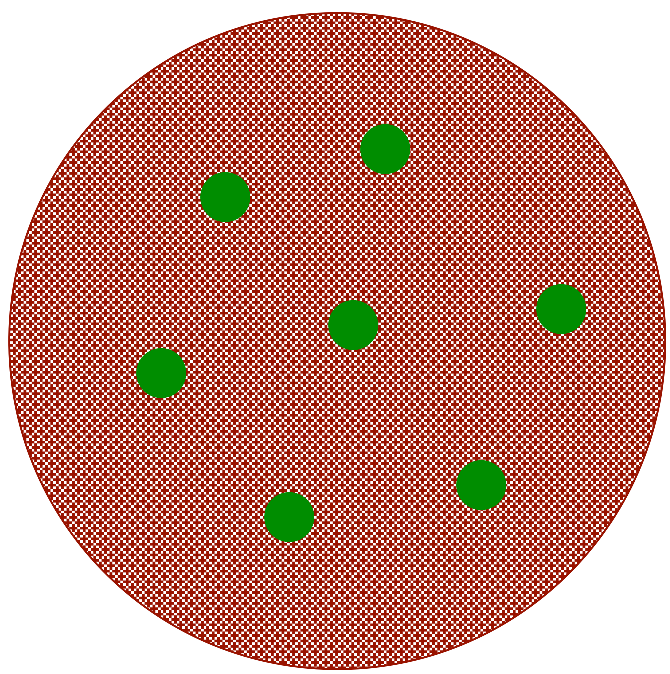
1. Electrons as constituents.
2. Some positive charges to neutralize the negative charges of the electrons.
3. Some scheme to account for the various different atomic weights.
4. Something to account for the different chemical properties of atoms

The "Plum Pudding" Model

(J.J. Thomson, 1904)

In this model, "a number of negatively-charged corpuscles were enclosed in a sphere of uniform positive electrification"

- A blob of positive "pudding" with electron "plums". The charges cancel.



Rutherford Scattering

- Rutherford (1907), with Geiger and Marsden, set out to study scattering of □-rays.

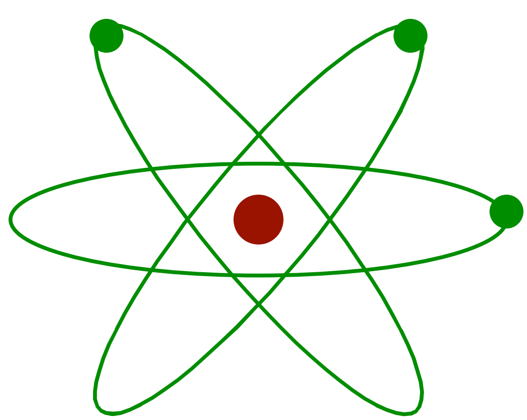
- □-rays can be observed through scintillation. A screen coated with ZnS emits a short, faint flash of light when struck by an □-ray.

- To the amazement of all, the □-rays were scattered through very large angles. (1 in 20,000 even bounced back in the direction from which they had come - a scattering angle of 180°.) This was an incredible result!

-In 1910 Rutherford calculated how close the positive □-ray must get to the positive charge in the gold atom for it to stop and reverse direction.

- The calculation showed a distance of about 1×10^{-14} m, about 1/10,000 the size of the atom.

The "plum pudding" model was wrong. The positive charge in the atom must be concentrated only at the very center.



Rutherford Scattering

Experiment of Geiger and Marsden:

α-rays scattered from thin gold target at large angles.

1911 - Rutherford:

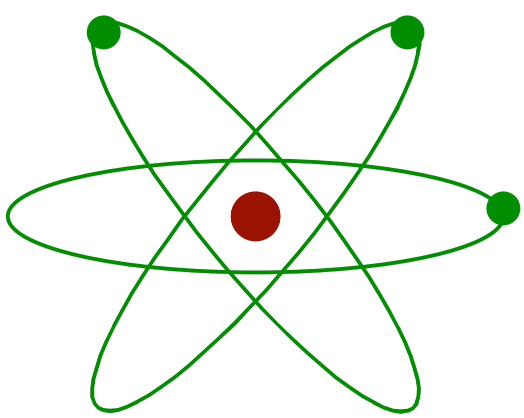
- Results inconsistent with scattering from a homogeneous structure.
- Atom must contain central charge in small volume.

Rutherford worked out the scattering expected for the α-rays as a function of angle, thickness of material, velocity, and charge.

Rutherfords formulae verified by Geiger and Marsden in 1913.

Rutherford coins the term nucleus: central positively-charged core of the atom.

Popular picture of atom today is due to Rutherford:



Impact parameter b vs. scattering angle θ:

$$b = (r_{min}/2)\cot(\theta/2)$$

where

$$r_{min} = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 K}$$

is the distance of closest approach for head-on collision.

Any particle hitting an area πb^2 around the nucleus will be scattered through an angle of θ or greater.

This area is called the cross section (for angle > θ). It is written:

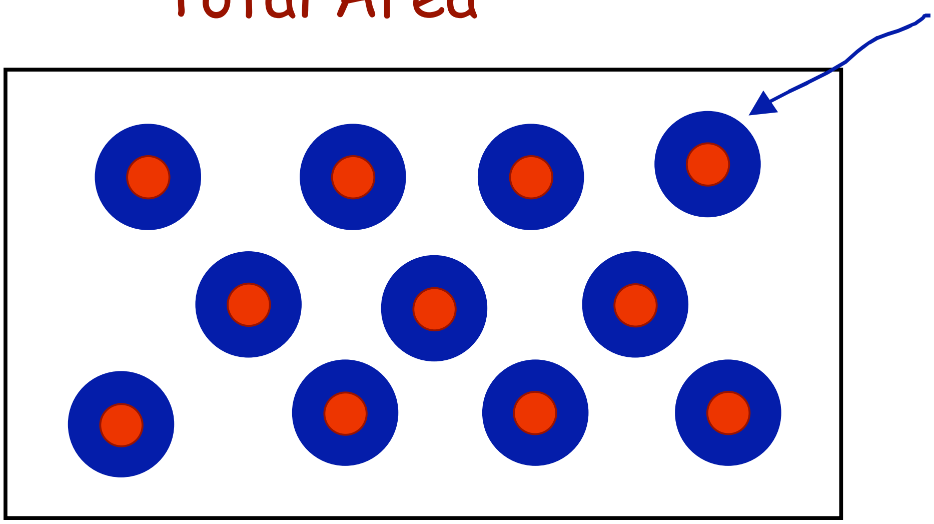
$$\sigma = \pi b^2$$

(common unit of σ : barn = 10^{-28} m^2
= 100 fm^2)

Probability of scattering (with angle > $\square$) is

$$f = \frac{(\#\ \text{atoms})\ \square}{\text{Total Area}}$$

$\square b^2$



\# atoms/Area =

(# atoms/Volume) x thickness

= n t

$$n = \left(N_A \frac{\text{atoms}}{\text{mole}}\right)\left(\frac{1}{A}\,\frac{\text{mole}}{\text{gm}}\right)\left(\square\ \frac{\text{gm}}{\text{cm}^3}\right)$$

$$= \frac{\square N_A}{A}$$

So

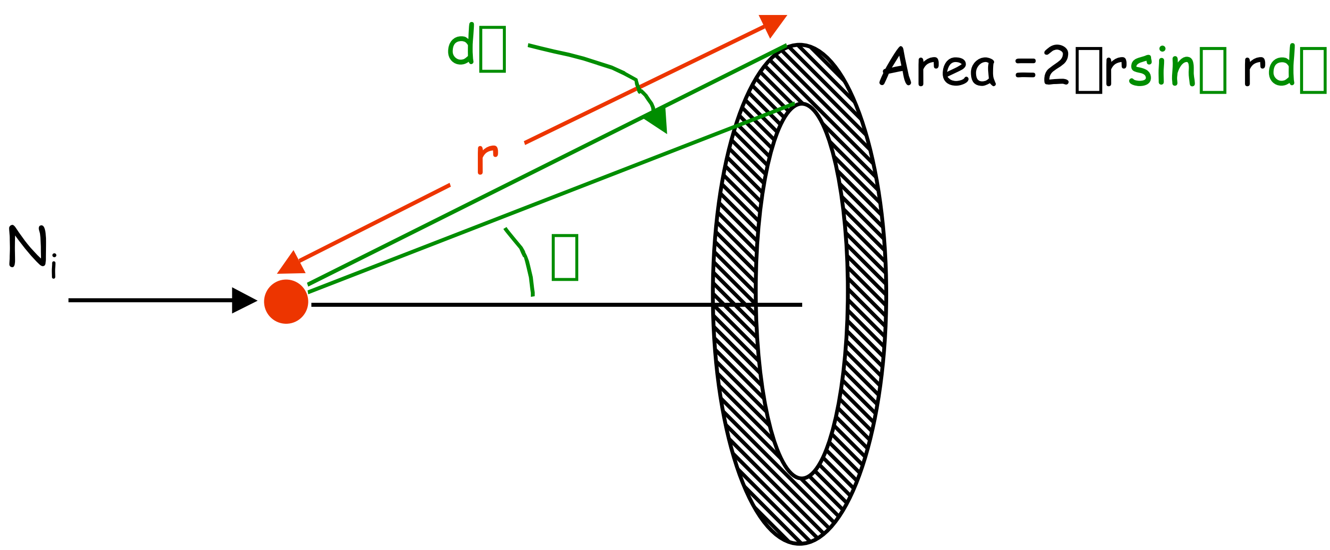
$$f = nt\square b^2 = (\square nt/4)\ r_{min}^2 \cot^2(\square/2)$$

In practice, events are measured in range θ to $\theta + d\theta$

Differential probability:

$$df = -(\pi n t/4)\, r_{min}^2 \cot(\theta/2)\,\mathrm{cosec}^2(\theta/2)\, d\theta$$

For N_i incident particles, # scattered into ring between θ and $\theta + d\theta$ is $N_i|df|$.



per unit area between θ and $\theta + d\theta$

$$= \frac{N_i\,|df|}{2\pi r^2 \sin\theta\, d\theta}$$

$$N(\theta) = \frac{N_i\, nt}{16\, r^2} (r_{min})^2 \frac{1}{\sin^4(\theta/2)}$$

$$= \frac{N_i\, nt}{16\, r^2} (e^2/4\pi\epsilon_0)^2 \frac{Z_1^2\, Z_2^2}{K^2 \sin^4(\theta/2)}$$

The Rutherford Scattering Formula

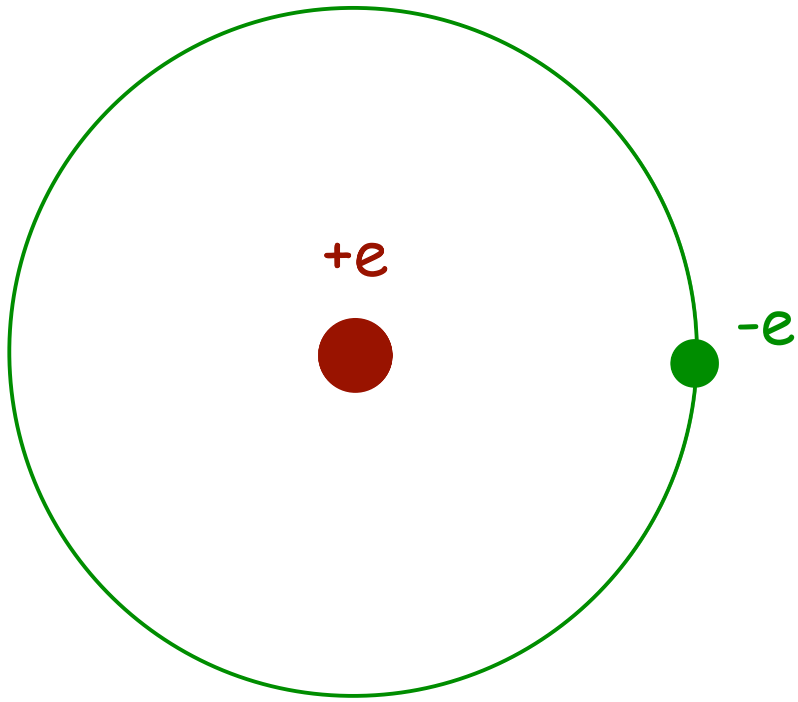
a) Proportional to Z_1^2 and Z_2^2

b) Proportional to $1/K^2$

c) Proportional to $1/\sin^4(\theta/2)$

d) Proportional to thickness t
(for thin targets)

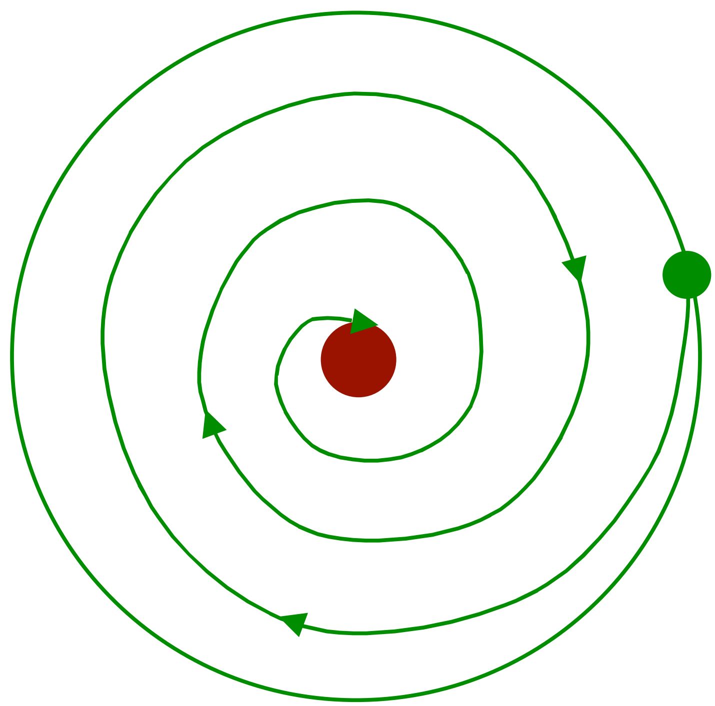
Rutherford's Model of the Atom



Hydrogen atom

But there are problems . . .

- Accelerating electrons radiate energy. The electron should spiral into the nucleus (in ~10^{-9} seconds!)



- Atoms with > 1 electron are unstable, due to electron repulsion.

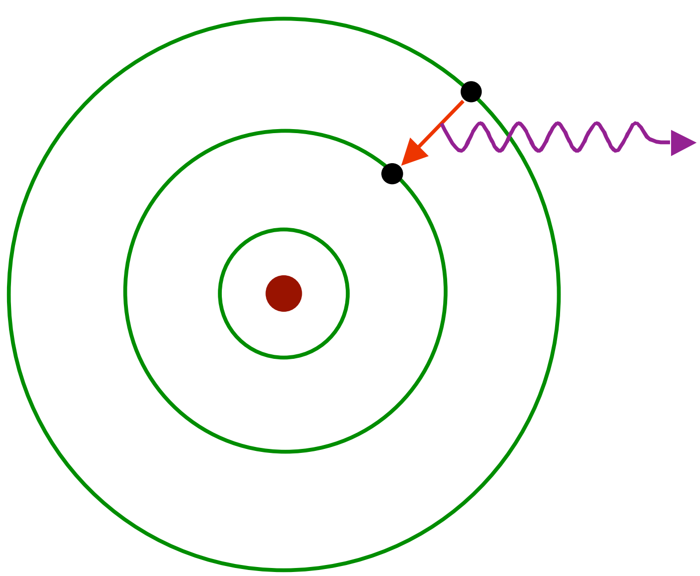
Bohr's Model of the Hydrogen Atom

Neils Bohr modified Rutherford's model with some ad-hoc assumptions:

1) Electrons only in special orbits: "Stationary States".

2) An electron in a stationary state obeys classical mechanics (Newton's laws).

3) Transitions between stationary states (i->f) do not obey classical mechanics. They are accompanied by the emission or absorption of radiation of fixed energy $E = E_i - E_f$.



4) The angular momentum of a stationary state is an integer multiple of $h/(2\pi)$. I.e. $L = n\, h/(2\pi)$, where $n = 0,1,2,\ldots$

The combination $h/(2\pi)$ occurs so frequently that it is *given* a special symbol:

$$\hbar = h/(2\pi)$$

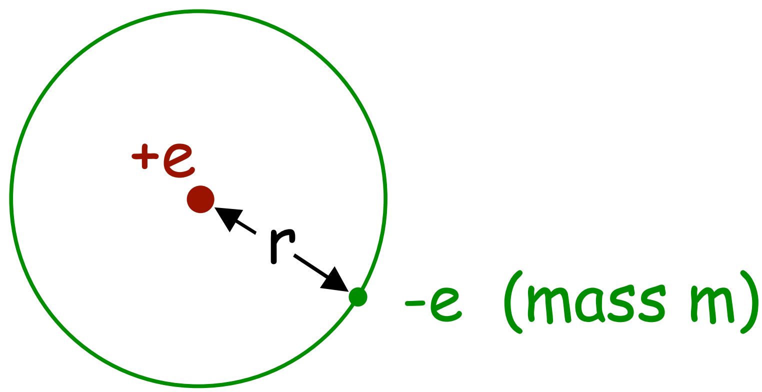
$$\hbar = 1.05 \times 10^{-34}\ \mathrm{J{\cdot}s}$$

Assumption 4 (quantization of angular momentum) was the most controversial.

But...

- it accounts for stability of Hydrogen,
- it leads to Rydberg formula for line spectra of Hydrogen.

Hydrogen with electron in circular orbit:



Quantization of L gives:

$$L = mvr = n\hbar$$

$$\Rightarrow \quad v = n\hbar / (mr) \qquad (1)$$

Centripetal force due to Coulomb attraction:

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = m\,a_{cent} = m\,\frac{v^2}{r}$$

$$\Rightarrow \quad v^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{mr} \qquad (2)$$

Plug (1) into (2) and solve for radius r:

$$\Rightarrow \quad r = 4\pi\varepsilon_0 \frac{n^2 \hbar^2}{m e^2}$$

$\Rightarrow$ Each stationary state orbits at specific radius, identified by integer n.

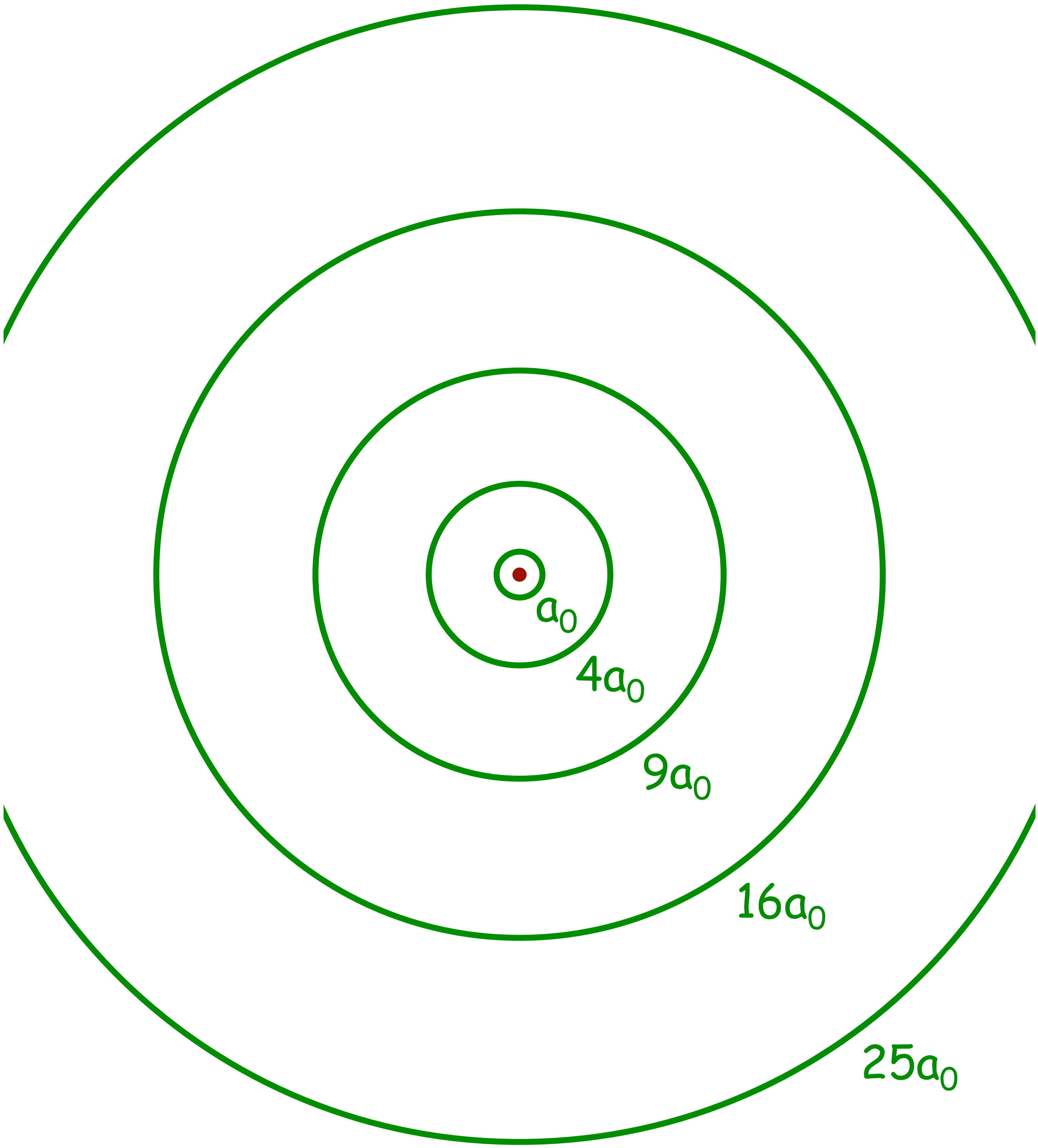
The smallest radius ($n=1$) is called the “Bohr radius:

$$a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{m e^2} = 5.3 \times 10^{-11}\ m$$

Other radii related by:

$$r_n = n^2 a_0.$$

Hydrogen atom is stable.



a0
4a0
9a0
16a0
25a0

Calculation of Energy levels

$$E \quad = \quad KE \quad + \quad PE$$

$$= \frac{1}{2} mv^2 \quad - \frac{e^2}{4\pi\varepsilon_0 r}$$

$$= \frac{1}{2} m\left(\frac{e^2}{4\pi\varepsilon_0 mr}\right) \quad - \frac{e^2}{4\pi\varepsilon_0 r}$$

$$= -\frac{1}{2} \frac{e^2}{4\pi\varepsilon_0 r}$$

Substituting for r:

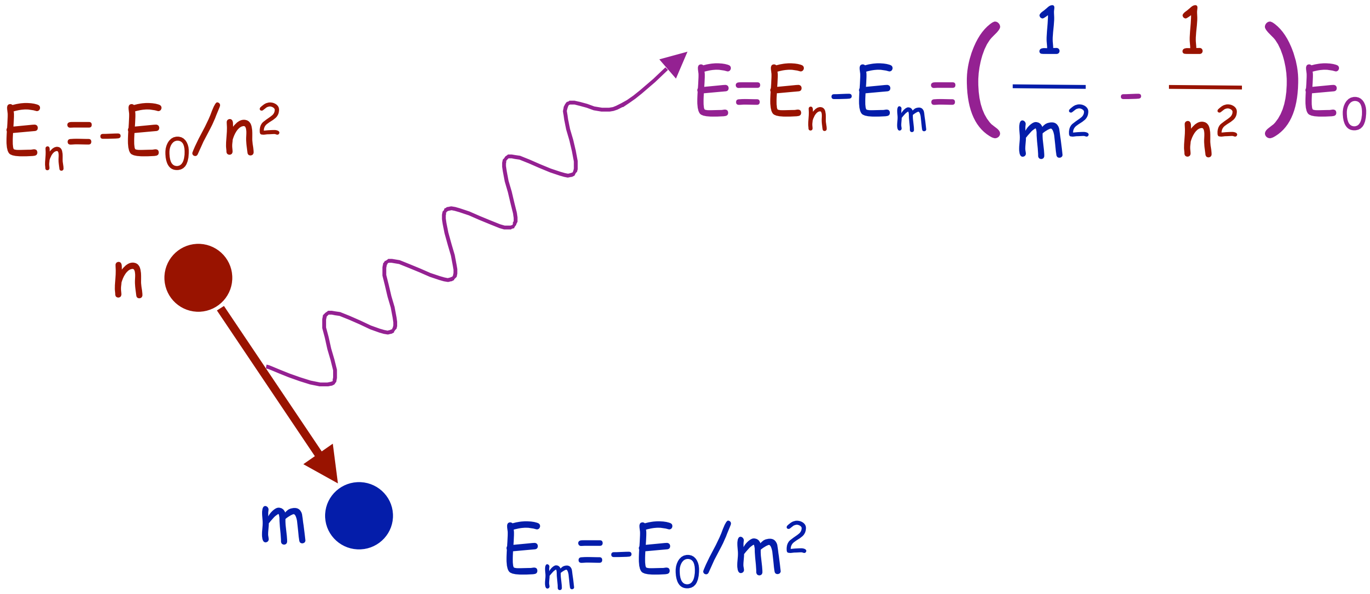
$$\Rightarrow \quad E = -E_0 / n^2$$

where

$$E_0 = \alpha^2 m c^2/2 = 13.6 \text{ eV}$$

The dimensionless constant α is called the "fine structure constant":

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} = \frac{1}{137.0}$$


$E_n=-E_0/n^2$
$E=E_n-E_m=\left(\frac{1}{m^2}-\frac{1}{n^2}\right)E_0$
n
m
$E_m=-E_0/m^2$

Emission and Absorption of Radiation

Energy of emitted radiation:

$$E = E_n - E_m$$

$$= E_0 \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$

Using the Planck formula $E = h\nu = hc/\lambda$,
leads to Rydberg formula:

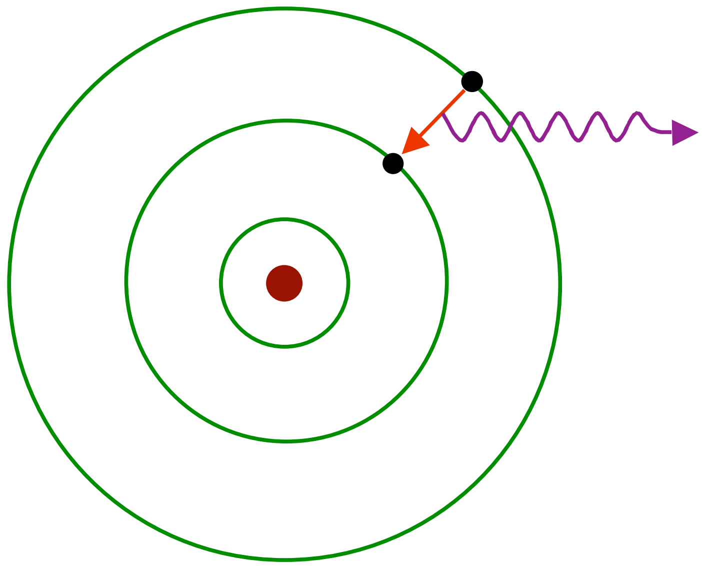
$$\frac{1}{\lambda} = R_H \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$

With

$$R_H = E_0/(hc) = 13.6 \text{ eV}/(1243 \text{ eV} \cdot \text{nm})$$
$$= 1.09 \times 10^{-7} \text{ m}$$

This predicts the Rydberg constant exactly!

Summary of the Bohr Model of Hydrogen



- L quantized: $L = n \hbar = n\, h/(2\pi)$
- Stationary State orbits: $r = a_0\, n^2$
- Stationary State energies: $E = -E_0 / n^2$
- Predicts Rydberg formula and constants: a_0, E_0, R_H

Generalization of the Bohr Model

Applies to any single electron atom (H, He^{+}, Li^{++}, ...) by changing nuclear charge from $+e$ to $+Ze$.

Radius of orbit is now

$$r_n = n^2 a_0 / Z$$

and energy is

$$E_n = - Z^2 E_0 / n^2.$$

Stronger electric fields:

- Smaller orbits
- More tightly bound electrons