Physics 321 – Spring 2005

Homework #6, Due at beginning of class Wednesday Mar 2.

1. [4 pts] A hook is at height y above the floor, where y is constant for all negative times: $y = y_0$ for $t < 0$. For positive times, y oscillates: $y = y_0 + A \sin \omega t$ for $t > 0$. A mass M hangs from an ideal spring attached to this hook. The mass is at height x above the floor. The mass hangs motionless at $x = x_0 = y_0 - Mg/k$ for $t < 0$, where k is the spring constant. Let $\omega_0 = \sqrt{k/M}$ as usual.

   (a) Find the motion $x(t)$ of the mass for $t > 0$ if $\omega = 2\omega_0$.

   (b) Find the motion $x(t)$ of the mass for $t > 0$ if $\omega = \omega_0$. (You can do this by first finding $x(t)$ for arbitrary ω and then carefully taking the limit $\omega \to \omega_0$; or if you're chicken, you can set $\omega \to \omega_0$ in the equation of motion and solve it.)

2. [4 pts] A driven harmonic oscillator obeys the equation $\ddot{x} + x = t\,(A - t)$ for $0 < t < A$. Given the initial conditions $x = \dot{x} = 0$ at $t = 0$, find the subsequent motion $x(t)$ during the time interval $0 < t < A$.

3. [4 pts] Marion & Thornton, problem 3-20 (Same in 4th edition). Do this problem by hand (i.e., using algebra, not using a computer). You need to find the two angular frequencies on either side of the resonance (call them ω_1 and ω_2) where the velocity amplitude is equal to the maximum velocity (on resonance) divided by $\sqrt{2}$, so the kinetic energy has half of its maximum value. This procedure finds the "Full Width at Half Maximum" (FWHM) of the resonance, $\omega_1 - \omega_2$, which is a common way to characterize its width of a resonance peak.

4. [4 pts] A damped driven harmonic oscillator obeys the equation $\ddot{x} + 2\beta\dot{x} + x = t\,e^{-\alpha t}$ for $t > 0$, where $0 < \beta < 1$ and α is a positive constant.

   Given the initial conditions $x = \dot{x} = 0$ at $t = 0$, find the subsequent motion $x(t)$. Hint: as is so often the case, the easiest way to solve the differential equation is to guess the answer.

5. [4 pts] Marion & Thornton, problem 3-28 (problem 3-32 in 4th edition).

6. [4 pts] Marion & Thornton, problem 3-39 (problem 3-43 in 4th edition).