# Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 3



## Review from Yesterday

- There are positive charges and negative charges
- **Law of Charges**
  - Like charges repel and opposite charges attract
- The unit of charge is the coulomb defined as
  - 1 C = 1 A•s
- **Law of charge conservation**
  - The total charge of an isolated system is strictly conserved



## Electrostatic Charging

- There are two ways to charge an object
  - Conduction
  - Induction
- Charging by conduction
  - We can charge an object by connecting a source of charge directly to the object and then disconnecting the source of charge
    - The object will remain charged
      - Conservation of charge



## Charging by Induction

- We can also charge an object without physically connecting to it
  - First we charge a paddle with negative charge
  - Then we ground the object to be charged
    - Connecting the object to the Earth provides an effectively infinite sink for charge
  - We bring the charged paddle close to the object but do not touch it
  - We remove the ground connection and move the paddle away
  - The object will be charged by induction

## Electric Force - Coulomb's Law

- Consider two electric charges: $q_1$ and $q_2$
- The electric force F between these two charges separated by a distance r is given by Coulomb's Law

$$F = k\frac{q_1 q_2}{r^2}$$

- The constant k is called Coulomb's constant and is given by

$$k = 8.99 \cdot 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$$



## Coulomb's Law (2)

- We can get a feeling for how big a coulomb of charge is if we calculate the force between two 1 C charges 1 meter apart

$$F = \left(8.99 \cdot 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\frac{1 \text{ C} \cdot 1 \text{ C}}{(1 \text{ m})^2} = 8.99 \cdot 10^9 \text{ N}$$

which is the weight of 450 Space Shuttles at launch

- The coulomb constant is also written as

$$k = \frac{1}{4\pi\varepsilon_0} \qquad \varepsilon_0 = 8.85 \cdot 10^{-12} \ \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}$$

- $\varepsilon_0$ is the electric permittivity of free space
  - Fundamental constant



## Electric Force

- The electric force is given by $F = k\frac{q_1 q_2}{r^2}$
- The electric force, unlike the gravitational force, can be positive or negative
  - If the charges are the opposite sign, the force is negative
    - Attractive
  - If the charges are the same sign, the force is positive
    - Repulsive
- We can also write the electric force in vector form

$$\vec{F}_{2\to 1} = k\frac{q_1 q_2}{r^3}(\vec{r}_2 - \vec{r}_1)$$



## Example - The Helium Nucleus

- The nucleus of a helium atom has two protons and two neutrons. These four nucleons are bound together by the strong force. What is the magnitude of the electric force between the two protons in the helium nucleus?

Each proton has charge $q = 1.602 \cdot 10^{-19}$ C

The distance between the two protons is approximately $2.0 \cdot 10^{-15}$ m

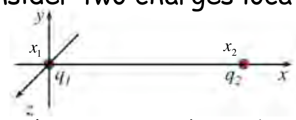
The force is given by

$$F = k\frac{q_1 q_2}{r^2} = \left(8.99 \cdot 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\frac{\left(1.602 \cdot 10^{-19} \text{ C}\right)^2}{\left(2.0 \cdot 10^{-15} \text{ m}\right)^2} = 58 \text{ N}$$

Considering that the mass of a proton is $1.67 \cdot 10^{-27}$ kg this force is huge

## Example - Equilibrium Position

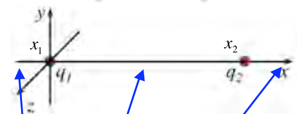
- Consider two charges located on the x axis




- The charges are described by
  - $q_1 = 0.15\ \mu C \quad x_1 = 0.0$ m
  - $q_2 = 0.35\ \mu C \quad x_2 = 0.40$ m
- Where do we need to put a third charge for that charge to be at an equilibrium point?
  - At the equilibrium point, the force from each of the two charges will cancel



## Example - Equilibrium Position (2)




- We can see that the equilibrium point must be along the x-axis
- Let's consider three regions along the x-axis where we might place our third charge
  - $x_3<x_1$
  - $x_1<x_3<x_2$
  - $x_2<x_3$



## Example - Equilibrium Position (3)


- $x_3<x_1$
  - Here the forces from $q_1$ and $q_2$ will always point in the same direction (to the left for a positive test charge)
    - No equilibrium
- $x_2<x_3$
  - Here the forces from $q_1$ and $q_2$ will always point in the same direction (to the right for a positive test charge)
    - No equilibrium



## Example - Equilibrium Position (4)


- $x_1<x_3<x_2$
  - Here the forces from $q_1$ and $q_2$ can balance

$$k\frac{q_1 q_3}{(x_3-x_1)^2} = k\frac{q_3 q_2}{(x_2-x_3)^2} \quad q_3 \text{ cancels}$$

$$\frac{q_1}{(x_3-x_1)^2} = \frac{q_2}{(x_2-x_3)^2} \Rightarrow$$

$$q_1(x_2-x_3)^2 = q_2(x_3-x_1)^2 \Rightarrow$$

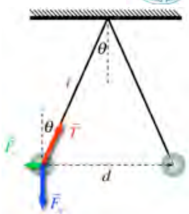
$$\sqrt{q_1}(x_2-x_3) = \sqrt{q_2}(x_3-x_1) \Rightarrow$$

$$x_3 = \frac{\sqrt{q_1}x_2 + \sqrt{q_2}x_1}{\sqrt{q_1}+\sqrt{q_2}}$$

$$x_3 = \frac{\sqrt{q_1}x_2 + \sqrt{q_2}x_1}{\sqrt{q_1}+\sqrt{q_2}} = \frac{\sqrt{0.15\ \mu\text{C}}\cdot(0.4\text{ m})}{\sqrt{0.15\ \mu\text{C}}+\sqrt{0.35\ \mu\text{C}}} = 0.16\text{ m}$$

# Example - Charged Balls

- Consider two identical charged balls hanging from the ceiling by strings of equal length 1.5 m. Each ball has a charge of 25 μC. The balls hang at an angle θ = 25° with respect to the vertical.
- What is the mass of each ball?



The distance between the balls is

$d = 2\ell \sin\theta = 2(1.5 \text{ m})\sin 25° = 1.27 \text{ m}$

The coulomb force between the balls is

$F_c = k\dfrac{q_1 q_2}{r^2} = k\dfrac{q^2}{d^2}$

The gravitation force on each ball points down

$F_g = mg$

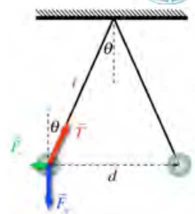


# Example - Charged Balls (2)

Looking at the left ball

x direction: $k\dfrac{q^2}{d^2} = T\sin\theta$

y directon: $mg = T\cos\theta$



Dividing these two equations we get

$\dfrac{kq^2}{mgd^2} = \dfrac{T\sin\theta}{T\cos\theta} = \tan\theta \Rightarrow$

$m = \dfrac{kq^2}{\tan\theta g d^2} = \dfrac{(8.99\cdot 10^9 \text{ N m}^2\text{C}^{-2})(2.5\cdot 10^{-5} \text{ C})^2}{\tan 25°(9.81 \text{ m/s}^2)(1.27 \text{ m})^2} = 0.76 \text{ kg}$

A similar analysis applies to the right ball



# Electric Force and Gravitational Force

- Coulomb's Law that describes the electric force and Newton's gravitational law have a similar functional form

$$F_{electric} = k\frac{q_1 q_2}{r^2} \qquad F_{gravity} = G\frac{m_1 m_2}{r^2}$$

- Both forces vary as the inverse square of the distance between the objects
- Gravitation is always attractive
- k and G give the strength of the forces



# Example - Forces between Electrons

- What is relative strength of the force of gravity compared with the electric force for two electrons?

$F_{electric} = k\dfrac{q_e^2}{r^2}$

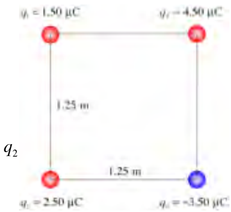
$F_{gravity} = G\dfrac{m_e^2}{r^2}$

$\dfrac{F_{electric}}{F_{gravity}} = \dfrac{kq_e^2}{Gm_e^2} = \dfrac{(8.99\cdot 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.602\cdot 10^{-19} \text{ C})^2}{(6.67\cdot 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(9.109\cdot 10^{-31} \text{ kg})^2} = 4.2\cdot 10^{42}$

- So the electric force is always very much larger than the gravitational force
- Macroscopic objects are usually uncharged so only gravity plays a role
  - Motion of the planets
- Gravity is irrelevant for sub-atomic processes

# Example - Four Charges

- Consider four charges placed at the corners of a square with sides of length 1.25 m as shown on the right. What is the magnitude of the electric force on $q_4$ resulting from the electric force from the remaining three charges?




Assume an x-y coordinate system with its origin located at $q_2$

x-direction

$$F_x = k\frac{q_1 q_4}{d^2} + k\frac{q_2 q_4}{\left(\sqrt{2}d\right)^2}\cos 45° = \frac{kq_4}{d^2}\left(q_1 + \frac{q_2}{2}\cos 45°\right)$$

y-direction

$$F_y = k\frac{q_2 q_4}{\left(\sqrt{2}d\right)^2}\sin 45° + k\frac{q_3 q_4}{d^2} = \frac{kq_4}{d^2}\left(\frac{q_2}{2}\sin 45° + q_3\right)$$

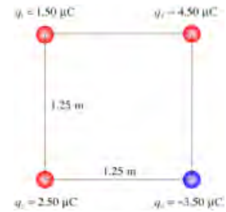


# Example - Four Charges (2)

$$F = \sqrt{F_x^2 + F_y^2}$$

$$F = \sqrt{\left(\frac{kq_4}{d^2}\left(q_1 + \frac{q_2}{2}\cos 45°\right)\right)^2 + \left(\frac{kq_4}{d^2}\left(\frac{q_2}{2}\sin 45° + q_3\right)\right)^2}$$

$$F = \frac{kq_4}{d^2}\sqrt{\left(q_1 + \frac{q_2}{2}\cos 45°\right)^2 + \left(\frac{q_2}{2}\sin 45° + q_3\right)^2}$$




$$\frac{q_2}{2}\sin 45° = \frac{q_2}{2}\cos 45° = \frac{2.50\ \mu\text{C}}{2\cdot\sqrt{2}} = 0.884\ \mu\text{C}$$

$$F = \frac{\left(8.99\cdot 10^9\right)\left(4.50\ \mu\text{C}\right)}{(1.25\ \text{m})^2}\sqrt{(1.50\ \mu\text{C} + 0.884\ \mu\text{C})^2 + (0.884\ \mu\text{C} - 3.50\ \mu\text{C})^2}$$

$$F = 0.0916\ \text{N}$$