Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 8

Electric Potential

- We have been studying the electric field
- Now we will begin our study of the electric potential
- We showed the similarity between the gravitational force and the electric force
- We demonstrated that gravitation could be described in terms of a gravitational potential and we will show that the electric potential is analogous
- We will show that the electric potential is can be related to energy and work
- We will show that that we can calculate the electric potential from the electric field and vice versa

Electric Potential Energy

- The electric force, like the gravitational force, is a conservative force
- Thus we can define an electric potential energy, *U*, in terms of the work done by the electric field, W_e, when a system changes its configuration from some initial configuration to some final configuration

Change in electric potential energy = -Work done by electric field

$$\Delta U = U_f - U_i = -W_e$$

U_i is the initial electric potential energy

U_f is the final electric potential energy

Electric Potential Energy (2)

- Like gravitational or mechanical potential energy, we must define a reference point from which to define the electric potential energy
- We define the electric potential energy to be zero when all charges are infinitely far apart
- We can then write a simpler definition of the electric potential taking the initial potential energy to be zero

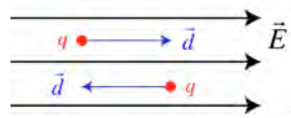
$$\Delta U = U_f - 0 = U = -W_e$$

- The negative sign on the work signifies that the electric force is doing work on the charges as they are brought in from infinity

Constant Electric Field

- Let's look at the electric potential when we move a charge *q* a distance *d* in a constant electric field
- The definition of work is

  $W = \vec{F} \bullet \vec{d}$



- For a constant electric field the force is

  $\vec{F} = q\vec{E}$

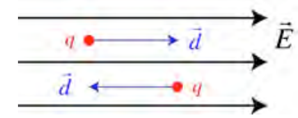
- So the work done by the electric field on the charge is

  $W = q\vec{E} \bullet \vec{d} = qEd\cos\theta$

  θ is the angle between the electric field and the displacement

Constant Electric Field - Special Cases

- Displacement is in the same direction as the electric field



  $W = qEd$

  $\Delta U = -W$

  - Charge loses potential energy when it moves in the direction of the electric field
- Displacement is in the opposite direction from the electric field

  $W = -qEd$

  $\Delta U = W$

  - Charge gains potential energy when it move in the opposite direction from the electric field

Definition of the Electric Potential

- The electric potential energy of a charged particle in an electric field depends not only on the electric field but on the charge of the particle
- We want to define a quantity to probe the electric field that is independent of the charge of the probe
- We define the electric potential as

  $$V = \frac{U}{q}$$

- Unlike the electric field, which is a vector, the electric potential is a scalar
  - The electric potential has a value everywhere in space but has no direction

Electric Potential Difference

- The electric potential difference between an initial point and final point f can be expressed in terms of the electric potential energy at each point

  $$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q}$$

- We can relate the change in electric potential to the work done by the electric field on the charge

  $$\Delta V = -\frac{W_e}{q}$$

Electric Potential Difference (2)

- Taking the electric potential energy to be zero at infinity we get

$$V = -\frac{W_{e,\infty}}{q}$$

where $W_{e,\infty}$ is the work done by the electric field on the charge as it is brought in from infinity

- The electric potential can positive, negative, or zero, but it does not have a direction
- The SI unit for electric potential is joules/coulomb

The Volt

- The commonly encountered unit joules/coulomb is called the volt, abbreviated V, after the Italian physicist Allesandro Volta (1745 - 1827)

$$1 \text{ V} = \frac{1 \text{ J}}{1 \text{ C}}$$

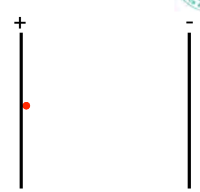
- With this definition of the volt, we can express the units of the electric field as

$$[E] = \frac{[F]}{[q]} = \frac{1 \text{ N}}{1 \text{ C}} = \left(\frac{1 \text{ N}}{1 \text{ C}}\right)\frac{1 \text{ V}}{\left(\frac{1 \text{ J}}{1 \text{ C}}\right)}\left(\frac{1 \text{ J}}{1 \text{ N} \cdot 1 \text{ m}}\right) = \frac{1 \text{ V}}{1 \text{ m}}$$

- For the remainder of our studies, we will use the unit V/m for the electric field

Example - Energy Gain of a Proton

- A proton is placed between two parallel conducting plates in a vacuum as shown. The potential difference between the two plates is 450 V. The proton is released from rest close to the positive plate.
- What is the kinetic energy of the proton when it reaches the negative plate?



The potential difference between the two plates is 450 V

We can relate the potential difference between the plates to the change in potential energy of the proton

$$\Delta V = \frac{\Delta U}{q}$$

Example - Energy Gain of a Proton (2)

We can relate the change in potential energy to the change in kinetic energy

$$\Delta K = \Delta U = q\Delta V$$

Because the proton started at rest

$$K = q\Delta V$$

$$K = \left(1.60 \cdot 10^{-19} \text{ C}\right)\left(450 \text{ V}\right) = 7.20 \cdot 10^{-17} \text{ J}$$

- Because the acceleration of a charged particle across a potential difference is often used in nuclear and high energy physics, the energy unit electron-volt (eV) is common
- An eV is the energy gained by a charge 1 particle accelerated across an electric potential of 1 volt

$$1 \text{ eV} = 1.6022 \cdot 10^{-19} \text{ J}$$

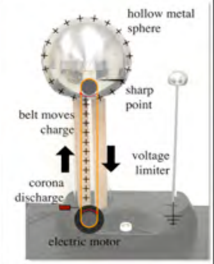
- The proton in this example would gain an energy of 450 eV = 0.450 keV

The Van de Graaff Generator

- One way to make a high electric potential is to use a Van de Graaff generator
- The Van de Graaff generator was invented by Robert J. Van de Graaff, an American physicist (1901 - 1967)
- Van de Graaff generators can produce electric potentials up to many 10s of millions of volts
- Van de Graaff generators can be used to produce particle accelerators
- We have been using a Van de Graaff generator in our lecture demonstrations and we will continue to use it, so here's how it works

The Van de Graaff Generator (2)

- The Van de Graaff generator works by applying a positive charge to a non-conducting moving belt using a corona discharge.
- The moving belt driven by an electric motor carries the charge up into a hollow metal sphere where the charge is taken from the belt by a pointed contact connected to the metal sphere.
- The charge that builds up on the metal sphere distributes itself uniformly around the outside of the sphere.
- For this particular Van de Graaff generator, a voltage limiter is used to keep the Van de Graaff generator from producing sparks larger than desired.



The Tandem Van de Graaff Accelerator

- One use of a Van de Graaff generator is to accelerate particles for condensed matter and nuclear physics studies
  - A Van de Graaff accelerator was used to test the Mars Pathfinder
- One particularly clever design is the tandem Van de Graaff accelerator
- In this design, a large positive electric potential is created by a huge Van de Graaff generator
- Negative ions are produced by adding an electron to a neutral atom
- The ions gain energy when they move from the ion source to terminal of the Van de Graaff
- Inside the terminal, the negative ions pass through a thin carbon foil where the electrons are removed from the ions, creating positively charged particles that are then gain energy again when they move away from the positive terminal

Example - Energy of Tandem Accelerator

- Suppose we have a tandem Van de Graaff accelerator that has a terminal voltage of 10 MV (10 million volts). We want to accelerate ^{12}C nuclei using this accelerator.
- What is the highest energy we can attain for carbon nuclei?
- What is the highest speed we can attain for carbon nuclei?
- There are two stages to the acceleration
  - The carbon ion with a -1e charge gains energy accelerating toward the terminal
  - The stripped carbon ion with a +6e charge gains energy accelerating away from the terminal


15 MV Tandem Van de Graaff at Brookhaven

Example - Energy of Tandem Accelerator (2)

$K = \Delta U = q_1 V + q_2 V$

$q_1 = 1e$

$q_2 = 6e$

$K = 7e \cdot V = 7 \cdot 1.602 \cdot 10^{-19}\ \text{C} \cdot 10 \cdot 10^{6}\ \text{V} = 1.12 \cdot 10^{-11}\ \text{J}$

$K = 7e \cdot 10 \cdot 10^{6}\ \text{V} = 7 \cdot 10^{7}\ \text{eV} = 70\ \text{MeV}\ \ (5.9\ \text{MeV/nucleon})$

The mass of a ^{12}C nucleus is $1.99 \cdot 10^{-26}$ kg

$$K = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \cdot 1.12 \cdot 10^{-11}\ \text{J}}{1.99 \cdot 10^{-26}\ \text{kg}}} = 3.36 \cdot 10^{7}\ \text{m/s}$$

$v = 11\%$ of the speed of light