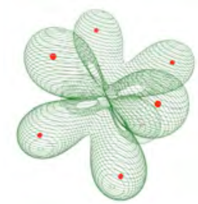
Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 9

Equipotential Surfaces and Lines

- When an electric field is present, the electric potential has a given value everywhere in space
- Points close together in space form an equipotential surface
- Charged particles can move along equipotential surfaces without having any work done on them by the electric field
- Equipotential surfaces exist in three dimensions
- We will often take advantage of symmetries in the electric potential and represent the equipotential surfaces as equipotential lines in a plane



Equipotential surface from eight point charges fixed at the corners of a cube

General Considerations

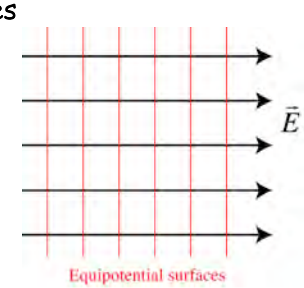
- Electric charges can move perpendicular to electric field lines without have any work done on them by the electric field because the scalar product of the electric field and the displacement is zero
- If the work done by the electric field is zero, then the electric potential must be constant

$$\Delta V = -\frac{W_e}{q} = 0 \Rightarrow V \text{ is constant}$$

- Thus equipotential surfaces and lines must always be perpendicular to the electric field lines

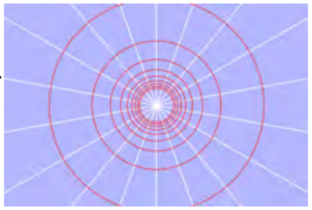
Constant Electric Field

- A constant electric field has straight, evenly space electric field lines
- The equipotential surfaces in the case of a constant electric field are equally spaced planes in three dimensions or equally spaced lines in two dimensions



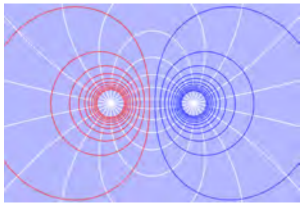
Electric Field from a Single Point Charge

- We have shown that the electric field lines from a single point charge are radial lines emanating from the point charge
- The equipotential surfaces for a point charge are concentric spheres in three dimensions and concentric circles in two dimensions



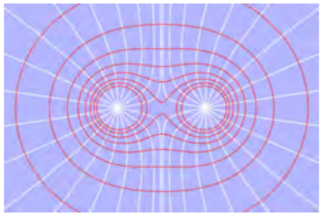
Electric Field from Two Oppositely Charged Point Charges

- The electric field lines from two oppositely charge point charges are more complicated
- The electric field lines originate on the positive charge and terminate on the negative charge
- The equipotential lines are always perpendicular to the electric field lines
- The red lines represent positive electric potential
- The blue lines represent negative electric potential
- Close to each charge, the equipotential lines resemble those from a point charge



Electric Field from Two Identical Point Charges

- The electric field lines from two identical point charges are also complicated
- The electric field lines originate on the positive charge and terminate on negative charge at infinity
- Again, the equipotential lines are always perpendicular to the electric field lines
- There are only positive potentials
- Close to each charge, the equipotential lines resemble those from a point charge



Calculating the Potential from the Field

- To calculate the electric potential from the electric field we start with the definition of the work *dW* done on a particle with charge *q* by a force *F* over a displacement *dS*

$$dW = \vec{F} \bullet d\vec{s}$$

- In this case the force is provided by the electric field *F* = *qE*

$$dW = q\vec{E} \bullet d\vec{s}$$

- Integrating the work done by the electric force on the particle as it moves in the electric field from some initial point *i* to some final point *f* we obtain

$$W = \int_i^f q\vec{E} \bullet d\vec{s}$$

Calculating the Potential from the Field (2)

- Remembering the relation between the change in electric potential and the work done

$$\Delta V = -\frac{W_e}{q}$$

- We get

$$\Delta V = V_f - V_i = -\frac{W_e}{q} = -\int_i^f \vec{E} \bullet d\vec{s}$$

- Taking the convention that the electric potential is zero at infinity we can express the electric potential in terms of the electric field as

$$V = \int_i^\infty \vec{E} \bullet d\vec{s}$$

Electric Potential of a Point Charge

- We define the electric potential of a point charge q in terms of the change in electric potential required to bring a positive test charge to a distance R from infinity in the presence of the electric field generated by the point charge.
- Remember that the electric field from a point charge q at a distance r is given by

$$E = \frac{kq}{r^2}$$

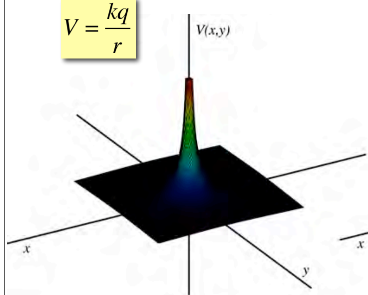
- The direction of the electric field from a point charge is always radial.
- Assuming that we integrate from a distance R from the point charge along a radial to infinity we obtain

$$V = \int_R^\infty \vec{E} \bullet d\vec{s} = \int_R^\infty \frac{kq}{r^2} dr = -\left[\frac{kq}{r}\right]_R^\infty = \frac{kq}{R}$$

Electric Potential of a Point Charge (2)

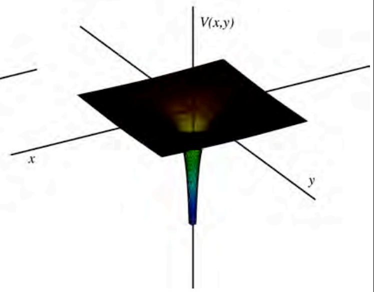
- The electric potential V from a point charge q at a distance r is then

$$V = \frac{kq}{r}$$



Positive point charge

Negative point charge



Electric Potential from a System of Charges

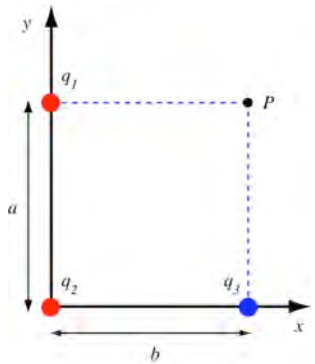
- We can calculate the electric potential from a system of n point charges by adding the potential from each charge at each point in space

$$V = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} \frac{kq_i}{r_i}$$

- This summation produces an electric potential at all points in space that has a value but no direction
- Calculating the electric potential from a group of point charges is usually much simpler than calculating the electric field

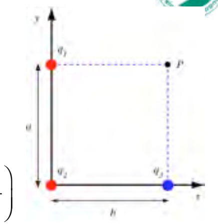
Example - Superposition of Electric Potential

- Assume we have a system of three point charges:
  q_1 = +1.50 μC
  q_2 = +2.50 μC
  q_3 = -3.50 μC.
- q_1 is located at (0,a)
  q_2 is located at (0,0)
  q_3 is located at (b,0)
  a = 8.00 m and b = 6.00 m.
- What is the electric potential at point P located at (b,a)?



Example - Superposition of Electric Potential (2)

- The electric potential at point P is given by the sum of the electric potential from the three charges



$$V = \sum_{i=1}^{3} \frac{kq_i}{r_i} = k\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}+\frac{q_3}{r_3}\right) = k\left(\frac{q_1}{b}+\frac{q_2}{\sqrt{a^2+b^2}}+\frac{q_3}{a}\right)$$

$$V = \left(8.99\cdot 10^{9}\ \text{N/C}\right)\left(\frac{1.50\cdot 10^{-6}\ \text{C}}{6.00\ \text{m}}+\frac{2.50\cdot 10^{-6}\ \text{C}}{\sqrt{(8.00\ \text{m})^2+(6.00\ \text{m})^2}}+\frac{-3.50\cdot 10^{-6}\ \text{C}}{8.00\ \text{m}}\right)$$

$$V = 562\ \text{V}$$

Calculating the Field from the Potential

- We can calculate the electric field from the electric potential starting with

$$V = -\frac{W_{e,\infty}}{q} \qquad dW = q\vec{E}\bullet d\vec{s}$$

- Which allows us to write

$$-qdV = q\vec{E}\bullet d\vec{s} \qquad \vec{E}\bullet d\vec{s} = -dV$$

- If we look at the component of the electric field along the direction of ds, we can write the magnitude of the electric field as the partial derivative

$$E_S = -\frac{\partial V}{\partial s}$$

Calculating the Field from the Potential (2)

- We can calculate any component of the electric field by taking the partial derivative of the potential along the direction of that component
- We can write the components of the electric field in terms of partial derivatives of the potential as

$$E_x = -\frac{\partial V}{\partial x};\ E_y = -\frac{\partial V}{\partial y};\ E_z = -\frac{\partial V}{\partial z}$$

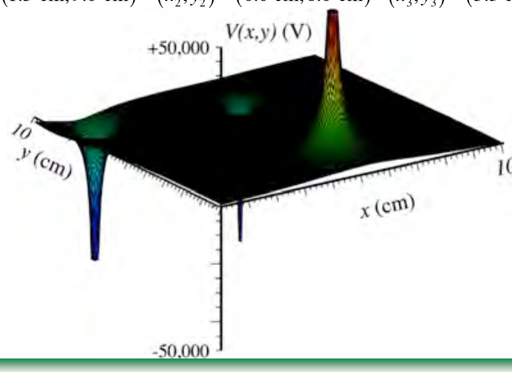
- In terms of graphical representations of the electric potential, we can get an approximate value for the electric field by measuring the gradient of the potential perpendicular to an equipotential line

Example - Graphical Extraction of the Field from the Potential

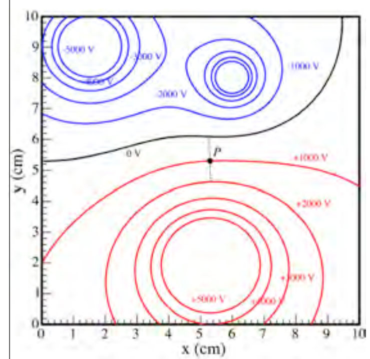
- Assume a system of three point charges

$q_1 = -6.00\ \mu\text{C}$ $\quad$ $q_2 = -3.00\ \mu\text{C}$ $\quad$ $q_3 = +9.00\ \mu\text{C}$

$(x_1, y_1) = (1.5\ \text{cm}, 9.0\ \text{cm})$ $\quad$ $(x_2, y_2) = (6.0\ \text{cm}, 8.0\ \text{cm})$ $\quad$ $(x_3, y_3) = (5.3\ \text{cm}, 2.0\ \text{cm})$



Example - Graphical Extraction of the Field from the Potential (2)



- We calculate the magnitude of the electric field at point P
- To perform this task, we draw a line through point P perpendicular to the equipotential line reaching from the equipotential line of 0 V to the line of 2000V
- The length of this line is 1.5 cm. So the magnitude of the electric field can be approximated as

$$|E_s| = \left| -\frac{\Delta V}{\Delta s} \right| = \left| \frac{(+2000\ \text{V}) - (0\ \text{V})}{1.5\ \text{cm}} \right| = 1.3 \cdot 10^5\ \text{V/m}$$

- The direction of the electric field points from the positive equipotential line to the negative potential line