Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 13

Review

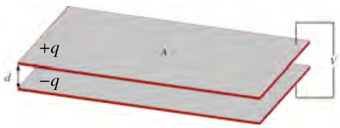
- Capacitors are devices that can store electrical energy
- The definition of capacitance is

$$C = \frac{q}{V}$$

  - C is the capacitance
  - q is the the charge on the capacitor
    - $+q$ on one plate
    - $-q$ on the other plate
  - V is the voltage across the plates
- The unit of capacitance is the farad (abbreviated F)

$$1\text{ F} = \frac{1\text{ C}}{1\text{ V}}$$

Review (2)



- The capacitance C of a parallel plate capacitor is given by

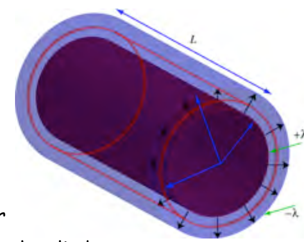
$$C = \frac{\varepsilon_0 A}{d}$$

- Where
  - A is the area of each plate
  - d is the distance between the plates
  - ε_0 is the electric permittivity of free space
- Note that this result for the capacitance of a parallel plate capacitor depends only on the geometry of the capacitor and not on the amount of charge or the voltage across the capacitor

Review (3)

- The capacitance of a cylindrical capacitor is

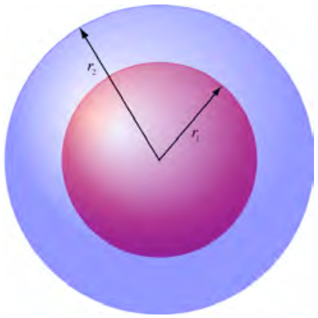
$$C = \frac{2\pi\varepsilon_0 L}{\ln\left(r_2 / r_1\right)}$$



  - L is the length of the cylinder
  - r_1 is the radius of the inner cylinder
  - r_2 is the radius of the outer cylinder
  - λ is the charge per unit length for both cylinders
  - $+q$ is the charge on the inner cylinder
  - $-q$ is the charge on the outer cylinder

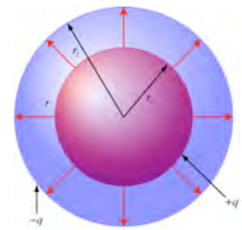
Spherical Capacitor

- Consider a spherical capacitor formed by two concentric conducting spheres with radii r_1 and r_2



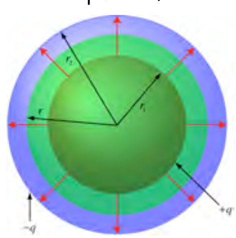
Spherical Capacitor (2)

- Let's assume that the inner sphere has charge +q and the outer sphere has charge -q
- The electric field is perpendicular to the surface of both spheres and points radially outward



Spherical Capacitor (3)

- To calculate the electric field, we use a Gaussian surface consisting of a concentric sphere of radius r such that $r_1 < r < r_2$



- The electric field is always perpendicular to the Gaussian surface so

$$\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = \varepsilon_0 EA = \varepsilon_0 E\left(4\pi r^2\right) = q$$

- Which reduces to

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

Spherical Capacitor (4)

- To get the electric potential we follow a method similar to the one we used for the cylindrical capacitor and integrate from the negatively charged sphere to the positively charged sphere

$$V = \int_{r_2}^{r_1} E dr = \int_{r_2}^{r_1} \frac{q}{4\pi\varepsilon_0 r^2} dr = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- Using the definition of capacitance we get

$$C = \frac{q}{V} = \frac{q}{\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} = \frac{4\pi\varepsilon_0}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

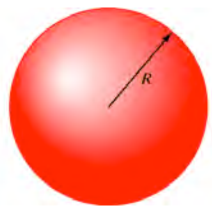
- The capacitance of a spherical capacitor is then

$$C = 4\pi\varepsilon_0 \frac{r_1 r_2}{r_2 - r_1}$$

Capacitance of an Isolated Sphere

- We can obtain the capacitance of a single conducting sphere by taking our result for a spherical capacitor and moving the outer spherical conductor infinitely far away
- Using our result for a spherical capacitor with $r_2 = \infty$ and $r_1 = R$ we get

$$C = 4\pi\varepsilon_0 R$$

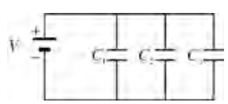


Capacitors in Circuits

- A circuit is a set of electrical devices connected with conducting wires
- Capacitors can be wired together in circuits in parallel or series
  - Capacitors in circuits connected by wires such that the positively charged plates are connected together and the negatively charged plates are connected together, are connected in parallel
  - Capacitors wired together such that the positively charge plate of one capacitor is connected to the negatively charged plate of the next capacitor are connected in series.

Capacitors in Parallel

- Consider an electrical circuit with three capacitors wired in parallel



- Each of three capacitors has one plate wired directly to the positive terminal of a battery with voltage V and one plate wired directly to the negative terminal
- The potential difference V across each capacitor is the same
- We can write the charge on each capacitor as

$$q_1 = C_1 V \quad q_2 = C_2 V \quad q_3 = C_3 V$$

Capacitors in Parallel (2)

- We can consider the three capacitors as one equivalent capacitor C_{eq} that holds a total charge q given by

$$q = q_1 + q_2 + q_3 = C_1 V + C_2 V + C_3 V = (C_1 + C_2 + C_3)V$$

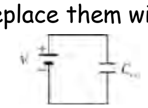
- We can now define C_{eq} as

$$q = C_{eq} V \qquad C_{eq} = C_1 + C_2 + C_3$$

- A general result for n capacitors in parallel is

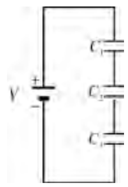
$$C_{eq} = \sum_{i=1}^{n} C_i$$

- If we can identify capacitors in a circuit that are wired in parallel, we can replace them with an equivalent capacitance



Capacitors in Series

- Consider a circuit with three capacitors wired in series
- The positively charged plate of C_1 is connected to the positive terminal of the battery
- The negatively charge plate of C_1 is connected to the positively charged plate of C_2
- The negatively charged plate of C_2 is connected to the positively charge plate of C_3
- The negatively charge plate of C_3 is connected to the negative terminal of the battery
- The battery produces an equal charge q on each capacitor because the battery induces a positive charge on the positive place of C_1, which induces a negative charge on the opposite plate of C_1, which induces a positive charge on C_2, etc.



Capacitors in Series (2)

- Knowing that the charge is the same on all three capacitors we can write

$$V = V_1 + V_2 + V_3 = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

- We can express an equivalent capacitance C_{eq} as

$$V = \frac{q}{C_{eq}} \qquad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

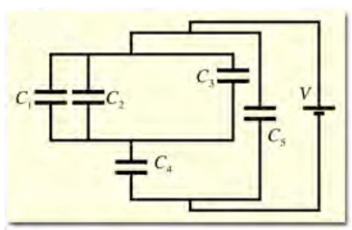
- We can generalize to n capacitors in series

$$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$$

- If we can identify capacitors in a circuit that are wired in series, we can replace them with an equivalent capacitance

Example - System of Capacitors

- Let's analyze a system of five capacitors



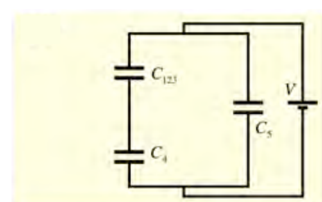
- If each capacitor has a capacitance of 5 nF, what is the capacitance of this system of capacitors?

System of Capacitors (2)

- We can see that C_1 and C_2 are in parallel and that C_3 is also in parallel with C_1 and C_2
- We can define

$$C_{123} = \sum_{i=1}^{3} C_i = C_1 + C_2 + C_3$$

- And make a new drawing

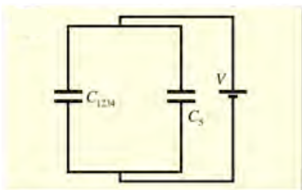


System of Capacitors (3)

- We can see that C_4 and C_{123} are in series
- We can define

$$\frac{1}{C_{1234}} = \frac{1}{C_{123}} + \frac{1}{C_4} \Rightarrow C_{1234} = \frac{C_{123}C_4}{C_{123}+C_4}$$

- And make a new drawing

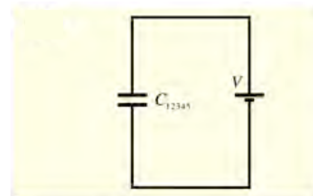


System of Capacitors (4)

- We can see that C_5 and C_{1234} are in parallel
- We can define

$$C_{12345} = C_{1234} + C_5 = \frac{C_{123}C_4}{C_{123}+C_4} + C_5 = \frac{(C_1+C_2+C_3)C_4}{C_1+C_2+C_3+C_4} + C_5$$

- And make a new drawing



System of Capacitors (5)

- We can now find the equivalent capacitance of our system of capacitors

$$C_{12345} = \left(\frac{(5+5+5)5}{5+5+5+5} + 5 \right) \text{ nF} = 8.75 \text{ nF}$$

- More than one half of the total capacitance of this arrangement is provided by C_5 alone
- This result makes it clear that one has to be careful how one arranges capacitors in circuits