# Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 14

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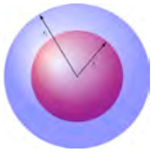
# Review

- The capacitance of a spherical capacitor is

$$C = 4\pi\varepsilon_0 \frac{r_1 r_2}{r_2 - r_1}$$

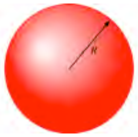
  - $r_1$ is the radius of the inner sphere
  - $r_2$ is the radius of the outer sphere



- The capacitance of an isolated spherical conductor is

$$C = 4\pi\varepsilon_0 R$$

  - *R* is the radius of the sphere



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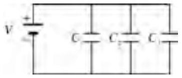
# Review (2)

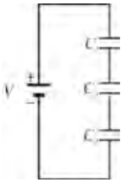
- The equivalent capacitance for *n* capacitors in parallel is

$$C_{eq} = \sum_{i=1}^{n} C_i$$



- The equivalent capacitance for *n* capacitors in series is

$$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$$



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# Energy Stored in Capacitors

- A battery must do work to charge a capacitor
- We can think of this work as changing the electric potential energy of the capacitor
- The differential work *dW* done by a battery with voltage *V* to put a differential charge *dq* on a capacitor with capacitance *C* is

$$dW = Vdq = \frac{q}{C}dq$$

- The total work required to bring the capacitor to its full charge q is

$$W_t = \int dW = \int_0^{q_t} \frac{q}{C}dq = \frac{1}{2}\frac{q_t^2}{C}$$

- This work is stored as electric potential energy

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}qV$$

## Energy Density in Capacitors

- We can define the energy density, *u*, as the electric potential energy per unit volume

$$u = \frac{U}{\text{volume}}$$

- Taking the special case of a parallel plate capacitor that has no fringe field, the volume between the plates is the area of each plate times the distance between the plates, *Ad*

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{CV^2}{2Ad}$$

- Inserting our formula for the capacitance of a parallel plate capacitor we get

$$u = \frac{\left(\frac{\varepsilon_0 A}{d}\right)V^2}{2Ad} = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2$$

## Energy Density in Capacitors (2)

- Recognizing that *V*/*d* is the magnitude of the electric field, *E*, we obtain an expression for the electric potential energy density for parallel plate capacitor

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$$

- This result, which is specific to the parallel plate capacitor, is in fact much more general
- This equation holds for all electric fields produced in any way
  - Can be used to describe the electric potential energy stored in an electric field per unit volume occupied by that field

## Example: Thundercloud

- Suppose a thundercloud with a width of 2.0 km and a length of 3.0 km hovers over a flat area, at an altitude of 500 m and carries a charge of 160 C.

- *Question 1:*
  - What is the potential difference between the cloud and the ground?
- *Question 2:*
  - Knowing that lightning strikes require electric field strengths of approximately 2.5 MV/m, are these conditions sufficient for a lightning strike?
- *Question 3:*
  - What is the total electrical energy contained in this cloud?

## Example: Thundercloud (2)

- *Question 1*
- We can approximate the cloud-ground system as a parallel plate capacitor whose capacitance is

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \cdot 10^{-12}\text{ F/m})(2000\text{ m})(3000\text{ m})}{500\text{ m}} = 0.11\ \mu\text{F}$$

- The charge carried by the cloud is 160 C, which means that each "plate" of the capacitor has a charge of 80 C

$$V = \frac{q}{C} = \frac{80\text{ C}}{0.11\ \mu\text{F}} = 7.2 \cdot 10^8\text{ V}$$

- 720 million volts!

# Example: Thundercloud (3)

- *Question 2*
- We know the potential difference between the cloud and ground so we can calculate the electric field

$$E = \frac{V}{d} = \frac{7.2 \cdot 10^8 \text{ V}}{500 \text{ m}} = 1.5 \text{ MV/m}$$

- Which is lower than 2.5 MV/m, so no lightning cloud to ground
  - May have lightning to radio tower or tree….
- *Question 3*
- The total energy stored in a parallel plate capacitor is

$$U = \frac{1}{2} qV = 0.5(80 \text{ C})(7.2 \cdot 10^8 \text{ V}) = 2.9 \cdot 10^{10} \text{ J}$$

  - Enough energy to run a 1500 W hair dryer for more than 5000 hours

# Capacitors with Dielectrics

- We have been discussing capacitors with air or vacuum between the plates
- However, most real-life capacitors have an insulating material, called a dielectric, between the two plates
- The dielectric serves several purposes:
  - Provides a convenient way to maintain mechanical separation between the plates
  - Provides electrical insulation between the plates
  - Allows the capacitor to hold a higher voltage
    - Increases the capacitance of the capacitor
    - Takes advantage of the molecular structure of the dielectric material

# Capacitors with Dielectrics (2)

- Placing a dielectric between the plates of a capacitor increases the capacitance of the capacitor by a numerical factor called the dielectric constant, $\kappa$
- We can express the capacitance of a capacitor with a dielectric with dielectric constant $\kappa$ between the plates as

$$C = \kappa C_{air}$$

- Where $C_{air}$ is the capacitance of the capacitor without the dielectric
- Placing the dielectric between the plates of the capacitor has the effect of lowering the electric field between the plates and allowing more charge to be stored in the capacitor.

# Parallel Plate Capacitor with Dielectric

- Placing a dielectric between the plates of a parallel plate capacitor modifies the electric field as

$$E = \frac{E_{air}}{\kappa} = \frac{q}{\kappa \varepsilon_0 A} = \frac{q}{\varepsilon A}$$

- The constant $\varepsilon_0$ is the electric permittivity of free space
- The constant $\varepsilon$ is the electric permittivity of the dielectric material

$$\varepsilon = \kappa \varepsilon_0$$

# Dielectric Strength

- The dielectric strength of a material measures the ability of that material to withstand voltage differences
- If the voltage across a dielectric exceeds the breakdown potential, the dielectric will break down and begin to conduct charge between the plates
- Real-life dielectrics enable a capacitor provide a given capacitance and hold the required voltage without breaking down
- Capacitors are usually specified in terms of their capacitance and rated voltage

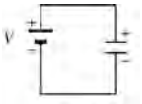
# Dielectric Constant

- The dielectric constant of vacuum is defined to be one
- The dielectric constant of is close to one and we will use the dielectric constant of air as one in our problems
- The dielectric constants and dielectric strengths of common materials are

| Material | Dielectric Constant κ | Dielectric Strength (kV/mm) |
|---|---|---|
| Air (1 atm) | 1.00059 | 2.5 |
| Polystyrene | 2.6 | 20 |
| Mylar | 3.1 | 280 |
| Paper | 3.0 | 8 |
| Water | 80.4 | 3.1 |

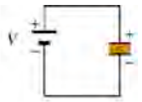
# Capacitor with Dielectric

- *Question 1:*
- Consider a parallel plate capacitor with capacitance $C$ = 2.00 μF connected to a battery with voltage V = 12.0 V as shown. What is the charge stored in the capacitor?



$$q = CV = \left(2.00 \cdot 10^{-6}\ \mathrm{F}\right)\left(12.0\ \mathrm{V}\right) = 2.40 \cdot 10^{-5}\ \mathrm{C}$$

- *Question 2:*
- Now insert a dielectric with dielectric constant $\kappa$ = 2.5 between the plates of the capacitor. What is the charge on the capacitor?



$C = \kappa C_{air}$ The capacitance of the capacitor is increased

$$q = \kappa CV = \left(2.50\right)\left(2.00 \cdot 10^{-6}\ \mathrm{F}\right)\left(12.0\ \mathrm{V}\right) = 6.00 \cdot 10^{-5}\ \mathrm{C}$$

The additional charge is provided by the battery

# Capacitor with Dielectric (2)

- *Question 3:*
- We isolate the charged capacitor with a dielectric by disconnecting it from the battery. We remove the dielectric, keeping the capacitor isolated.
- What happens to the charge and voltage on the capacitor?

- The charge on the isolated capacitor cannot change because there is nowhere for the charge to flow
- The voltage on the capacitor will be

$$V = \frac{q}{C} = \frac{6.00 \cdot 10^{-5}\ \mathrm{C}}{2.00 \cdot 10^{-6}\ \mathrm{F}} = 30.0\ \mathrm{V}$$

- The voltage went up because removing the dielectric increased the electric field and the resulting potential difference between the plates

# Capacitor with Dielectric (3)

- *Question 4:*
- Does removing the dielectric from the isolated capacitor change the energy stored in the capacitor?

- The energy stored in the capacitor before the dielectric was removed was

$$U=\frac{1}{2}CV^2=\frac{1}{2}\kappa C_{air}V^2=\frac{1}{2}(2.50)\left(2.00\cdot10^{-6}\ \mathrm{F}\right)(12\ \mathrm{V})^2=3.60\cdot10^{-4}\ \mathrm{J}$$

- After the dielectric is removed, the energy is

$$U=\frac{1}{2}C_{air}V^2=\frac{1}{2}\left(2.00\cdot10^{-6}\ \mathrm{F}\right)(30\ \mathrm{V})^2=9.00\cdot10^{-4}\ \mathrm{J}$$

- The energy increase results from the energy required to pull the dielectric out from between the plates of the capacitor