Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 16

Direct Current

- This week we will study charges in motion
- Electric charge moving from one region to another is called electric current
- Current is all around us
- Current is flowing through light bulbs, iPods, and lightning strikes
- Current consists of mobile electrons traveling in conducting materials
- Direct current is defined as a current that flows in one direction.

Electric Current

- We define the electric current i as the net charge passing a given point in a given time
- Random motion of electrons in conductors or the flowing of electrically neutral atoms are not current in spite of the fact that large amounts of charge are moving past a given point
- If net charge dq passes a point in time dt we define the current i to be

$$i = \frac{dq}{dt}$$

Electric Current (2)

- The amount of charge q passing a given point in time t is the integral of the current with respect to time given by

$$q = \int dq = \int_0^t i\,dt$$

- We will use charge conservation, implying that charge flowing in a conductor is never lost
- Therefore the same amount of charge must flow through one end of the conductor that exits from the other end of the conductor.

The Ampere

- The unit of current is coulombs per second, which has been given the unit ampere, named after French physicist André Ampère, (1775-1836)
- The ampere is abbreviated as A and is given by

$$1\ \text{A} = \frac{1\ \text{C}}{1\ \text{s}}$$

- Some typical currents are
  - Flashlight - 1 A
  - The starter in your car - 200 A
  - iPod - 50 mA
  - In a lightning strike (for a short time) - 100,000 A

Batteries

- In the following days we will make extensive use of batteries as devices that provide direct currents in circuits
- If you examine a battery, you will find its voltage written on it
- This voltage is the potential difference it can provide to a circuit
- You will also find their ratings in units of mAh
- This rating provides information on the total charge that they can deliver when fully charged
- The quantity mAh is another unit of charge:

$$1\ \text{mAh} = (10^{-3}\ \text{A})(3600\ \text{s}) = 3.6\ \text{As} = 3.6\ \text{C}$$

Current

- Current is a scalar
- Current has a sign but not a direction
- This week we will represent the direction of the current flowing in a conductor using an arrow
- This arrow represents whether the net current is positive or negative in a conductor at a given point but does not represent a direction in three dimensions
- Physically the charge carriers in a conductor are electrons that are negatively charged
- However, as is conventionally done, we define positive current as the net flow of positive charge carriers past a given point per unit time.

Current Density

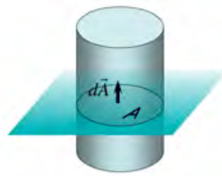
- Let's consider current flowing in a conductor
- Taking a plane through the conductor, we can define the current per unit area flowing through the conductor at that point as the current density $\vec{J}$
- We take the direction of $\vec{J}$ as the direction of the velocity of the charges crossing the plane
- If the cross sectional area is small, the magnitude of $\vec{J}$ will be large
- If the cross section area is large, the magnitude of $\vec{J}$ will be small.

Current Density (2)

- The current flowing through the surface is

$$i = \int \vec{J} \cdot d\vec{A}$$

- where is the differential area element perpendicular to the surface.
- If the current is constant and perpendicular to the surface, then and we can write an expression for the magnitude of the current density



$$J = \frac{i}{A}$$

Electron Drift Velocity

- In a conductor that is not carrying current, the conduction electrons move randomly
- When current flows through the conductor, the electrons still move randomly but with an added drift velocity, v_d
- The magnitude of the velocity of random motion is on the order of 10^6 m/s while the magnitude of the drift velocity is on the order of 10^{-4} m/s
- We can relate the current density J to the drift velocity v_d of the moving electrons

Electron Drift Velocity (2)

- Consider a conductor with cross sectional area *A* and electric field *E*
- Suppose that there are *n* electrons per unit volume.
- The negatively charged electrons will drift in a direction opposite to the electric field by definition
- We assume that all the electrons have the same drift velocity vd and that the current density J is uniform
- In a time interval *dt* each electron moves a distance $v_d t$
- The volume is then $Av_d t$ and the number of electrons is $nAv_d t$

Electron Drift Velocity (3)

- Each electron has charge *e* so that the charge *dq* that flows through the differential area in time *dt* is

$$dq = nev_d dt$$

- And the current is

$$i = \frac{dq}{dt} = nev_d A$$

- The current density is

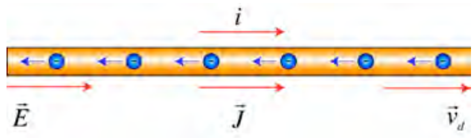
$$J = \frac{i}{A} = nev_d$$

- The current density and the drift velocity are parallel vectors, pointing in the same direction, and we can write

$$\vec{J} = (ne)\vec{v}_d$$

Electron Drift Velocity (4)

- Consider a wire carrying a current



- The physical current carriers are negatively charged electrons
- These electrons are moving to the left in this drawing
- However, the electric field, current density, drift velocity, and current are all to the right because of the convention that these quantities refer to positive charges

Resistance and Resistivity

- Some materials conduct electricity better than others
- If we apply a given voltage across a conductor, we get a large current
- If we apply the same voltage across an insulator, we get little current
- The property of a material that describes its ability to conduct electric currents is called the *resistivity*, ρ
- The property of a particular device or object that describes it ability to conduct electric currents is called the *resistance*, *R*

Resistance and Resistivity (2)

- If we apply an electric potential difference *V* across a conductor and measure the resulting current *i* in the conductor, we can define the resistance *R* of that conductor as

$$R = \frac{V}{i}$$

- The unit of resistance is volt per ampere
- In honor of German physicist George Simon Ohm (1789-1854) resistance has been given the unit ohm, Ω

$$1\ \Omega = \frac{1\ \text{V}}{1\ \text{A}}$$

Resistance and Resistivity (3)

- The resistance of a conductor can depend on the direction the current flows in the conductor
  - For example, semiconductors
- We will assume that the resistance of the device is uniform for all directions of the current.
- The resistance of a conductor depends on the material from which the conductor is constructed as well as the geometry of the conductor
- First we discuss the effects of the material of the conductor and then we will discuss the effects of geometry on resistance.

Resistivity

- The conducting properties of a material are characterized in terms of its resistivity
- We define the resistivity, ρ, of a material in terms of the magnitude of the applied electric field, E, and the magnitude of the resulting current density, J, as

$$\rho = \frac{E}{J}$$

- The units of resistivity are

$$\frac{\left(\frac{\text{V}}{\text{m}}\right)}{\left(\frac{\text{A}}{\text{m}^2}\right)} = \frac{\text{Vm}}{\text{A}} = \Omega\text{m}$$

Typical Resistivities

- The resistivities of some representative conductors at 20° C are listed in the table below

Material	Resistivity ρ (Ωm)	Resistivity ρ ($\mu\Omega\cdot$cm)
Silver	$1.59\cdot10^{-8}$	1.59
Copper	$1.72\cdot10^{-8}$	1.72
Gold	$2.44\cdot10^{-8}$	2.44
Aluminum	$2.82\cdot10^{-8}$	2.82
Nickel	$6.84\cdot10^{-8}$	6.84
Mercury	$95.8\cdot10^{-8}$	95.8

- *As you can see, typical values for the resistivity of conductors used in wires are on the order of 10^{-8} Ω*