Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 17

Review

- Electric current i is the net charge passing a given point in a given time

$$i = \frac{dq}{dt}$$

- The ampere is abbreviated as A and is given by

$$1\ \mathrm{A} = \frac{1\ \mathrm{C}}{1\ \mathrm{s}}$$

- The current per unit area flowing through a conductor is the current density

$$\vec{J}$$

Review (2)

- If the current is constant and perpendicular to the surface, then and we can write an expression for the magnitude of the current density

$$J = \frac{i}{A}$$

- The current density and the drift velocity are parallel vectors, pointing in the same direction, and we can write

$$\vec{J} = (ne)\vec{v}_d$$

- The property of a material that describes its ability to conduct electric currents is called the *resistivity, ρ*

Review (3)

- The property of a particular device or object that describes it ability to conduct electric currents is called the *resistance, R*
- The resistance R of that conductor is define as

$$R = \frac{V}{i}$$

- The unit of resistance is the ohm, Ω

$$1\ \Omega = \frac{1\ \mathrm{V}}{1\ \mathrm{A}}$$

Resistivity

- Knowing the resistivity of a material, we can then calculate the resistance of a conductor given its geometry
- For a homogeneous conductor of length L and constant cross sectional area A, we can relate the electric field and the electric potential as

$$E = \frac{V}{L}$$

- Remembering our definition for the current density

$$J = \frac{i}{A}$$

Resistivity

- Taking our definition for resistivity we obtain

$$\rho = \frac{E}{J} = \frac{\left(\frac{V}{L}\right)}{\left(\frac{i}{A}\right)} = \frac{V}{i}\frac{A}{L}$$

- Using the definition for resistance and rearranging terms we get an expression for the resistance of a conductor in terms of the resistivity of its constituent material, length, and cross sectional area

$$R = \rho \frac{L}{A}$$

Example: Resistance of a Copper Wire

- Standard wires that electricians put into residential housing have actually fairly low resistance
- *Question:*
- What is the resistance of a length of 100 m of standard 12-gauge copper wire, typically used in household wiring for electrical outlets?
- *Answer:*
- The American Wire Gauge (AWG) size convention specifies wire cross sectional area on a logarithmic scale.
- The higher the gauge number is, the thinner is the wire
- Every reduction by 3 gauges doubles the cross-sectional area

Example: Resistance of a Copper Wire (2)

- The formula to convert from the AWG size to the wire diameter is

$$d = 0.127 \cdot 92^{(36-AWG)/39} \text{ mm}$$

- So a 12-gauge copper wire has a diameter of 2.05 mm
- Its cross sectional area is then

$$A = \tfrac{1}{4}\pi d^2 = 3.3 \text{ mm}^2$$

- Using the value of the resistivity for copper we then find the resistance of the wire to be

$$R = \rho\frac{L}{A} = (1.72 \cdot 10^{-8} \ \Omega\text{m})\frac{100 \text{ m}}{3.3 \cdot 10^{-6} \text{ m}^2} = 0.52 \ \Omega$$

Resistors

- In many electronics applications one needs a range of resistances in various parts of the circuits
- For this purpose one can use commercially available resistors

- Resistors are commonly made from carbon, inside a plastic cover that looks like a medicine capsule, with two wires sticking out at the two ends for electrical connection
- The value of the resistance is indicated by four color-bands on the capsule
- The first two bands are numbers for the mantissa, the third is a power of ten, and the fourth is a tolerance for the range of values.

Resistors (2)

- The number associated with the colors are:
  - black = 0
  - brown = 1
  - red = 2
  - orange = 3
  - yellow = 4
  - green = 5
  - blue = 6
  - purple = 7
  - gray = 8
  - white = 9
- In the tolerance band
  - gold means 5%
  - silver means 10%
  - no tolerance band means 20%

For example, the single resistor shown here has colors (top to bottom)
brown, green, brown and gold
Using our table, we can see that the resistance is $15 \cdot 10^1\ \Omega = 150\ \Omega$
with a tolerance of 5%

Temperature Dependence of Resistivity

- The values of resistivity and resistance vary with the temperature
- For metals, this dependence on temperature is linear over a broad range of temperatures
- An empirical relationship for the temperature dependence of the resistivity of metals is given by

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$

  - ρ is the resistivity at temperature T
  - ρ_0 is the resistivity at temperature T_0
  - α is the temperature coefficient of electric resistivity for the material under consideration

Temperature Dependence of Resistance

- In everyday applications we are interested in the temperature dependence of the resistance of various devices
- The resistance of a device depends on the length and the cross sectional area
- These quantities depend on temperature
- However, the temperature dependence of linear expansion is much smaller than the temperature dependence of resistivity of a particular conductor
- We can then write the temperature dependence of the resistance of a conductor can be approximately written as

$$R - R_0 = R_0 \alpha (T - T_0)$$

Temperature Dependence

- Note that our equations for temperature dependence deal with relative temperatures so that one can use °C as well as K
- Values of α for representative conductors are shown below

Material	Temperature Coefficient of Resistivity, α (K^{-1})
Silver	$4.1 \cdot 10^{-3}$
Copper	$4.3 \cdot 10^{-3}$
Aluminum	$4.4 \cdot 10^{-3}$
Iron	$6.5 \cdot 10^{-3}$

Other Temperatue Dependence

- At very low temperatures the resistivity of some materials goes to exactly zero
- These materials are called superconductors
  - Many applications including MRI
- The resistance of some semiconducting materials actually decreases with increasing temperature
- These materials are often found in high-resolution detection devices for optical measurements or particle detectors
- These devices must be kept cold to keep their resistance high using refrigerators or liquid nitrogen.

Ohm's Law

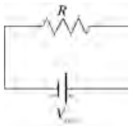
- To make current flow through a resistor one must establish a potential difference across the resistor
- This potential difference is termed an electromotive force, emf
- A device that maintains a potential difference is called an emf device and does work on the charge carriers
- The emf device not only produces a potential difference but supplies current
- The potential difference created by the emf device is termed V_{emf}
- We will assume that emf devices have terminals that we can connect and the emf device is assumed to maintain V_{emf} between these terminals

Ohm's Law (2)

- Examples of emf devices are
  - Batteries that produce emf through chemical reactions
  - Electric generators that create emf from mechanical motion
  - Solar cells that convert energy from the Sun to electric energy.
- In this chapter we will assume that the source of emf is a battery.
- A circuit is an arrangement of electrical components connected together with conducting wires with no resistance
- Electrical components can be sources of emf, capacitors, resistors, or other electrical devices
- We will begin with simple circuits that consist of resistors and sources of emf

Ohm's Law (3)

- Consider a simple circuit of the form shown below



- Here a source of emf provides a voltage *V* across a resistor with resistance *R*
- The relationship between the voltage and the resistance in this circuit is given by Ohm's Law

$$V_{emf} = iR$$

- where *i* is the current in the circuit

Ohm's Law (4)

- Now let's visualize the same circuit in a different way, making it clearer where the potential drop happens and what part of the circuit is at which potential
- The top part of this drawing is just our original circuit diagram
- In the bottom part we show the same circuit, but now the vertical dimension represents the voltage drop around the circuit
- The voltage is supplied by the source of emf and the entire voltage drop occurs across the single resistor.