Physics for Scientists & Engineers 2

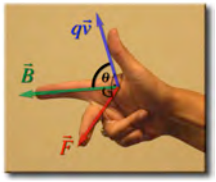
Spring Semester 2005

Lecture 21

Review

- The force that a magnetic field exerts on a charge moving with velocity v is given by

$$\vec{F}_B = q\vec{v} \times \vec{B}$$



- The magnitude of the force exerted by a magnetic field on a moving charge is

$$F_B = qvB\sin\theta$$

- If the charge moves perpendicular to the magnetic field then

$$F = qvB$$

Review (2)

- The unit of magnetic field strength the tesla (T)

$$1\ \mathrm{T} = 1\ \frac{\mathrm{Ns}}{\mathrm{Cm}} = 1\frac{\mathrm{N}}{\mathrm{Am}}$$

- Another unit of magnetic field strength that is often used but is not an SI unit is the gauss (G)

$$1\ \mathrm{G} = 10^{-4}\ \mathrm{T} \qquad 10\ \mathrm{kG} = 1\ \mathrm{T}$$

- Typically the Earth's magnetic field is about 0.5 G at the surface
- The NSCL K1200 superconducting cyclotron has a magnetic field of 5.5 T

Orbits in a Constant Magnetic Field

- Consider the situation in which you tie a string to a rock and twirl it at constant speed in a circle over your head
- The tension of the string provides the centripetal force that keeps the rock moving in a circle
- The tension on the string always points to the center of the circle and creates a centripetal acceleration
- A similar physical situation occurs when a particle with charge *q* and mass *m* moves with velocity *v* perpendicular to a uniform magnetic field *B*
- In this case the particle will move in a circle with a constant speed *v* and the magnetic force *F* = *qvB* will keep the particle moving in a circle

Orbits in a Constant Magnetic Field (2)

- For motion perpendicular to the magnetic field, the force required to keep the particle moving in a circle with radius *r* is the centripetal force

$$F = \frac{mv^2}{r}$$

- Setting this centripetal force equal to the magnetic force we obtain

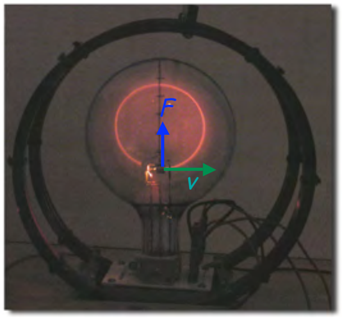
$$vBq = \frac{mv^2}{r}$$

- Rearranging we get an expression for the radius of the circle in which the particle is traveling

$$\boxed{r = \frac{mv}{qB}}$$

Example - Moving Electrons

- In this photo, an electron beam is accelerated by an electric field
- After acceleration, the electrons move in a circle perpendicular to the constant magnetic field created by a pair of Helmholtz coils



Is the magnetic field into the page or out of the page?

Remember that the magnetic force on an electron is opposite that on a proton

The magnetic field is out of the page

Mass Spectrometer

- One common application of moving charged particles in magnetic field is a mass spectrometer
- In a magnetic spectrometer the magnetic field is fixed and the radius of curvature of the trajectory of the particles is measured by various means such as a photographic plate or electronic detectors
- Remembering that the momentum of a particle is $p = mv$ we can write

$$\boxed{Br = \frac{p}{q}}$$

- The fixed magnetic field implies that all particles with the same radius of curvature have the same ratio of momentum to charge
- Thus by measuring the charge and the radius of the trajectory of a charged particle moving in a constant magnetic field, one can measure the momentum, or energy, of the particle

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Example: The Time Projection Chamber

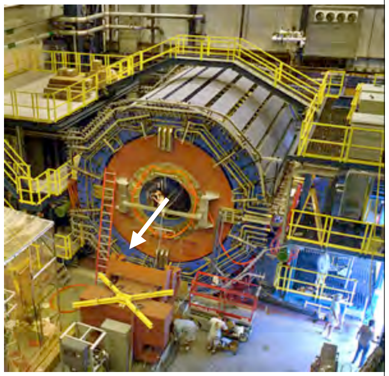
- In high-energy nuclear physics, new forms of matter are studied by colliding gold nuclei at very high energies
- In particle physics, new elementary particles are created and studied by colliding protons and anti-protons at the highest energies
- In these collisions, many particles are created that stream away from the interaction point at high speeds
- A simple particle detector is not sufficient to measure and identify these particles
- A device that can help physicists study these collisions is the time projection chamber (TPC).

Example: The Time Projection Chamber (2)

- The STAR TPC consists of a large cylinder filled a carefully chosen gas (90% argon, 10% methane) that allows free electrons to drift without recombining
- As created charged particles pass through the gas, the particles ionize the atoms of the gas, releasing free electrons
- An electric field is applied between the center of the TPC and the caps of the cylinder that exerts an electric force on these freed electrons, making them drift to the end-caps of the TPC, where they are recorded electronically
- Using the drift time and the recording positions, the computer software reconstructs the trajectories that the produced particles took through the TPC.

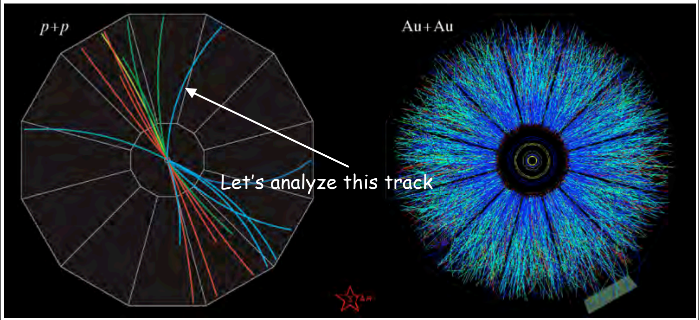
Example: The Time Projection Chamber (3)

- The TPC sits inside a giant solenoid magnet shown to the right with the magnetic field pointing along the beam direction
- The produced charged particles have a component of their velocity that is perpendicular to the magnetic field and thus have circular trajectories when viewed end-on
- From the radius of curvature one can extract the particles' momenta



Events in the TPC

- Here are two events in the STAR TPC at Brookhaven National Lab
- Magnetic field is directed into the screen

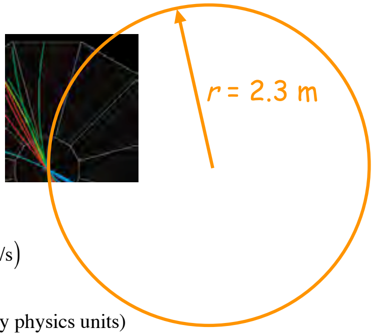


Example - Momentum of a Track

- Calculate the momentum of this track

$$Br = \frac{p}{q}$$
$$p = qBr$$
$$r = 2.3\ \text{m}$$
$$B = 0.50\ \text{T}$$
$$q = 1.6 \cdot 10^{-19}\ \text{C}$$
$$p = 1.8 \cdot 10^{-19}\ \text{kg m/s}$$

$$1\ \text{MeV} = 1.602 \cdot 10^{-13}\ \text{J}$$
$$pc = \left(1.8 \cdot 10^{-19}\ \text{kg m/s}\right)\left(3.0 \cdot 10^{8}\ \text{m/s}\right)$$
$$pc = 5.53 \cdot 10^{-11}\ \text{J}$$
$$p = 345\ \text{MeV/c (nuclear/high energy physics units)}$$



Orbits in a Constant Magnetic Field

- If a particle performs a complete circular orbit inside a constant magnetic field, then the period of revolution of the particle is just the circumference of the circle divided by the speed

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

- From the period we can get the frequency and angular frequency

$$f = \frac{1}{T} = \frac{qB}{2\pi m} \qquad \omega = 2\pi f = \frac{qB}{m}$$

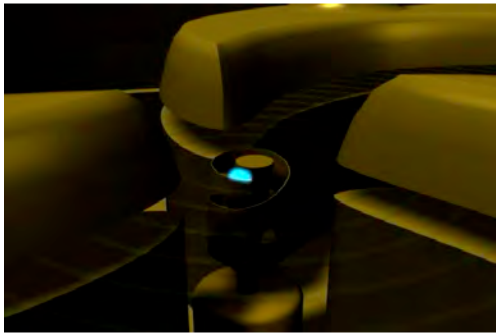
- The frequency of the rotation is independent of the speed of the particle
  - Isochronous orbits
    - Basis for cyclotrons

Cyclotrons

- A cyclotron is a particle accelerator
- The golden D-shaped pieces of metal in the upcoming video (descriptively called “dees”) have alternating electric potentials applied to them such that a positively charged particle always sees a negatively charged dee ahead when it emerges from under the previous dee, which is now positively charged
- The resulting electric field accelerates the particle
- Because the cyclotron sits in a strong magnetic field, the trajectory is curved
- The radius of the trajectory is proportional to the momentum, so the accelerated particle spirals outward.

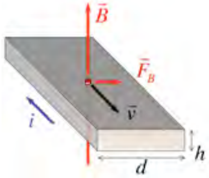
K500 Superconducting Cyclotron

- Movie from Nova program “The Nucleus Factory”



The Hall Effect

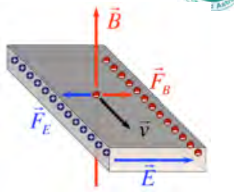
- Consider a conductor carrying a current *i* perpendicular to a magnetic field *B* as illustrated below



- The electrons in the conductor will be moving with a velocity in a direction opposite to the current
- The moving electrons experience a force perpendicular to their velocity, making the electrons move toward one edge of the conductor

The Hall Effect (2)

- After some time, many electrons move to one edge of the conductor leaving a net positive charge on the opposite edge of the conductor
- This charge distribution creates an electric field, *E*, that exerts a force on the electrons in a direction opposite to that exerted by the magnetic field
- When the magnitude of the force exerted on the electrons by the electric field is equal to the magnitude of the force exerted by the magnetic field, the net number of electrons no longer changes with time



The Hall Effect (3)

- The potential difference V_H between the edges of the conductor in the steady state is termed the Hall potential difference and is given by

  $$V_H = Ed$$
- where *d* is the width of the conductor and *E* is the magnitude of the created electric field.
- The Hall effect can be used to demonstrate that the charge carriers in metals are negatively charged
- If the charge carriers in a metal were positive and moving in the direction of the current shown previously, the positive charges would collect on the left edge of the conductor giving an electric field with an opposite sign
- Experimentally we observe that the charge carriers in conductors are negatively charged electrons.

The Hall Effect (4)

- The Hall effect can be used to measure magnetic fields by applying a known current in the conductor and measuring the resulting electric field across the conductor
- To obtain this result quantitatively we start by expressing the equal magnitudes of the magnetic and electric forces on an electron

  $$F_E = F_B \Rightarrow eE = vBe \Rightarrow B = \frac{E}{v} = \frac{V_H}{dv}$$
- Earlier we had found that the drift speed of an electron in a conductor can be related to the current density *J* in the strip

  $$J = \frac{i}{A} = nev$$
- where *A* is the cross sectional area of the conductor given by *A* = *dh* where *h* is the thickness of the conductor, and *n* is the number of electrons per unit volume in the conductor

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The Hall Effect (5)

- Solving for the drift velocity gives us

  $$v = \frac{i}{Ane} = \frac{i}{dhne}$$
- Which allows us to write an expression for the magnetic field

  $$B = \frac{V_H}{dv} = \frac{V_H dhne}{di} = \frac{V_H hne}{i}$$
- in terms of the measured Hall voltage V_H and knowing the height *h* and density of charge carriers *n* of the conductor

Example: Hall Effect

- Suppose we put a Hall probe into a constant magnetic field. The Hall probe is constructed of copper and has a width of 2.00 mm. We measure a voltage of 0.250 mV across the Hall probe when we run a current 1.25 A through the probe.
- What is the magnitude of the magnetic field?

The magnetic field is given by

$$B = \frac{V_H}{dv} = \frac{V_H dhne}{di} = \frac{V_H hne}{i}$$

Example: Hall Effect (2)

$$B = \frac{V_H}{dv} = \frac{V_H dhne}{di} = \frac{V_H hne}{i}$$

the density of electrons is calculated by

$$n = \frac{\text{\# electrons}}{\text{volume}}$$

$$\rho_{Cu} = 8.96\ \text{g/cm}^3 = 8960\ \text{kg/m}^3$$

$6.02 \cdot 10^{23}$ copper atoms have a mass of 63.5 g

Each copper atom has 1 free electron

$$n = \frac{1\ \text{electron}}{\text{atom}} \frac{6.02 \cdot 10^{23}\ \text{atom}}{63.5\ \text{g}} \frac{8.96\ \text{g}}{\text{cm}^3} \frac{1000\ \text{cm}^3}{\text{m}^3} = 8.49 \cdot 10^{25}\ \frac{\text{electrons}}{\text{m}^3}$$

$$B = \frac{0.00025\ \text{V} \cdot 0.002\ \text{m} \cdot 8.49 \cdot 10^{25}\ \frac{\text{electrons}}{\text{m}^3} \cdot 1.602 \cdot 10^{-19}\ \text{C}}{1.25\ \text{A}} = 5.44\ \text{T}$$