Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 22

Review

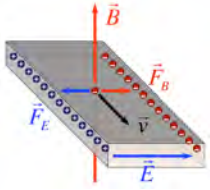
- A charged particle with charge *q* and mass *m* moving with speed *v* perpendicular to a constant magnetic field with magnitude *B* will travel in a circle with radius *r* given by

$$r = \frac{mv}{qB}$$

- For the same conditions we can relate the momentum *p* and the charge *q* to the magnitude of the magnetic field *B* and the radius *r* of the circular motion

$$Br = \frac{p}{q}$$

Review (2)



- If we run a current *i* through a conductor of width *h* in a constant magnetic field *B*, we induce a voltage V_H across the conductor that is given by

$$B = \frac{V_H}{dv} = \frac{V_H dhne}{di} = \frac{V_H hne}{i}$$

- where *n* is the number of electrons per unit volume and *e* is the charge of an electron
- Hall Effect

Magnetic Fields

- Let's address the problem of calculating magnetic fields generated by moving charges
- Remember that we calculated the electric field in terms of the electric charge using the form

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

- where *dq* is a charge element
- The electric field points radially from the electric charge so that we can write

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^3}\vec{r}$$

Magnetic Fields (2)

- The magnetic field has an added complication because the current that produces the magnetic field has a direction
- The electric field is produced by charge that is a scalar
- We can write the magnetic field produced by a current element *ids* as

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{s} \times \vec{r}}{r^3}$$

- This formula is the Biot-Savart Law
- μ_0 is the magnetic permeability of free space whose value is

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{Tm}}{\text{A}}$$

Magnetic Fields (3)

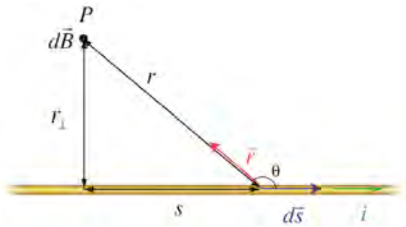
- The direction of the magnetic field produced by the current element is perpendicular to both the radial direction and to the current element
- The magnitude of the magnetic field is given by

$$dB = \frac{\mu_0}{4\pi}\frac{ids\sin\theta}{r^2}$$

- Where θ is the angle between the radial direction and the current element
- Let's calculate the magnetic field for various current element distributions

Magnetic Field from a Long, Straight Wire

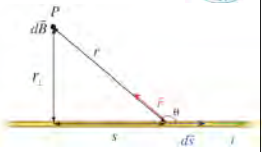
- For our first application, we will calculate the magnetic field from an infinitely long straight wire
- We calculate the magnetic field *dB* at a point *P* at a distance *r* from the wire as illustrated below



- The magnitude of the magnetic field will be given by the Biot-Savart Law and the direction will be out of the page

Magnetic Field from a Long, Straight Wire (2)

- We will calculate the magnetic field from the right half of the wire and multiply by two to get the magnetic field from the whole wire



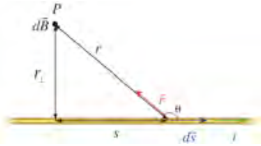
- The magnitude of the magnetic field from the right side of the wire is given by

$$B = 2\int_0^\infty dB = 2\int_0^\infty \frac{\mu_0}{4\pi}\frac{ids\sin\theta}{r^2} = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{ds\sin\theta}{r^2}$$

- We can relate *r*, *s*, and θ by

$$r = \sqrt{s^2 + r_\perp^2} \qquad \sin\theta = \frac{r_\perp}{\sqrt{s^2 + r_\perp^2}}$$

Magnetic Field from a Long, Straight Wire (3)



- Substituting we get

$$B = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{r_\perp ds}{\left(s^2 + r_\perp^2\right)^{3/2}}$$

- Carrying out this integral we get

$$B = \frac{\mu_0 i}{2\pi}\left[\frac{1}{r_\perp^2}\frac{r_\perp s}{\left(s^2+r_\perp^2\right)^{1/2}}\right]_0^\infty = \frac{\mu_0 i}{2\pi r_\perp}\left(\frac{s}{\left(s^2+r_\perp^2\right)^{1/2}}\right)_{s\to\infty} - 0$$

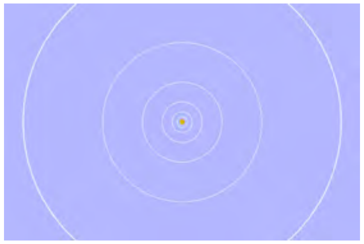
- So our resulting equation for the magnetic field at a perpendicular distance from a long, straight wire carrying a current is

$$\boxed{B(r_\perp) = \frac{\mu_0 i}{2\pi r_\perp}}$$

Magnetic Field from a Long, Straight Wire (4)

- The direction of the magnetic field is given by the right hand rule
- If you grab the wire such that your thumb points in the direction of the current, your fingers will point in the direction of the magnetic field as shown

- Looking down the wire, the magnetic field lines form circles



Magnetic Field from a Loop

- Let's calculate the magnetic field at the center of a circular loop of wire carrying current
- We start with

$$dB = \frac{\mu_0}{4\pi}\frac{ids\sin\theta}{r^2}$$

- and apply it to this case
- We can see that $r = R$ and $\theta = 90°$ for every current element along the loop
- For the magnetic field from each current element we get

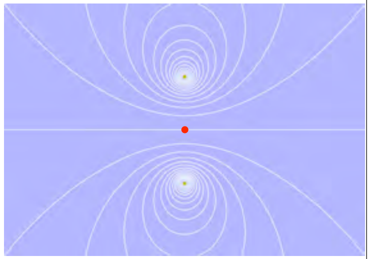
$$dB = \frac{\mu_0}{4\pi}\frac{ids\sin 90°}{R^2} = \frac{\mu_0}{4\pi}\frac{ids}{R^2}$$

Magnetic Field from a Loop (2)

- Going around the loop, we can relate an angle ϕ to the current element by $dS = Rd\phi$ allowing us to calculate the magnetic field at the center of the loop

$$\boxed{B = \int dB = \int_0^{2\pi}\frac{\mu_0}{4\pi}\frac{iRd\phi}{R^2} = \frac{\mu_0 i}{2R}}$$

- Please keep in mind that this calculation only gives us information on the value of the magnetic field at the center of the loop
- Using other techniques, we can calculate the magnetic field everywhere is shown to the right



Ampere's Law

- Recall that we can calculate the electric field resulting from any distribution of electric charge using

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^3}\vec{r}$$

- If the charge distribution were complicated, we would be faced with a difficult integral
- However, if the charge distribution had cylindrical or spherical symmetry, we could apply Gauss' Law

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q$$

- and obtain the electric field in an elegant manner

Ampere's Law (2)

- In a similar way we can calculate the magnetic field from an arbitrary distribution of current elements using

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{s} \times \vec{r}}{r^3}$$

- However, we again may be faced with a difficult integral
- In cases where the distribution of current elements has cylindrical or spherical symmetry, we can apply Ampere's Law to calculate the magnetic field from a distribution of current elements with much less effort than using a direct integration
- Ampere's Law is

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

- Where the integral is carried out around an Amperian loop and i_{enc} is the current enclosed by the loop

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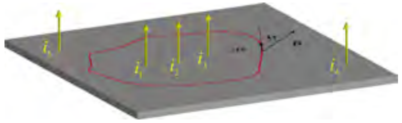
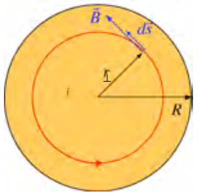
Ampere's Law (3)

- As an example of Ampere's Law, consider the five currents shown below



- The currents are perpendicular to the plane shown
- We can draw an Amperian loop represented by the red line
- This loop encloses currents i_1, i_2, and i_3 and excludes i_4 and i_5
- A direction of integration is shown above along with the resulting magnetic field
- The sign of the contributing currents can be determined using a right hand rule by pointing your fingers along the direction of integration and pointing your palm at the current
  - your thumb will indicate the positive direction

Magnetic Field inside Long Wire



- Consider a current i flowing out of the page in a wire with a circular cross section of radius R
- This current is uniformly distributed over the cross sectional area of the wire
- To find the magnetic field we use an Amperian loop with radius represented by the red circle
- The magnetic field is tangential to this Amperian loop so we can write the left side of Ampere's Law as

$$\oint \vec{B} \bullet d\vec{s} = B\oint d\vec{s} = B2\pi r_\perp$$

Magnetic Field inside Long Wire (2)

- The right hand side of Ampere's Law contains the enclosed current
- The enclosed current can be calculated from the ratio of the area of the Amperian loop to the cross sectional area of the wire

$$i_{enc} = i\frac{A_{loop}}{A_{wire}} = i\frac{\pi r_\perp^2}{\pi R^2}$$

- Equating the left and right sides we get

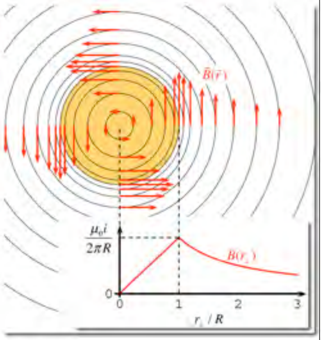
$$i\frac{r_\perp^2}{R^2} = B2\pi r_\perp$$

- or

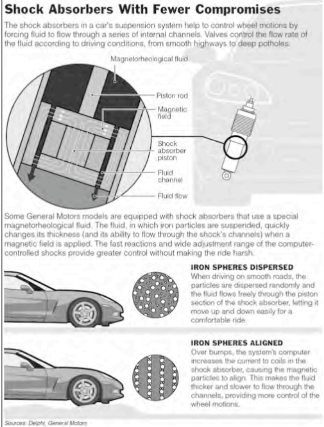
$$B(r_\perp) = \left(\frac{\mu_0 i}{2\pi R^2}\right) r_\perp$$

Magnetic Field inside Long Wire (3)

- We can look at the magnetic field as a function of the distance from the center of the wire, $r_\perp$
- The magnitude of the magnetic field varies linearly with the distance from the center of the wire until we reach the radius of the wire, R
- For distances from the center of wire greater than the radius of the wire, we get our previous result for a long straight wire (varies with the distance from the center)



Cool Technology with Magnetism



Shock Absorbers With Fewer Compromises

The shock absorbers in a car's suspension system help to control wheel motions by forcing fluid to flow through a series of internal channels. Valves control the flow rate of the fluid according to driving conditions, from smooth highways to deep potholes.

Some General Motors models are equipped with shock absorbers that use a special magnetorheological fluid. The fluid, in which iron particles are suspended, quickly changes its thickness (and its ability to flow through the shock's channels) when a magnetic field is applied. The fast reactions and wide adjustment range of the computer-controlled shocks provide greater control without making the ride harsh.

IRON SPHERES DISPERSED
When driving on smooth roads, the particles are dispersed randomly and the fluid flows freely through the piston section of the shock absorber, letting it move up and down easily for a comfortable ride.

IRON SPHERES ALIGNED
Over bumps, the system's computer increases the current to coils in the shock absorber, causing the magnetic particles to align. This makes the fluid thicker and slower to flow through the channels, providing more control of the wheel motions.

Sources: Delphi, General Motors

- New Shock Absorbers
- Stiffness regulated with magnetic fields that line up iron spheres!
- Suspension adjusts itself once per millisecond!
- http://www.nytimes.com/2005/02/21/automobiles/21CARS.html?ex=1109739600&en=740eded3bb7ad783&ei=5070