Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 23

Review

- μ_0 is the magnetic permeability of free space whose value is

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{Tm}}{\text{A}}$$

- The magnitude of the magnetic field at a distance r from a long, straight wire carrying currrent i is given by

$$B(r) = \frac{\mu_0 i}{2\pi r}$$

- The magnitude of the magnetic field at the center of a loop with radius R carrying current i is given by

$$B = \frac{\mu_0 i}{2R}$$

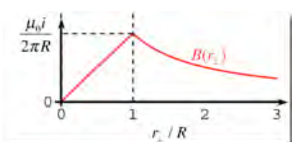
Review (2)

- Ampere's Law is

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

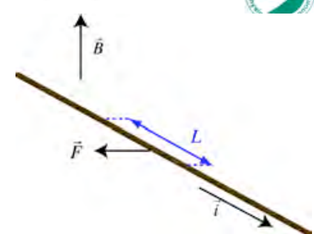
- where the integral is carried out around an Amperian loop and i_{enc} is the current enclosed by the loop
- The magnitude of the magnetic field inside a long wire with radius R carrying a current i at a radius $r_\perp$ is given by

$$B(r_\perp) = \left(\frac{\mu_0 i}{2\pi R^2}\right) r_\perp$$



Force on a Current Carrying Wire

- Consider a long, straight wire carrying carrying a current i in a constant magnetic field B
- The magnetic field will exert a force on the moving charges in the wire
- The charge q flowing in the wire in a given time t in a length L of wire is given by



$$q = ti = \frac{L}{v} i$$

- where v is the drift velocity of the electrons

Force on a Current Carrying Wire (2)

- The magnitude of the magnetic force is then

$$F = qvB\sin\theta = \left(\frac{L}{v}i\right)vB = iLB\sin\theta$$

- θ is the angle between the current and the magnetic field
- The direction of the force is perpendicular to both the current and the magnetic field and is given by the right hand rule
- This equation can be expressed as a vector cross product

$$\vec{F} = i\vec{L}\times\vec{B}$$

- iL represents the current in a length L of wire

Parallel Current Carrying Wires

- Consider the case in which two parallel wires are carrying current
- The two wires will exert a magnetic force on each other because the magnetic field of one wire will exert a force on the moving charges in the second wire
- The magnitude of the magnetic field created by a current carrying wire is given by

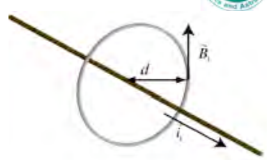
$$B(r) = \frac{\mu_0 i}{2\pi r}$$

- This magnetic field is always perpendicular to the wire with a direction given by the right hand rule.

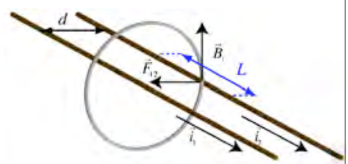
Parallel Current Carrying Wires (2)

- Let's start with wire one carrying a current i_1 to the right
- The magnitude of the magnetic field a distance d from wire one is

$$B_1 = \frac{\mu_0 i_1}{2\pi d}$$



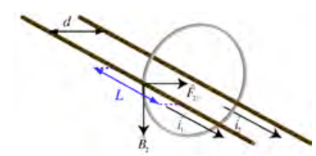
- Now consider wire two carrying a current i_2 in the same direction as i_1 placed a distance d from wire one
- The magnetic field due to wire one will exert a magnetic force on the moving charges in the current flowing in wire two



Parallel Current Carrying Wires (3)

- The charge q_2 flowing in wire two in a given time t in a length L of wire is given by

$$q_2 = ti_2 = \frac{L}{v}i_2$$



- where v is the drift speed of the charge carriers
- The magnetic force is then

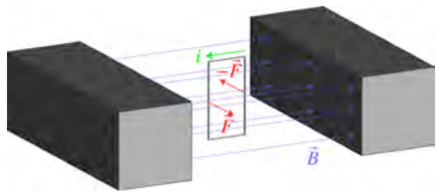
$$F = qvB = \left(\frac{L}{v}i_2\right)vB_1 = i_2LB_1$$

- Putting in our expression for B_1 we get

$$F_{12} = i_2L\left(\frac{\mu_0 i_1}{2\pi d}\right) = \frac{\mu_0 i_1 i_2 L}{2\pi d}$$

Torque on a Current-Carrying Loop

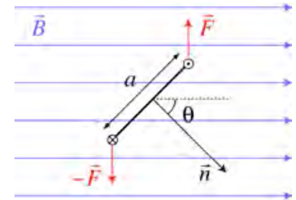
- Electric motors rely on the magnetic force exerted on a current carrying wire
- This force is used to create a torque that turns a shaft
- A simple electric motor is depicted below consisting of a single loop carrying current *i* in a constant magnetic field *B*



- The two magnetic forces, *F* and -*F*, shown in the figure are of equal magnitude and opposite direction
- These forces create a torque that tends to rotate the loop around its axis

Torque on a Current-Carrying Loop (2)

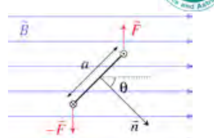
- As the coil turns in the field, the forces on the sides of the loop perpendicular to the magnetic field will change
- The forces on the square loop with sides are illustrated below where θ is the angle between a normal vector, *n*, and the magnetic field *B*



- The normal vector is perpendicular to the plane of the wire loop and points in a direction given by the right hand rule based on the current flowing in the loop

Torque on a Current-Carrying Loop (3)

- Here the current is flowing upward in the top segment and downward in the lower segment as illustrated by the arrow feathers and arrowhead



- The force each of the vertical segments is
  $F = iaB$
- The force on the other two sides is parallel or anti-parallel to the axis of rotation and cannot cause a torque
- The sum of the torque on the upper side plus the torque on the lower side gives the torque exerted on the coil about the center of the loop

$$\tau_1 = (iaB)\left(\frac{a}{2}\right)\sin\theta + (iaB)\left(\frac{a}{2}\right)\sin\theta = ia^2 B\sin\theta = iAB\sin\theta$$

- where $A = a^2$

Magnetic Dipole Moment

- If we replace this loop with *N* loops wound close together we can write

$$\tau = N\tau_1 = NiAB\sin\theta$$

- Although we derived this expression for a square loop, this express applies to circular loops as well as long as the magnetic field is uniform
- We can describe this coil with one parameter consisting of information about the coil only, combined with information about the magnetic field
- We define the magnitude of the magnetic dipole moment of the coil above to be

$$\mu = NiA$$

Magnetic Dipole Moment (2)

- The direction of the magnetic dipole moment, μ, is given by the right hand rule and points in the direction of the surface normal vector *n*

- We can rewrite our expression for the torque as

$$\tau = (NiA)B\sin\theta = \mu B\sin\theta$$

- which we can generalize to

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

- The torque will always be perpendicular the magnetic field magnetic dipole moment and the magnetic field

Potential Energy of a Magnetic Dipole

- A magnetic dipole has a potential energy in an external magnetic field
  - If the magnetic dipole is aligned with the magnetic field, it is in its minimum energy condition
  - If the magnetic dipole oriented in a direction opposite to the external field, the dipole is in its maximum energy condition
- The magnetic potential energy *U* of a magnetic dipole in an external magnetic field *B* can be written as

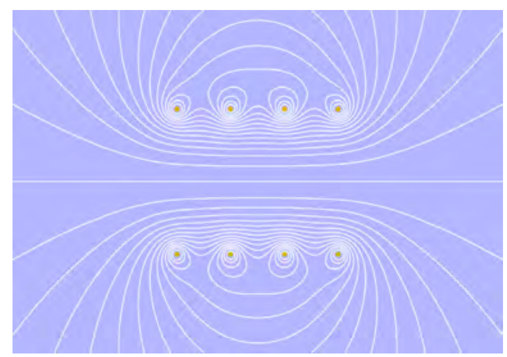
$$U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\theta$$

- where θ is the angle between the magnetic dipole moment and the external field.
- This potential energy of orientation can be applied to many physical situations concerning magnetic dipoles in external magnetic fields

Magnetic Fields of Solenoids

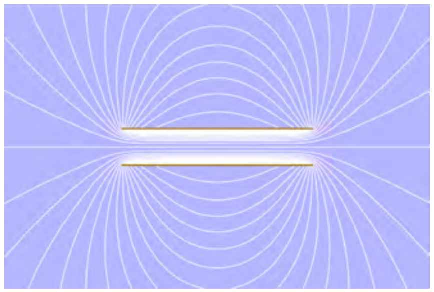
- Current flowing through a single loop of wire produces a magnetic field that is not very uniform
- Applications often require a uniform magnetic field
- A common first step toward a more uniform magnetic field is the Helmholtz coil
- A Helmholtz coil consists of two sets of coaxial wire loops
- Each set of coaxial loops acts like a single loop
- Carrying the idea of multiple loops one step farther, we could attempt to generate a constant magnetic field lines from four loops
- Let's look at the progression…

Magnetic Fields of Solenoids (2)



Magnetic Fields of Solenoids (3)

- To create a uniform magnetic field, a solenoid is used consisting of many loops wound close together
- Solenoids have many applications and are found in everyday life
- The magnetic field lines from a solenoid with 600 turns are shown below

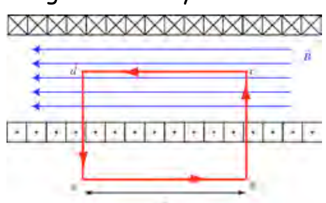


Ideal Solenoids

- The field from a real-life solenoid has fringe fields near the ends of the solenoid
  - The field is constant away from the ends of the solenoid
  - There is a small fringe field outside the solenoid near the ends of the solenoid
- An ideal solenoid is assumed to have a constant magnetic field B inside the solenoid and zero field outside the solenoid
- We will calculate the magnetic field of an ideal solenoid by applying Ampere's Law to a section of the solenoid far from the ends of the solenoid

Ideal Solenoids (2)

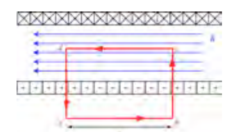
- We first define an Amperian Loop over which to carry out the required integral shown by the red line below



$$\oint \vec{B} \bullet d\vec{s} = \int_a^b \vec{B} \bullet d\vec{s} + \int_b^c \vec{B} \bullet d\vec{s} + \int_c^d \vec{B} \bullet d\vec{s} + \int_d^a \vec{B} \bullet d\vec{s}$$

$$\oint \vec{B} \bullet d\vec{s} = 0 + 0 + Bh + 0$$

Ideal Solenoids (3)



- The enclosed current is the current in the enclosed turns of the solenoid
- The current is the same in each turn
- Thus the enclosed current is

$$i_{enc} = nhi$$

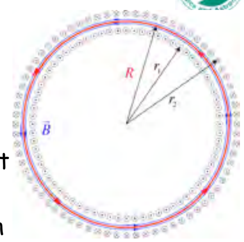
- where n is the number of turns per unit length
- The magnetic field inside an ideal solenoid is

$$B = \mu_0 in$$

- Note that this expression is only valid away from the ends of a real-world solenoid
- Note that there is no dependence on position inside the solenoid
  - An ideal solenoid creates a uniform magnetic field everywhere inside the solenoid and zero magnetic field outside the solenoid

Toroids

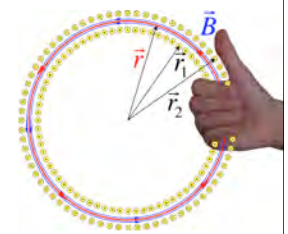
- One can create a toroidal magnet by "bending" a solenoid magnet such that the two ends meet as illustrated here



- The wire is wound around the doughnut shape forming a series of loops, each with the same current flowing through it
- Just like for the ideal solenoid, the magnetic field outside the coils of the ideal toroid is zero
- The magnetic field inside the toroid coil volume can be calculated by using Ampere's Law

Toroids (2)

- We assume an Amperian loop in the form of a circle with radius *r* such that $r_1 < r < r_2$
- The magnetic field is always directed tangential to the Amperian loop, so we can write
  $$\oint \vec{B} \bullet d\vec{s} = 2\pi r B$$
- The enclosed current is the number of turns *N* in the toroid times the current *i* in each loop, so Ampere's law gives us
  $$2\pi r B = \mu_0 N i$$
- So we find that the magnetic field of a toroid is given by
  $$B = \frac{\mu_0 N i}{2\pi r}$$



- Note that the magnitude of the electric field depends on r
- The direction is given by the right hand rule