Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 31

Electromagnetic Waves

- This week we will study electromagnetic waves
- We will see that light is an electromagnetic wave
- Electromagnetic waves have electric and magnetic fields
- We will see Maxwell's Equations that describe electromagnetic phenomena
- We will see that the speed of light is constant and can be related to ε_0 and μ_0
- We will see that electromagnetic waves can transport energy and momentum
- Electromagnetic waves can be polarized

Induced Magnetic Fields

- We have seen that a changing magnetic field induces and electric field
- Faraday's Law of Induction tells us

$$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$$

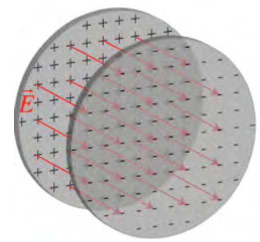
- In a similar manner, a changing electric field induces a magnetic field
- Maxwell's Law of Induction tells us

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

- where B is the magnetic field induced in a closed loop by a changing magnetic field by a changing electric flux Φ_E in that loop

Circular Capacitor

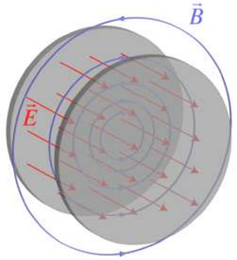
- To illustrate induced magnetic fields, consider the example of a circular parallel plate capacitor
- We charge the capacitor and disconnect the battery



- The charge is constant and the electric field between the plates is constant
- There is no magnetic field

Circular Capacitor (2)

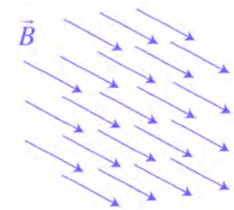
- Now let's increase the charge as a function of time

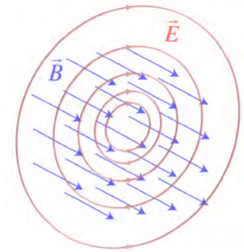


- A magnetic field is induced as indicated by the blue lines in the direction indicated
- The magnitude of the induced magnetic field is the same along each line and the direction is tangential to the line

Circular Capacitor (3)

- Now let's consider a constant magnetic field
- Now let's increase the magnitude of the magnetic while keeping the magnetic field uniform in space and in the same direction
- An electric field is induced as shown by the red loops
- The magnitude of the electric field is constant along each loop and the direction is tangential to each loop
- Note that the induced electric field points in the opposite direction from the magnetic field induced by a changing electric field





Circular Capacitor (4)

- We now recall Ampere's Law

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

- relating the integral around a loop of the dot product of the magnetic field and the integration direction to the current flowing through the loop
- We see that we can combine Maxwell's Law of Induction and Ampere's Law to produce a description of magnetic fields created by moving charges and by changing electric fields

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

- Which is called the Maxwell-Ampere Law (not surprisingly!)
  - For the case of constant current, such as current flowing in a conductor, this equation reduces to Ampere's Law
  - For the case of a changing electric field without current flowing, such as the electric field between the plates of a capacitor, this equation reduces to the Maxwell Law of Induction

Displacement Current

- Looking at the Maxwell-Ampere Law

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

- one can see that the quantity $\varepsilon_0 \frac{d\Phi_E}{dt}$ must have the units of current
- This term has been called the displacement current

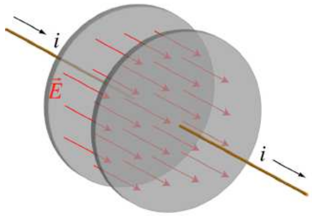
$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

- Note however that no actual current is being displaced
- We can then rewrite the Maxwell-Ampere Law as

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 (i_d + i_{enc})$$

Displacement Current (2)

- Now let's consider a parallel plate capacitor with circular plates as we did earlier
- We place the capacitor in a circuit in which a current *i* is flowing while the capacitor is charging



- For a parallel plate capacitor with area *A* we can relate the charge *q* to the electric field *E*

$$q = \varepsilon_0 AE$$

Displacement Current (3)

- We can get the current by taking the time derivative of the charge

$$i = \frac{dq}{dt} = \varepsilon_0 A \frac{dE}{dt}$$

- Assuming that the electric field between the plates of the capacitor is uniform we can obtain an expression for the displacement current

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d(AE)}{dt} = \varepsilon_0 A \frac{dE}{dt}$$

- The current in the circuit is the same as the displacement current i_d
- Although there is no actual current flowing between the plates of the capacitor in the sense that no actual charges flow across the capacitor gap from one plate to the other, we can use the concept of displacement current to calculate the induced magnetic field

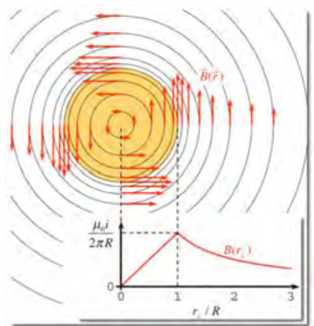
Displacement Current (4)

- To calculate the magnetic field between the two plates of the capacitor, we assume that the volume between the two plates can be replaced with a conductor of radius *R* carrying current i_d
- Thus from chapter 27 we know that the magnetic field at a distance from the center of the capacitor is given by

$$B = \left(\frac{\mu_0 i_d}{2\pi R^2}\right) r$$

- Outside the capacitor we can treat the system as a current-carrying wire
- The magnetic field is

$$B = \frac{\mu_0 i_d}{2\pi r}$$



Maxwell's Equations

- The Maxwell-Ampere Law completes the explanation of the four equations known as Maxwell's Equations that describe electromagnetic phenomena
- We have used these equations to describe electric fields, magnetic fields, and circuits
- We now will apply these equations to electromagnetic waves

Name	Equation	Description
Gauss' Law for Electric Fields	$\oint \vec{E} \bullet d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$	Relates the net electric flux to the net enclosed electric charge
Gauss' Law for Magnetic Fields	$\oint \vec{B} \bullet d\vec{A} = 0$	States that the net magnetic flux is zero (no magnetic charge)
Faraday's Law	$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$	Relates the induced electric field to the changing magnetic flux
Ampere-Maxwell Law	$\oint \vec{B} \bullet d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$	Relates the induced magnetic field to the changing electric flux and to the current

Wave Solutions to Maxwell's Equations

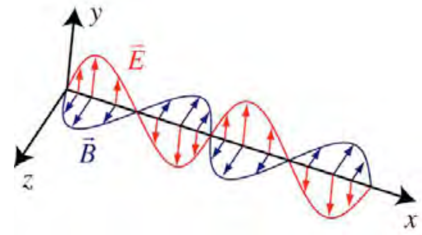
- It is possible to derive a general wave equation from Maxwell's Equations
- Here we will assume that electromagnetic waves propagating in vacuum (no moving charges or currents) have a certain form and show that this form satisfies Maxwell's Equations
- We will make the *Ansatz* that the magnitude of the electric and magnetic fields in electromagnetic waves are given by the form

$$E(\vec{r},t) = E_{\max} \sin\left(kx - \omega t\right)$$
$$B(\vec{r},t) = B_{\max} \sin\left(kx - \omega t\right)$$

- where $k = 2\pi/\lambda$ is the angular wave number and $\omega = 2\pi f$ is the angular frequency of a wave with wavelength λ and frequency f
- note that there is no dependence on the *y*- or *z*-coordinates, only on the *x*-coordinate and time
- This type of wave is called a plane wave

Wave Solutions to Maxwell's Equations (2)

- Implicit in our *Ansatz* is the result that our electromagnetic wave is traveling in the +*x* direction

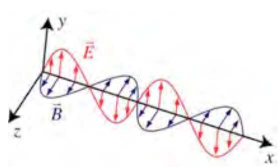


- We assume that the electric field is in the *y* direction and the magnetic field is in the *z* direction

$$\vec{E}(\vec{r},t) = E(\vec{r},t)\vec{e}_y$$
$$\vec{B}(\vec{r},t) = B(\vec{r},t)\vec{e}_z$$

Wave Solutions to Maxwell's Equations (3)

- The electric field is always perpendicular to the direction the electromagnetic wave is traveling and is always perpendicular to the magnetic field
- The electric and magnetic fields are in phase
- The wave shown is a snapshot in time
- The vectors shown represent the magnitude and direction for the electric and magnetic fields



- However, one must realize that these fields are not solid objects
- There is nothing actually moving left and right or up and down as the wave travels
- The vectors pointing left and right and up and down represent the abstract concepts of electric and magnetic fields
- Now we must show that our proposed solution satisfies Maxwell's Equations