Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 33

Midterm 2 Review

Review

- Electric current *i* is the net charge passing a given point in a given time

$$i = \frac{dq}{dt}$$

- The ampere is abbreviated as A and is given by

$$1\text{ A} = \frac{1\text{ C}}{1\text{ s}}$$

- The current per unit area flowing through a conductor is the current density

$$\vec{J}$$

Review (2)

- If the current is constant and perpendicular to the surface, then and we can write an expression for the magnitude of the current density

$$J = \frac{i}{A}$$

- The current density and the drift velocity are parallel vectors, pointing in the same direction, and we can write

$$\vec{J} = (ne)\vec{v}_d$$

- The property of a material that describes its ability to conduct electric currents is called the *resistivity, ρ*

Review (3)

- The property of a particular device or object that describes it ability to conduct electric currents is called the *resistance, R*
- The resistance *R* of that conductor is define as

$$R = \frac{V}{i}$$

- The unit of resistance is the ohm, Ω

$$1\ \Omega = \frac{1\text{ V}}{1\text{ A}}$$

Review (4)

- The resistance R of a device is given by

$$R = \rho \frac{L}{A}$$

  - ρ is resistivity of the material from which the device is constructed
  - L is the length of the device
  - A is the cross sectional area of the device
- The temperature dependence of the resistivity of metals is given by

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$

  - ρ is the resistivity at temperature T
  - ρ_0 is the resistivity at temperature T_0
  - α is the temperature coefficient of electric resistivity for the material under consideration

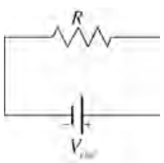
Review (5)

- The temperature dependence of the resistance of metals is given by

$$R - R_0 = R_0 \alpha (T - T_0)$$

  - R is the resistance at temperature T
  - R is the resistance at temperature T_0
  - α is the temperature coefficient of electric resistivity for the material under consideration
- Ohm's Law for a circuit consisting of a resistor and a battery is given by

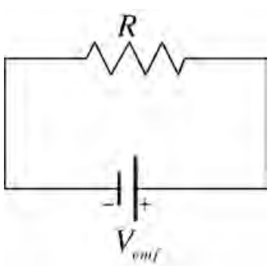
$$V_{emf} = iR$$

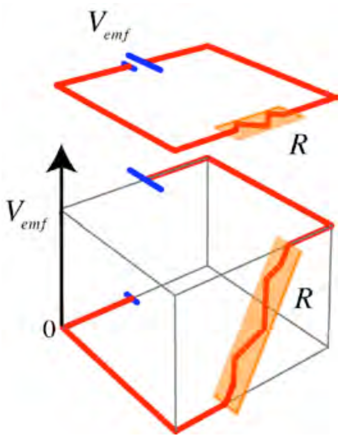


  - V_{emf} is the emf or voltage produced by the battery
  - i is the current
  - R is the resistance of the resistor

Review (6)

- We can visualize a circuit with a battery and a resistor in three dimensions





Review (7)

- n resistors in series can be replaced by an equivalent resistance given by the sum of the resistances of the resistors in series

$$R_{eq} = \sum_{i=1}^{n} R_i$$

- n resistors in parallel can be replaced by an equivalent resistance given by the sum of the resistances of the resistors in parallel

$$\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}$$

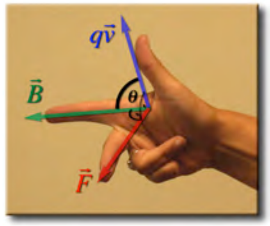
- The power dissipated in a circuit or circuit element is given by

$$P = iV = i^2 R = \frac{V^2}{R}$$

Review (8)

- The force that a magnetic field exerts on a charge moving with velocity v is given by

$$\vec{F}_B = q\vec{v} \times \vec{B}$$



- The magnitude of the force exerted by a magnetic field on a moving charge is

$$F_B = qvB\sin\theta$$

- If the charge moves perpendicular to the magnetic field then

$$F = qvB$$

Review (9)

- The unit of magnetic field strength the tesla (T)

$$1\ \mathrm{T} = 1\ \frac{\mathrm{Ns}}{\mathrm{Cm}} = 1\frac{\mathrm{N}}{\mathrm{Am}}$$

- *Another unit of magnetic field strength that is often used but is not an SI unit is the gauss (G)*

$$1\ \mathrm{G} = 10^{-4}\ \mathrm{T} \qquad 10\ \mathrm{kG} = 1\ \mathrm{T}$$

- Typically the Earth's magnetic field is about 0.5 *G* at the surface
- The NSCL K1200 superconducting cyclotron has a magnetic field of 5.5 T

Review (10)

- A charged particle with charge *q* and mass *m* moving with speed *v* perpendicular to a constant magnetic field with magnitude *B* will travel in a circle with radius *r* given by

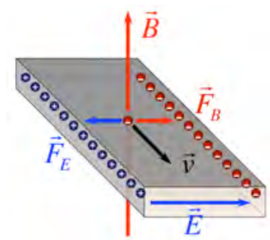
$$r = \frac{mv}{qB}$$

- For the same conditions we can relate the momentum *p* and the charge *q* to the magnitude of the magnetic field *B* and the radius *r* of the circular motion

$$Br = \frac{p}{q}$$

Review (11)

- If we run a current *i* through a conductor of width *h* in a constant magnetic field *B*, we induce a voltage V_H across the conductor that is given by



$$B = \frac{V_H}{dv} = \frac{V_H dhne}{di} = \frac{V_H hne}{i}$$

- where *n* is the number of electrons per unit volume and *e* is the charge of an electron
- Hall Effect

Review (12)

- μ_0 is the magnetic permeability of free space whose value is

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{Tm}}{\text{A}}$$

- The magnitude of the magnetic field at a distance *r* from a long, straight wire carrying currrent *i* is given by

$$B(r) = \frac{\mu_0 i}{2\pi r}$$

- The magnitude of the magnetic field at the center of a loop with radius *R* carrying current *i* is given by

$$B = \frac{\mu_0 i}{2R}$$

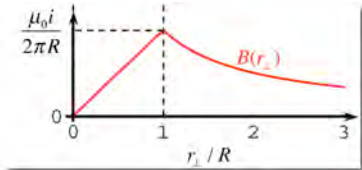
Review (13)

- Ampere's Law is

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

- where the integral is carried out around an Amperian loop and i_{enc} is the current enclosed by the loop
- The magnitude of the magnetic field inside a long wire with radius *R* carrying a current *i* at a radius $r_\perp$ is given by

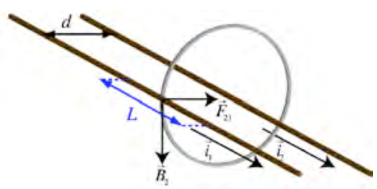
$$B(r_\perp) = \left(\frac{\mu_0 i}{2\pi R^2}\right) r_\perp$$



Review (14)

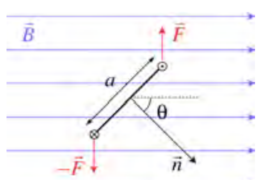
- The force between two current-carrying wires is given by

$$F_{12} = \frac{\mu_0 i_1 i_2 L}{2\pi d}$$



- The torque exerted by a magnetic field on a current-carrying loop is given by

$$\tau = iAB\sin\theta$$



Review (15)

- We define the magnitude of the magnetic dipole moment of a coil to be

$$\mu = NiA$$

- We can express the torque on a coil in a magnetic field as

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

- The magnetic potential energy of a magnetic dipole in a magnetic field is given by

$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos\theta$$

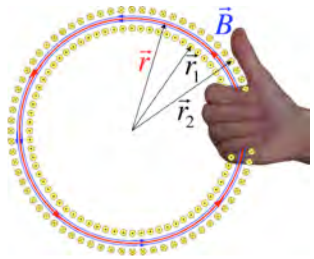
Review (16)

- The magnetic field inside an ideal solenoid is given by

$$B = \mu_0 i n$$

- The magnetic field inside an ideal toroidal magnet is given by

$$B = \frac{\mu_0 N i}{2\pi r}$$



Review (17)

- Faraday's Law of Induction in words is
  - *The magnitude of the V_{emf} induced in a conducting loop is equal to the time rate of change of the magnetic flux from the loop. This induced emf tends to oppose the flux change.*
- Faraday's Law of Induction in equation form is

$$V_{emf} = -\frac{d\Phi_B}{dt}$$

  - V_{emf} is the induced voltage
  - $d\Phi_B/dt$ is time rate change of the magnetic flux
  - The negative sign means that the induced voltage opposes the change in flux

Review (18)

- If we have a flat loop, we can keep two of the three variables (A,B,θ) constant, and vary the third, then we can have the following three special cases
  - We leave the area of the loop and its orientation relative to the magnetic field constant, but vary the magnetic field in time

    A,θ constant: $V_{emf} = -A\cos\theta \frac{dB}{dt}$
  - We leave the magnetic field as well as the orientation of the loop relative to the magnetic field constant, but change the area of the loop that is exposed to the magnetic field

    B,θ constant: $V_{emf} = -B\cos\theta \frac{dA}{dt}$
  - We leave the magnetic field constant and keep the area of the loop fixed as well, but allow the angle between the two to change as a function of time

    A, B constant: $V_{emf} = \omega AB\sin\theta$

Review (19)

- Lenz's law states that a current is induced in the loop that tends to oppose the change in magnetic flux
- The induced emf due to a changing magnetic field is given by

$$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$$

- The unit of inductance is the henry (H)

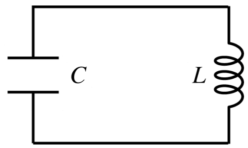
$$[L] = \frac{[\Phi_B]}{[i]} \Rightarrow 1\ \mathrm{H} = \frac{1\ \mathrm{Tm}^2}{1\ \mathrm{A}}$$

- The inductance of a solenoid of length l and area A with n turns per unit length an is given by

$$L = \mu_0 n^2 l A$$

Review (20)

- Consider a circuit consisting of an inductor L and a capacitor C



- The charge on the capacitor as a function of time is given by

$$q = q_{\max} \cos\left(\omega_0 t + \phi\right)$$

- The current in the inductor as a function of time is given by

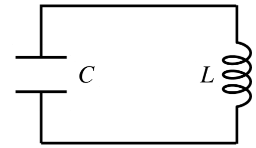
$$i = -i_{\max} \sin(\omega_0 t + \phi)$$

- where ϕ is the phase and ω_0 is the angular frequency

$$\omega_0 = \sqrt{\frac{1}{LC}} = \frac{1}{\sqrt{LC}}$$

Review (21)

- The energy stored in the electric field of the capacitor C as a function of time is



$$U_E = \frac{q_{\max}^2}{2C} \cos^2\left(\omega_0 t + \phi\right)$$

- The energy stored in the magnetic field of the inductor L as a function of time is

$$U_B = \frac{L}{2} i_{\max}^2 \sin^2(\omega_0 t + \phi)$$

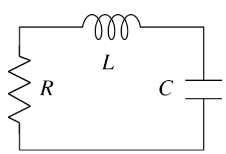
- The total energy stored in the circuit is given by

$$U = U_E + U_B = \frac{q_{\max}^2}{2C}$$

Review (22)

- If we have a single loop RLC circuit, the charge in the circuit as a function of time is given by

$$q = q_{\max} e^{-\frac{Rt}{2L}} \cos\left(\omega t + \phi\right)$$



- Where

$$\omega = \sqrt{\omega_0^2 - \left(\frac{R}{2L}\right)^2} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

- The energy stored in the capacitor as a function of time is given by

$$U_E = \frac{q_{\max}^2}{2C} e^{-\frac{Rt}{L}} \cos^2\left(\omega t + \phi\right)$$

Review (23)

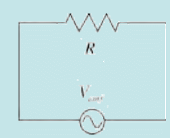
- Time-varying emf

$$V_{emf} = V_{\max} \sin \omega t$$

- Time-varying emf V_R with resistor

$$i_R = \frac{V_R}{R} = I_R \sin \omega t$$

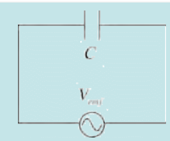
Resistance



- Time-varying emf V_C with capacitor

$$X_C = \frac{1}{\omega C} \qquad i_C = \frac{V_C}{X_C} \sin(\omega t + 90°)$$

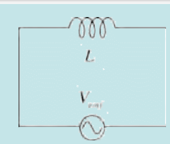
Capacitive Reactance



- Time-varying emf V_L with inductor

$$X_L = \omega L \qquad i_L = \frac{V_L}{X_L} \sin\left(\omega t - 90°\right)$$

Inductive Reactance



Review (24)