Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 35

Energy Transport

- The rate of energy transported by an electromagnetic wave is given by the Poynting vector

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

- The instantaneous power per unit area of the wave is given by

$$S = \left|\vec{S}\right| = \left(\frac{\text{power}}{\text{area}}\right)_{\text{instantaneous}}$$

- The intensity of the wave is given by the average power per unit area

$$I = S_{ave} = \left(\frac{\text{power}}{\text{area}}\right)_{ave}$$

Review (2)

- The energy in the electric and magnetic fields of the electromagnetic wave are equal

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = u_B = \frac{1}{2\mu_0}B^2$$

- The radiation pressure due to a totally absorbed electromagnetic wave is

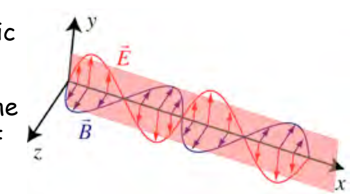
$$p_r = \frac{I}{c}$$

- The radiation pressure due to a reflected electromagnetic wave is just twice the absorbed value

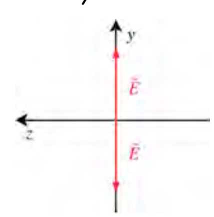
$$p_r = \frac{2I}{c}$$

Polarization

- Consider the electromagnetic wave shown
- The electric field for this electromagnetic wave always points along the *y*-axis
- Taking the *x*-axis as the direction that the wave is traveling, we can define a plane of oscillation for the electric field of the electromagnetic wave as shown

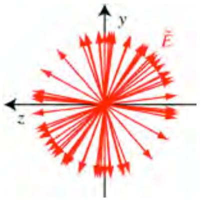


- This type of wave is called a plane-polarized wave in the *y* direction
- We can represent the polarization of an electromagnetic wave by looking at the electric field vector of the wave in the *x*-*z* plane, which is perpendicular to the direction the wave is traveling
    - The electric field oscillates in the *y*-plane



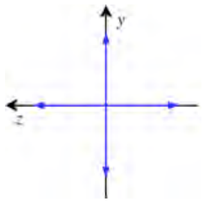
Polarization (2)

- The electromagnetic waves making up the light emitted by most common light sources such as an incandescent light bulb have random polarizations
- Each wave has its electric field vector oscillating in a different plane
- This light is called unpolarized light
- We can represent the polarization of the light from an unpolarized source by drawing many waves like the one shown on the previous page but with random orientations

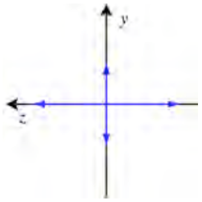


Polarization (3)

- We can represent light with many polarizations by summing the *y* components and summing the *z* components to produce the net y and *z* components
- For unpolarized light, we obtain equal components in the *y*- and *z*-directions



- If there is less net polarization in the *y* direction than in the *z* direction, then we say that the light is partially polarized in the *z* direction

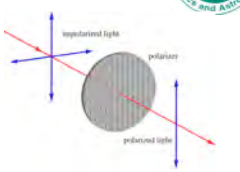


Polarization (4)

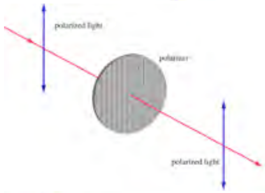
- We can change unpolarized light to polarized light by passing the unpolarized light through a polarizer
- A polarizer allows only one component of the polarization of the light to pass through
- One way to make a polarizer is to produce a material the consists of long parallel chains of molecules that effectively let components of the light pass with one polarization and block light with components perpendicular to that direction
- We will discuss polarizers without taking into account the details of the molecular structure
- Instead we will characterize each polarizer with a polarizing direction
- Unpolarized light passing through a polarizer will emerge polarized in the polarizing direction

Polarization (5)

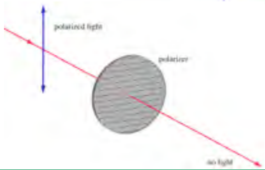
- The components of the unpolarized light that have same polarization as the polarizer are transmitted but the components of the light that are perpendicular to the polarizer are absorbed



- If polarized light with polarization parallel to the polarizing angle is incident on the polarizer, all the light passes through



- If polarized light with polarization perpendicular to the polarizing angle is incident on the polarizer, none of the light is transmitted



Polarization (6)

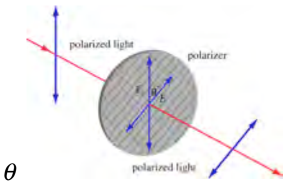
- Now let's consider the intensity of the light that passes through a polarizer
- We begin with unpolarized light with intensity I_0
- Unpolarized light has equal components of polarization in the *y* and *z* directions
- After passing through a vertical polarizer only the *y* component of the polarization remains
- The intensity *I* of the light passing through the polarizer is given by

$$I = \frac{1}{2} I_0$$

- because the unpolarized light had equal contribution from the *y* and *z* components and only the *y* components are transmitted by the vertical polarizer
- This factor of one half only applies to the case of unpolarized light passing through a polarizer

Polarization (7)

- Now let's assume that polarized light passes through a polarizer and that this light has a polarization that is not parallel or perpendicular to the polarizing direction of the polarizer



- The angle between the incident polarization is θ
- The component of the electric field *E* of the light that is transmitted is given by

$$E = E_0 \cos\theta$$

- where E_0 is the electric field of the incident polarized light
- The intensity of the light I_0 before the polarizer is given by

$$I_0 = \frac{1}{c\mu_0} E_{rms}^2 = \frac{1}{2c\mu_0} E_0^2$$

Polarization (8)

- After the light passes through the polarizer, the intensity *I* is given by

$$I = \frac{1}{2c\mu_0} E^2$$

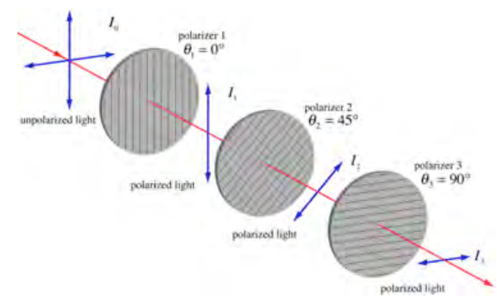
- The transmitted intensity in terms of the initial intensity is

$$I = \frac{1}{2c\mu_0} E^2 = \frac{1}{2c\mu_0} \left(E_0 \cos\theta\right)^2 = I_0 \cos^2\theta$$

- This result is called the Law of Malus
- This equation only applies to the case of polarized light incident on a polarizer
- Now we will do a specific example of the intensity of light passing through polarizers

Example: Three Polarizers

- Consider the case of unpolarized light with intensity I_0 incident on three polarizers



- The first polarizer has a polarizing direction that is vertical
- The second polarizer has a polarizing angle of 45° with respect to the vertical
- The third polarizer has a polarizing angle of 90° with respect to the vertical
- What is the intensity of the light passing through all the polarizers in terms of the initial intensity?

Example: Three Polarizers (2)

- The intensity of the unpolarized light is I_0
- The intensity of the light passing through the first polarizer is

$$I_1 = \frac{1}{2} I_0$$

- The intensity of the light passing the second polarizer is

$$I_2 = I_1 \cos^2(45^\circ - 0^\circ) = I_1 \cos^2(45^\circ) = \frac{1}{2} I_0 \cos^2(45^\circ)$$

- The intensity of the light passing the third polarizer is

$$I_3 = I_2 \cos^2(90^\circ - 45^\circ) = I_2 \cos^2(45^\circ) = \frac{1}{2} I_0 \cos^4(45^\circ) = I_0 / 8$$

- The fact that 1/8th of the intensity of the light is transmitted is somewhat surprising because polarizers 1 and 3 have polarizing angles that are perpendicular to each other
- The fact that polarizer 2 is in between these two polarizers allows light to pass through