Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 38

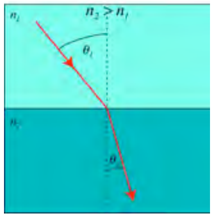
Review

- The index of refraction of an optical material, *n*, is given by

$$n = \frac{c}{v}$$

  - *c* is the speed of light in a vacuum
  - *v* is the speed of light in the optical material
  - $n \geq 1$, $n_{vacuum} = 1$
  - We will use $n_{air} = 1$
- The Law of Refraction or Snell's Law can be expressed as

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



Lenses

- When light is refracted crossing a curved boundary between two different media, the light rays follow the law of refraction at each point on the boundary
- The angle at which the light rays cross the boundary is different along the boundary, so the refracted angle is different at different points along the boundary
- A curved boundary between two optically transparent media is called a lens
- Light rays that are initially parallel before they strike the boundary are refracted in different directions depending on the part of the lens they strike
- Depending of the shape of the lens, the light rays can be focused or caused to diverge

Lens Maker Formula

- If the front surface of the lens is part of the surface of a sphere with radius R_1 and the back surface of the lens is part of the surface of a sphere with radius R_2, then we can calculate the focal length *f* of the lens using the lens-makers formula

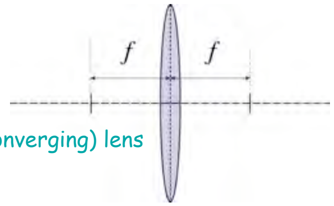
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

- Note that in this equation R_2 is negative because it has the opposite curvature from the front surface
- If we have a lens with the same radii on the front and back of the lens so that $R_1 = R_2 = R$, we get

$$\frac{1}{f} = \frac{2(n-1)}{R}$$

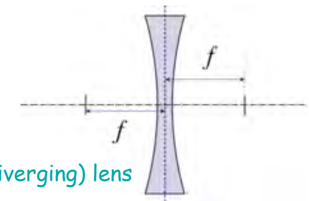
Lens Focal Lengths

- Unlike mirrors, lenses have a focal length on both sides
- Light can pass through lenses while mirrors reflect the light allowing no light on the opposite side



Convex (converging) lens

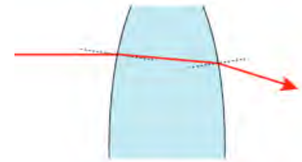
- The focal length of a convex (converging) lens is defined to be positive
- The focal length of a concave (diverging) lens is defined to be negative.

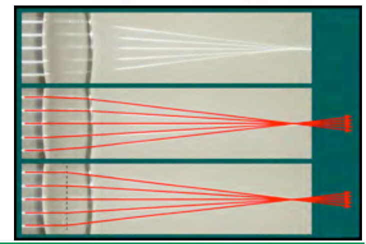


Concave (diverging) lens

Convex Lenses

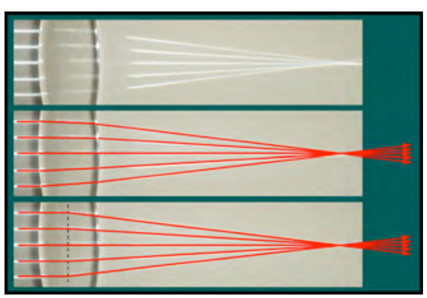
- A convex lens is shaped such that parallel rays will be focused by refraction at the focal distance *f* from the center of the lens
- In the drawing on the right, a light ray is incident on a convex glass lens
- At the surface of the lens, the light ray is refracted toward the normal
- When the ray leaves the lens, it is refracted away from the normal
- Let us now study the case of several horizontal light rays incident on a convex lens
- These rays will be focused to a point a distance *f* from the center of the lens on the opposite side from the incident rays





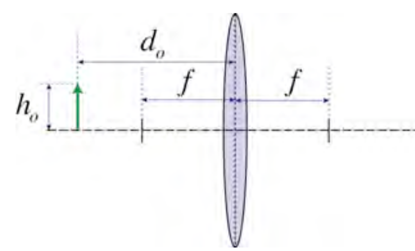
Thin Lenses

- The rays enter the lens, get refracted at the surface, traverse the lens in a straight line, and get refracted when they exit the lens
- In the third frame, we represent the thin lens approximation by drawing a black dotted line at the center of the lens
- Instead of following the detailed trajectory of the light rays inside the lens, we draw the incident rays to the centerline and then on to the focal point
- This real-life lens is a thick lens and there is displacement between the refraction at the entrance and exit surfaces
- We will treat all lenses as thin lenses and treat the lens as a line at which refraction takes place



Images with Convex Lenses

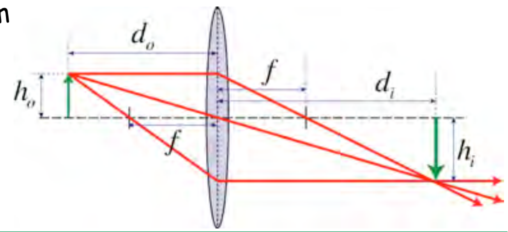
- Convex lenses can be used to form images
- We show the geometric construction of the formation of an image using a convex lens with focal length *f*



- We place an object standing on the optical axis represented by the green arrow
- This object has a height h_o and is located a distance d_o from the center of the lens such that $d_o > f$

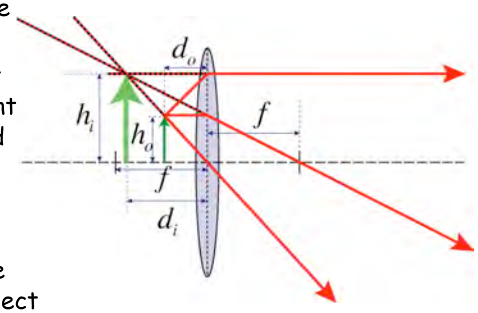
Images with Convex Lenses (2)

- We start with a ray along the optical axis of the lens that passes straight through the lens that defines the bottom of the image.
- A second ray is then drawn from the top of the object parallel to the optical axis
    - This ray is focused through the focal point on the other side of the lens
- A third ray is drawn through the center of the lens that is not refracted in the thin lens approximation
- A fourth ray is drawn from the top of the object through the focal point on the same side of the lens that is then directed parallel to the optical axis



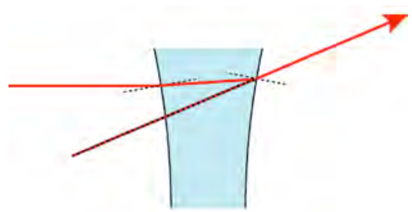
Images with Convex Lenses (3)

- Now let us consider the image formed by an object with height placed a distance from the center of the lens such that $d_o < f$
- The first ray again is drawn from the bottom of the object along the optical axis
- The second ray is drawn from the top of the object parallel to the optical axis and is focused through the focal point on the opposite side
- A third ray is drawn through the center of the lens
- A fourth line is drawn such that it originated from the focal point on the same side of the lens and is then focused parallel to the optical axis
    - These three rays are diverging
- A virtual image is located on the same side of the lens as the object



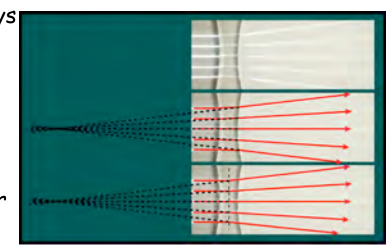
Concave Lenses

- A concave lens is shaped such that parallel rays will be caused to diverge by refraction such that their extrapolation would intersect at a focal distance from the center of the lens on the same side of the lens as the rays are incident
- Assume that a light ray parallel to the optical axis is incident on a concave glass lens
- At the surface of the lens, the light rays are refracted toward the normal
- When the rays leave the lens, they are refracted away from the normal as shown
- The extrapolated line shown as a red and black dashed line that points to the focal point on the same side of the lens as the incident ray



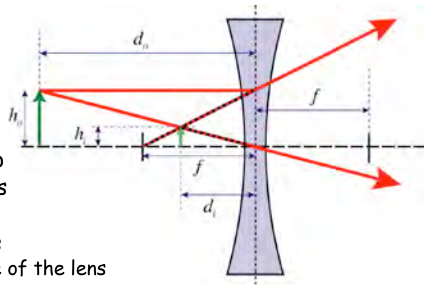
Concave Lenses (2)

- Let us now study several horizontal light rays incident on a concave lens
- After passing through the lens, the rays will diverge such that their extrapolations intersect at a point a distance *f* from the center of the lens on the same side of the lens as the incident rays
- To the right is a concave lens with five parallel lines of light incident in the surface of a concave lens from the left
- In the second panel we have drawn red lines representing light rays
- We can see that the light rays diverge after passing through the lens
- We have drawn red and black dashed lines to show the extrapolation of the diverging rays
- The extrapolated rays intersect a focal length away from the center of the lens
- In the third panel we draw the diverging rays using the thin lens approximation where the incident rays are drawn to the center of the lens



Images formed with Concave Lenses

- Here we show the formation of an image using a concave lens
- We place an object standing on the optical axis represented by the green arrow
- This object has a height h_o and is located a distance d_o from the center of the lens such that $d_o > f$
- We again start with a ray along the optical axis of the lens that passes straight through the lens that defines the bottom of the image
- A second ray is then drawn from the top of the object parallel to the optical axis
  - This ray is refracted such that its extrapolation of the diverging ray passes through the focal point on the other side of the lens
- A third ray is drawn through the center of the lens that is not refracted in the thin lens approximation
  - This ray is extrapolated back along its original path
- The image formed is virtual, upright, and reduced



The Lens equation

- The images formed by lenses are described by the lens equation

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

- This equation is the same relationship between focal length, image distance, and object distance that we had found for mirrors
- To treat all possible cases for lenses, we must define some conventions for distances and heights
  - We define the focal length f of a convex lens to be positive and the focal length of a concave lens to be negative
  - We define the object distance d_o to be positive
  - If the image is on the opposite side of the lens from the object, the image distance d_i is positive and the image is real
  - If the image is on the same side of the lens as the object, the image distance d_i is negative and the image is virtual
  - If the image is upright, then h_i is positive and if the image is inverted, h_i is negative

Special Cases

- For a convex lens, we find that for $d_o > f$ we always get a real, inverted image formed on the opposite side of the lens
- For a convex lens and $d_o < f$, we always get a virtual, upright, and enlarged image on the same side of the lens as the object
- The special cases for $d_o > f$ for a convex lens are

Case	Type	Direction	Magnification
$f < d_o < 2f$	Real	Inverted	Enlarged
$d_o = 2f$	Real	Inverted	Same size
$d_o > 2f$	Real	Inverted	Reduced

- For concave lenses, we always get an image that is virtual, upright, and reduced in size

Magnification for Lenses

- The magnification m of a lens is defined the same as as for a mirror

$$m = -\frac{d_i}{d_o} = -\frac{h_i}{h_o}$$

  - d_o is the object distance
  - h_o is the object height
  - d_i is the image distance
  - h_i is the image height
- If $|m| > 1$, the image is enlarged
- If $|m| < 1$, the image is reduced
- If $m < 0$, the image is inverted
- If $m > 0$, the image is upright

Power of Lenses

- The power of a lens is often quoted rather than its folca length
- The power of a lens, D (diopters), is given by the equation

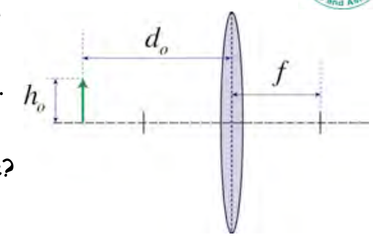
$$D = \frac{1\text{ m}}{f}$$

- For example, common reading glasses have a power of D = 1.5 diopters
- The focal length of these glasses is

$$f = \frac{1\text{ m}}{1.5\text{ diopters}} = 0.67\text{ m}$$

Example: Image formed by Convex Lens

- The focal length of a converging lens is 21 cm. An object is located a distance d_o = 32 cm from the center of the lens.
- Where is the image?
- What is the magnification of the image?



$$\text{convex lens} \quad \Rightarrow \quad f \text{ is positive}$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \Rightarrow \quad d_i = \frac{d_o f}{d_o - f}$$

$$d_i = \frac{(0.320\text{ m})(0.210\text{ m})}{(0.320\text{ m}) - (0.210\text{ m})} = 0.61\text{ m}$$

image will be located 61 cm to the right of the lens

$$m = -\frac{d_i}{d_o} = -\frac{h_i}{h_o} = -\frac{0.61\text{ m}}{0.21\text{ m}} = -2.9 \quad \Rightarrow \quad \text{enlarged, inverted}$$