



Physics for Scientists & Engineers 2

Spring Semester 2005
Lecture 41

Light as Waves



- In the previous chapter we discussed light as rays
- These rays traveled in a straight line except when they were reflected off a mirror or were refracted at the boundary between two optical media
- Now we will discuss the implications of the wave nature of light
- We know that light is an electromagnetic wave
- We normally do not think of light as a wave, because its wavelength is so short that we usually do not notice this wave behavior

Wave Optics

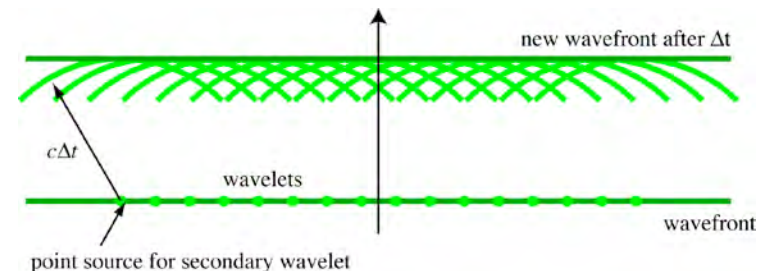


- One way to reconcile the wave nature of light with the geometric optical properties of light is to use **Huygens' Principle** developed by Christian Huygens, a Dutch physicist who proposed a wave theory of light in 1678 before Maxwell developed his theories of light
- This principle states that every point on a propagating wave front serves as a source of spherical secondary wavelets
- At a later time, the envelope of these secondary waves becomes a wave front
- If the original wave has frequency f and speed v , the secondary wavelets have the same f and v

Huygen's Constructions



- Diagrams of phenomena based on Huygens' Principle are called Huygens' constructions
- A Huygens' construction for a light wave traveling in a straight line is shown below



Huygen's Constructions (2)

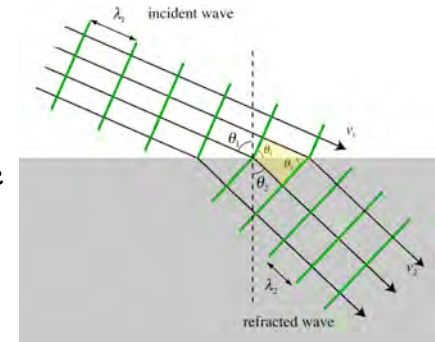


- We start with a wave front traveling at the speed of light c
- We assume point sources of spherical wavelets along the wave front
- These wavelets also travel at c so at a time Δt the wavelets have traveled a distance of $c\Delta t$
- If we assume many point sources along the wave front, we can see that the envelope of these wavelets forms a new wave front parallel to the original wave front
- Thus the wave continues to travel in a straight line with the original frequency and speed

Derivation of Snell's Law



- Now let's use a Huygens' construction to derive Snell's Law for refraction between two optical media with different indices of refraction
- Assume that we have a wave with wave fronts separated by a wavelength λ_1 traveling with speed v_1 in an optically clear medium incident on the boundary with a second optically clear medium as shown



Derivation of Snell's Law (2)



- The angle of the incident wave front with respect to the boundary is θ_1 , which is also the angle the direction of the wave makes with a normal to the boundary
- When the wave enters the second medium it travels with speed v_2
- According to Huygens' Principle the wave fronts are the result of wavelet propagation at the speed of the original wave so we can write the separation of the wave fronts in the second medium in terms of the wavelength in the second medium λ_2

Derivation of Snell's Law (3)



- Thus the time interval between wave fronts for the first medium is λ_1/v_1 and the time interval for the second medium is λ_2/v_2
- This time interval is the same at boundary so we can write

$$\frac{\lambda_1}{v_1} = \frac{\lambda_2}{v_2}$$

- which we can rewrite as

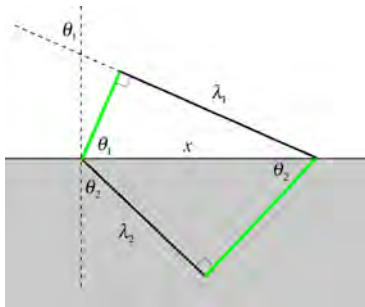
$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

- The wavelengths of the light in the two media are proportional to the speed of light in those media

Derivation of Snell's Law (4)



- We can get a relation between the angle of the incident wave fronts θ_1 with the boundary and the angle of the transmitted wave fronts θ_2 with the boundary by analyzing an expanded region of the Huygens drawing



Derivation of Snell's Law (5)



- We can see that

$$\sin \theta_1 = \frac{\lambda_1}{x} \quad \text{and} \quad \sin \theta_2 = \frac{\lambda_2}{x}$$

- Solving for x we get

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

- Remembering that $n = c/v$ we get

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1} \quad \text{or} \quad n_1 \sin \theta_1 = n_2 \sin \theta_2$$

- Which is Snell's Law!

Light Traveling in an Optical Medium



- We have seen that the wavelength of light changes when traveling in an optical medium with index of refraction greater than one
- Taking with case one as a vacuum and case two as a medium with index of refraction n we can write

$$\lambda_n = \lambda \frac{v}{c} = \frac{\lambda}{n}$$

- Remembering that $v = \lambda f$ we can write the frequency f_n of light traveling in a medium as

$$f_n = \frac{v}{\lambda_n} = \frac{c/n}{\lambda/n} = \frac{c}{\lambda} = f$$

Light Traveling in an Optical Medium (2)



- So the frequency of light traveling in an optical medium with $n > 1$ is the same as the frequency of that light traveling in a vacuum
- We perceive color by frequency rather than wavelength
- Thus placing an object under water does not change our perception of the color of the object
- We can demonstrate that fact by taking a colored object and putting it in a jar of water
- Water has index of refraction $n = 1.33$
- The object appears to have the same color under water as in air

Interference



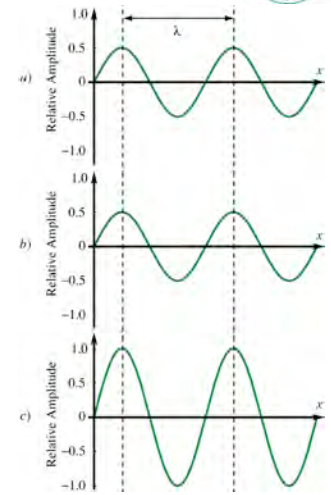
- Sunlight is composed of light containing a broad range of frequencies and corresponding wavelengths
- We often see different colors separated out of sunlight by refraction in rainbows
- We also sometimes see various colors from sunlight due to constructive and destructive interference phenomena on the surface of DVD's or CDs or in thin layers of oil or water



Constructive Interference



- The geometric optics of the previous chapter cannot be used to explain interference
- To understand these interference phenomena we must take into account the wave nature of light
- Interference takes place when light waves of the same wavelength are superimposed
- If the light waves are in phase, they interfere constructively, as shown to the right



Constructive Interference (2)



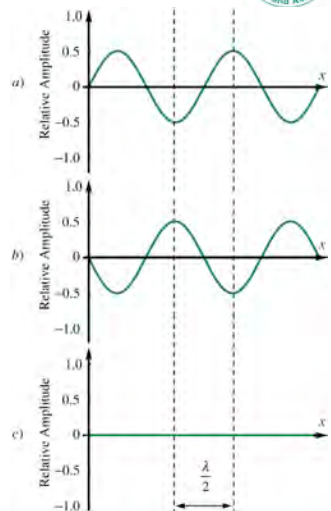
- The statement that the two waves interfere constructively is the same as saying that the phase difference between the two waves is zero
- A phase difference of 2π radians, 360° , or one wavelength will also produce two waves that are in phase
- If the light waves are traveling from some point, then the phase difference can be related to the path difference between the two waves
- The criterion for constructive interference is given by a path difference Δx given by

$$\Delta x = m\lambda \quad (m = 0, m = \pm 1, m = \pm 2, \dots)$$

Destructive Interference



- If the two light waves are out of phase, as shown to the right, the amplitudes of the waves will sum to zero everywhere and the two waves will destructively interfere
- Here the phase difference between the two waves is π radians, 180° , or $\lambda/2$



Destructive Interference (2)



- If we think of the two light waves as being emitted from the same source, the phase difference can be related to the path difference, and we can see that destructive interference will take place if the path difference is a half wavelength plus an integer times the wavelength

$$\Delta x = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, m = \pm 1, m = \pm 2, \dots)$$

- In the following lectures we will study phenomena that produce interference effects by producing light waves with the same wavelength that are initially in phase that must travel different distances or travel with different speeds to reach the same point