Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 47

Relativity

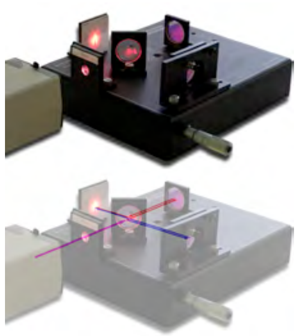
- Common wisdom: "everything is **relative** …"
- Is this really what the theory of relativity tells us?
- **Absolutely** not!

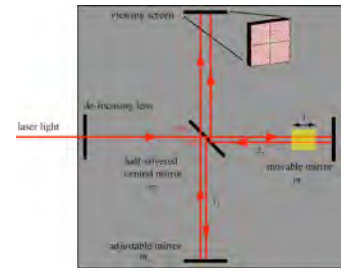
Interference in a Nutshell

- Path length difference of λ (n + 1/2): destructive interference
- Path length difference of nλ: constructive interference
- Interferometer: can be used to measure very small path length differences between light-"beams"
  - Appearance of fringes (light and dark)

Interferometer

- Simplified Michelson-Morley interferometer

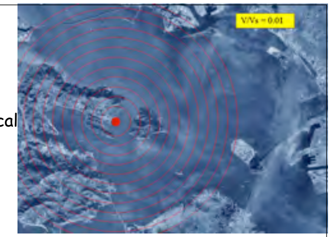




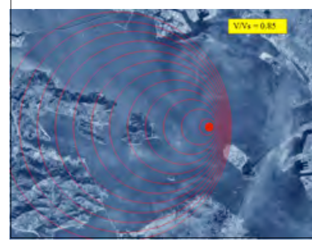
Waves in a Nutshell (from PHY183)

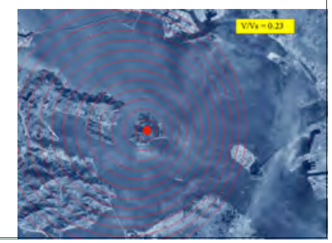
- The equation for all wave motion: $\frac{\partial^2}{\partial t^2} y(x,t) - v^2 \frac{\partial^2}{\partial x^2} y(x,t) = 0$
- Solutions: $y_1(x,t) = Y(x - vt)$, $y_2(x,t) = Y(x + vt)$ with Y = arbitrary function!
- Sinusoidal waves:
  - Moving to "right": $y(x,t) = A\sin(\omega t - \kappa x + \phi_0)$
  - Moving to "left": $y(x,t) = A\sin(\omega t + \kappa x + \phi_0)$
- Wave speed: $v = \frac{\omega}{\kappa} = \lambda f$

Wave in Medium: Doppler Effect



- Physical reason for Doppler Effect:
  - Stationary sound source emits spherical sound waves, spreading with v_s
  - Movement of sound source with v towards observer bunches up waves, causing more waves to move past the observer in a given time interval
  - => perceived higher frequency





Doppler Effect (2)

- General formula for observed frequency of sound f_o as a function of emitted frequency of sound f:
  - v_s = sound velocity
  - v_o = velocity of observer
  - v = velocity of source

$$f_o = f\,\frac{v_s \pm v_o}{v_s \pm v}$$

- +- signs for the following cases:
  - Source moves away from observer: +
  - Source moves towards observer: -
  - Observer moves away from source: -
  - Observer moves towards source: +
- All velocities are relative to the medium (air), which is assumed to be at rest

Electromagnetic Waves

- The speed of an electromagnetic wave can be expressed in terms of two fundamental constants related to electric fields and magnetic fields, the magnetic permeability and the electric permittivity of the vacuum

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

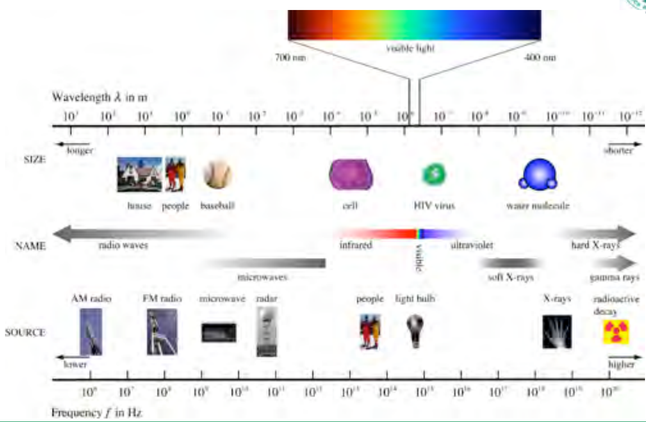
- For an electromagnetic wave, the wavelength and frequency of the wave are related to the speed of light

$$c = \lambda f$$

- Traveling electromagnetic waves

$$E(\vec{r},t) = E_{\max}\sin(kx - \omega t)$$
$$B(\vec{r},t) = B_{\max}\sin(kx - \omega t)$$

Electromagnetic Spectrum



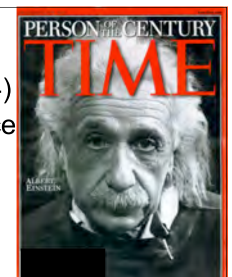
Ether

- So far we have not answered what medium light waves propagate in
  - Air? Not needed, because light from outside the atmosphere reaches us, too (sun, stars, moon)
  - Now we know: light can propagate in vacuum
  - Around 1900 this was not known, and the medium that light propagates in was postulated to be the "ether"
- Can one see something similar to the Doppler Effect for light, since the Earth moves relative to the ether?
  - Michelson&Morley built their interferometer to find out

Michelson & Morley's Result

- If there is motion of Earth relative to ether, then the observed speed of light should be different for light in different directions
- Michelson & Morley used their interferometer and found …
- **NOTHING**
- Light speed is the same in any direction
- ~300,000 km/s

Einstein



1879 Born in Ulm, Germany (March 14)
1902 Begins work at Swiss patent office
1905 Publishes the three seminal papers on theoretical physics
1916 Proposes general theory of relativity
1919 General relativity is proved correct during a solar eclipse
1922 Awarded Nobel Prize in Physics
1933 Emigrates to US (Princeton, N.J.)
1939 Urges F.D.R. to develop atom bomb
1955 Dies in his sleep (April 18)

Einstein's Miracle Year, 1905

- Three seminal papers:
  - A paper explaining the so-called *photoelectric effect* as due to the quantum nature of light. This earned him the Nobel Prize in 1922. Light a quantum particle (more on this in modern physics).
  - A paper explaining the effect of Brownian motion, the motion of very small particles in water or other solutions, as due to collisions with molecules and atoms, thus proving that atoms really exist. (This was not at all clear before his work.) This would have also been worth a Nobel Prize.
  - Theory of special relativity.
- 2005: Celebrate 100th anniversary

Einstein's Postulates

- **Postulate 1:**
  - The laws of physics are the same in each reference frame, independent of the motion of this reference frame.
- **Postulate 2:**
  - The speed of light, *c*, is the same in every reference frame.

Reference Frame

- What is a reference frame?
- It is any frame in which an observation can be made.
- Any point on Earth is a reference frame
- Reference frames can move with respect to each other
- Let's go back to moving reference frames, another review from PHY183

Moving Reference Frame - Classical

- Person walking with a velocity v_w, as measured by an observer moving along with him on the walkway.
- Walkway surface moves with v_{wt} relative to terminal.
- Two velocities add as vectors
- Velocity of person as measured by someone standing in the terminal:

$$v_t = v_{wt} + v_w$$

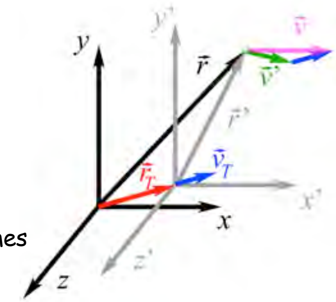
Galilean Transformation

- Two coordinate systems x,y,z and x',y',z' that have their axes parallel to each other and coincide at $t=0$.
- Origin of x',y',z' moves with constant velocity $\vec{v}_T$ relative to x,y,z
- After some time t, the origin of x',y',z' is located at

$$\vec{r}_T = \vec{v}_T t$$

- Vector addition gives us the transformation between frames

$$\boxed{\vec{r} = \vec{r}\,' + \vec{r}_T = \vec{r}\,' + \vec{v}_T t}$$



Galilean Transformation (2)

- Velocities:

$$\vec{r} = \vec{r}\,' + \vec{r}_T = \vec{r}\,' + \vec{v}_T t$$

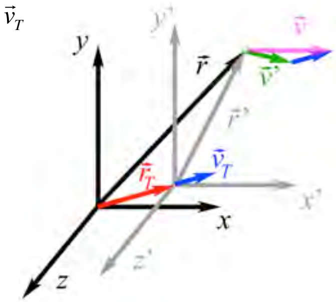
$$\Rightarrow \frac{d}{dt}\vec{r} = \frac{d}{dt}\left(\vec{r}\,' + \vec{v}_T t\right) = \frac{d}{dt}\vec{r}\,' + \vec{v}_T$$

$$\boxed{\vec{v} = \vec{v}\,' + \vec{v}_T}$$

- Accelerations:

$$\frac{d}{dt}\vec{v} = \frac{d}{dt}(\vec{v}\,' + \vec{v}_T) = \frac{d}{dt}\vec{v}\,' + 0$$

$$\boxed{\vec{a} = \vec{a}\,'}$$



Simple Example

- A spaceship moves towards Earth with speed $c/2$
- Someone in the spaceship shines a laser (speed c in the reference frame of the spaceship) towards Earth
- What is the speed of this laser light, as measured on Earth?
- Galilean answer: $v = c + c/2 = 1.5\,c$
- *Einstein's answer: the speed of light is the same in every reference frame. So the speed of the laser light measured on Earth is:* $v = c$

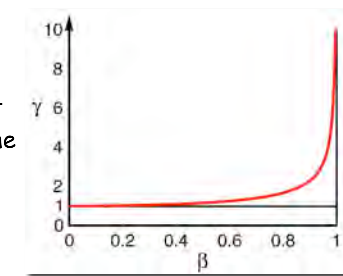
Beta and Gamma

- It is convenient to express speeds as a fraction of the speed of light.
- Introduce a variable beta as the ratio of the speed to the speed of light

$$\boxed{\beta = \frac{v}{c}}$$

- Another useful variable that depends on the speed and the speed of light

$$\boxed{\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(v/c)^2}}}$$



Example: Beta and Gamma

On their way to the moon, the Apollo spacecrafts reached speeds of 40,000 km/h, almost an order of magnitude faster than any airplane.

Question: What are the values of the relativistic factors β and γ in this case?

Answer: $v = 40,000\ \text{km/h} = 40,000 \cdot (1000\ \text{m})/(3600\ \text{s}) \approx 11\ \text{km/s}$

$$\beta = \frac{v}{c} = \frac{1.1 \cdot 10^4\ \text{m/s}}{2.9979 \cdot 10^8\ \text{m/s}} = 3.7 \cdot 10^{-5}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(3.7 \cdot 10^{-5})^2}} = 1 + 6.8 \cdot 10^{-10} = 1.00000000068$$

Main message: for all speeds we are used to in our daily lives, β is very close to 0, and γ is only slightly larger than 1.