Physics for Scientists & Engineers 2

Spring Semester 2005

Lecture 48

Einstein's Postulates

- **Postulate 1:**
  - The laws of physics are the same in each reference frame, independent of the motion of this reference frame.
- **Postulate 2:**
  - The speed of light, c, is the same in every reference frame.

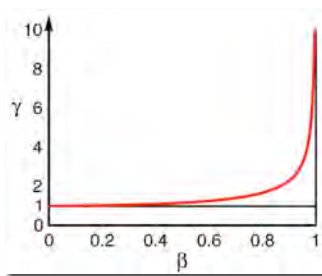
Beta and Gamma

- It is convenient to express speeds as a fraction of the speed of light.
- Introduce a variable beta as the ratio of the speed to the speed of light

$$\beta = \frac{v}{c}$$

- Another useful variable that depends on the speed and the speed of light

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(v/c)^2}}$$



Time Dilation

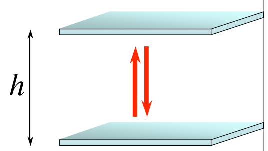
- Conventionally, we think of time as flowing at a uniform rate in one direction (past to the future)
- But if you accept that the speed of light is the same in any reference frame, then our old understanding of time runs into conceptual problems
- The time interval it takes for an event to happen as observed in a moving rest frame is dilated (= made bigger) as compared to the time interval for the event to occur in the frame where it is at rest

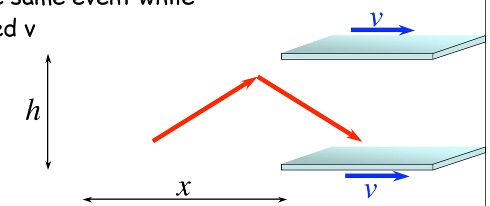
Time Dilation (2)

- Let's start with two parallel mirrors, distance h, and send a light beam from one to the other
- This takes a time

$$\Delta t_0 = \frac{h}{c}$$

- Now observe the same event while moving with speed v



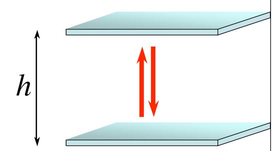


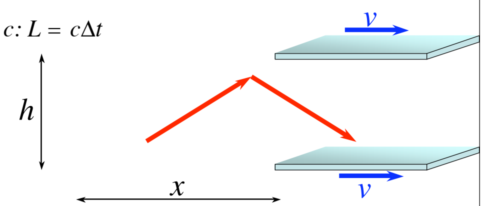
Time Dilation (3)

- Now the mirrors are moving a distance $x = v\Delta t$ in horizontal direction.
- Light beam has covered a distance (Pythagoras!)

$$L = \sqrt{h^2 + x^2}$$

- Light moves with c: $L = c\Delta t$





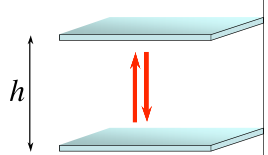
Time Dilation (4)

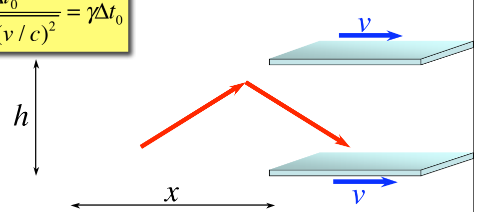
- Put this together

$$L = \sqrt{h^2 + x^2} \Rightarrow c\Delta t = \sqrt{(c\Delta t_0)^2 + (v\Delta t)^2}$$

- Solve for the time interval in the moving frame:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}} = \gamma \Delta t_0$$





Time Dilation (5)

- Once more, in words:
  The rate at which time flows is dependent on how fast the observer moves!
- Note: this is not just the subjective perception of time, but the objective measurable length of time!

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}} = \gamma \Delta t_0$$

Simply the most astonishing formula derived in the entire course up to now!

Example: Time Dilation

- Muon Decay
- Muons have a lifetime of 2.2 micro-seconds
- CERN experiment: muons produced with β = 0.9994

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.9994^2}} = 28.87$$

- Lifetime of these moving muons should be 28.87 times longer, = 63.5 micro-secs, than those at rest.
- During this time, the muons can move a distance

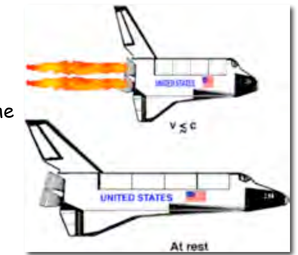
$$x = v\tau = v\gamma\tau_0 = 0.9994\ c \cdot 28.87 \cdot 2.200\ \mu\text{s}=19\text{ km}$$

- Without time dilation, this distance would only be 660 m
- => Direct observation of time dilation!

Length Contraction

- If you want to measure the length of a spaceship at rest, you can use a laser and measure the time it takes to move the length of the spaceship

$$L_0 = c\Delta t_0$$



- Now we redo the same experiment with the shuttle and the clock to measure the time inside it are moving by at speed v.
- This time the measurement then yields:

$$L = \frac{c\Delta t_0}{\gamma} = \frac{L_0}{\gamma}$$

- Length contraction by a factor γ!
- Note: length only changes in the direction of motion, not perpendicular

Twin Paradox

- Twin sisters Alice (astronaut) and Betty (stays home), both age 20
- Alice takes a space trip with γ = 10
- Alice flies to a nearby star and back
- (Note the direction of the velocity vector does not matter!)
- Alice flies for 5 years and is 25 when she returns home to Earth
- But in the meantime Betty has aged 5γ = 50 years and is now 70 when her sister returns!
- But wait a minute! We can also put ourselves into Alice's rest frame, and in this frame Betty (and the entire Earth) has moved with γ = 10, and so Alice would expect to be 25 on her return, but her sister should only be 20.5 years old.
- Big difference! … in particular for Betty …
- Which answer is correct?

Twin Paradox - Solution

- Alice is not really in an inertial frame, because she has to accelerate and slow down
  - True, but also not relevant
  - We can restrict the speeding up and slowing down phases to a very small fraction of the trip; so they will not change the answer
- Consider that the two endpoints of the trip are given in the coordinate frame of Earth, and that they are fixed: Earth on one end and a star 25 light-years away at the other end.
- In Alice's traveling frame that distance appears length contracted by a factor of 10, which is why she is able to make the roundtrip in 5 years.
- Consequently Betty is 70 and Alice 25 years old when they meet again.

Frequency Shift

- Time dilation modifies all other observables that are related to time.
- Example: frequency
- Relativistic frequency shift acts similarly to Doppler Effect, but has origin in time dilation, not in movement of the wave medium relative to observer or source

$$f = f_0\sqrt{\frac{c \mp v}{c \pm v}}$$

- Upper signs for observer and emitted moving away from each other, lower for moving toward each other

Wavelength Shift

- Since $c = \lambda f$, a shift in the frequency is also a shift in wavelength
- Observed wavelength

$$\lambda = \lambda_0\sqrt{\frac{c \pm v}{c \mp v}}$$

(again, upper signs for moving away from each other, lower for towards each other)

- Red-shifted: object moves away from us
- Blue-shift: object moves towards us

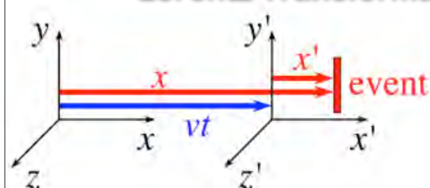
Red Shift

- Red-shift parameter z is defined as the ratio of the wave length shift of light divided by the wave length of the same light when the source is at rest
- Red shift

$$z = \frac{\Delta\lambda}{\lambda_0} = \frac{\lambda - \lambda_0}{\lambda_0} = \sqrt{\frac{c \pm v}{c \mp v}} - 1$$

- Observation: Practically all galaxies in the universe are red-shifted and thus move away from us
  - The further away they are, the more red-shifted

Lorentz Transformation



Note:

Lorentz transformation contains both time dilation and length contraction

Galilei	*Lorentz*	*Inverse*
$x' = x - vt$	$x' = \gamma(x - vt)$	$x = \gamma(x' + vt)$
$y' = y$	$y' = y$	$y = y'$
$z' = z$	$z' = z$	$z = z'$
$t' = t$	$t' = \gamma(t - vx/c^2)$	$t = \gamma(t' + vx'/c^2)$

Small v: Galilei special case of Lorentz

Relativistic Velocity Addition

- Galilean: $v_{1+2} = v_1 + v_2$
- Relativistic addition (without derivation):

$$v_{1+2} = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$$

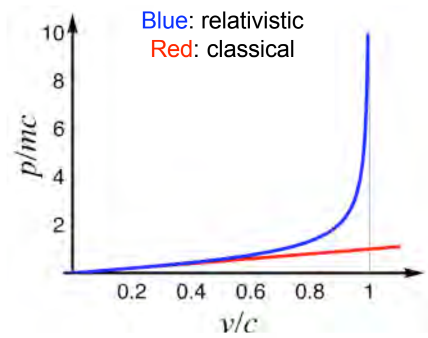
- Earlier example c “plus” $c/2$: $$v_{c+c/2} = \frac{c + c/2}{1 + \frac{c \cdot c/2}{c^2}} = c$$
- For speed small compared to c, we can neglect $v_1 v_2/c^2$ compared to 1, and we get back our Galilean result.

Relativistic Momentum

- Classical $\vec{p} = m\vec{v}$
- Relativistic correct

$$\vec{p} = \gamma\, m\vec{v}$$

- m is the mass of a particle as measured in its rest frame



Momentum and Force

- Newton’s 2nd Law in old form,

$$\vec{F} = m\vec{a} = md\vec{v} / dt$$

is not correct any more in the relativistic limit
- But the form

$$\vec{F} = \frac{d}{dt}\vec{p}$$

is still valid