Experiment 3

Simple Measurements and Error Estimation

Reading and problems:

Homework 3: turn in as part of your preparation for this experiment. **This is a difficult homework set, but a very important one. Allow adequate time to do a good job.** These are exercises on formulas you will use all term.

Read sections 3.1-3.10 of Taylor (you can skip 3.2). Read the handout on the important things in uncertainty calculations. Look carefully at the definition of independence in the handout.

Do the problems below. They and the analysis and discussion requested below will help prepare you for the uncertainty calculations needed for this lab, and indeed for uncertainty calculations all term. Using a spreadsheet is usually easier than a calculator. If you decide to do the additions in quadrature on a calculator, note that the conversion from rectangular to polar coordinates automatically calculates $\sqrt{(x^2 + y^2)}$ for given x and y.

1) If x has been measured as 4.0 ± 0.1 cm, what should I report for x^2 and x^3? Give percent and absolute uncertainties, as determined by rule (3.10) for a power.

2) A student measures a = 50 ± 5, c = 60 ± 2, e = 5.8 ± .3, all in cm, and calculates the sums a+c and a+e. Assuming the original errors were independent and random, find the uncertainties in her answers (using rule 3.16, "errors add in quadrature"). If she has reason to think the original errors are *not* independent, find the uncertainties in her answers (using rule 3.17, "errors add directly"). Summarize your calculations in a table. Useful headings might be Sum, Value, δq(independent), δq(not independent). Indicate with an asterisk those cases in which the second uncertainty (in c or e) can be entirely ignored, assuming the uncertainties are needed with only one significant figure. Comment on the comparative sizes of the uncertainties in each case.

3) A student makes the following measurements

a = 5 ± 1 cm, b = 18 ± 2 cm, c = 12 ± 1 cm, t = 3.0 ± 0.5 s, m = 18 ± 1 gm

Calculate the fraction uncertainty of each quantity (q=a,b,c,t,m) and put this in a table with headings

q value δq δq /q (%).

Compute the quantities q=a+b+c and q=m b/ t , their uncertainties, and percentage uncertainties for the two cases of independent uncertainties, and *not* independent uncertainties. Before you start, predict which errors will be most important in each case. Show the formulas you will use, and arrange your results in a table with headings as below.

		Independent		not independent		
q	value	δq	$\delta q/q$ (%)	δq	$\delta q/q$ (%)	most important variable

For the "most important variable", indicate which variable contributes most strongly to the uncertainty for q. Why did the importance of δb change between the q=a+b+c and the q = m b/ t cases?

Explain whether the uncertainty of a+b-c will be the same, or different from, the uncertainty of a+b+c. From this answer, explain whether the fractional uncertainty of a+b+c will be larger or smaller than the fractional uncertainty of a+b-c, and why.

4) A student is studying the properties of a resistor. She measures the current flowing through the resistor and the voltage across it as

$I = 2.10 \pm 0.02$ amps and $V = 1.02 \pm 0.01$ volts.

a) What should be her calculated value for the power delivered to the resistor, P = IV, with its uncertainty? b) What for the resistance R = V/I (in ohms)? Assume the original uncertainties are independent. With I in amps and V in volts, the power P comes out in watts and the resistance in ohms. Start by evaluating the fractional uncertainty of I, and V in percent, then calculate $\delta P/P$ in percent following the example on p 62. Show algebra, not just numbers. Finally, derive δP from $\delta P/P$. For b), see if you can avoid repeating the entire calculation.

5) In an experiment on the conservation of angular momentum, a student needs to find the angular momentum L of a uniform disc of mass M and radius R as it rotates with angular velocity ω. She makes the following measurements:

$M = 1.10 \pm .01$ kg, $R = .250 \pm .005$ m, $\omega = 21.5 \pm 0.4$ rad/s

And then calculates L as $L = \frac{1}{2} MR^2 \omega$. (The factor $\frac{1}{2} MR^2$ is just the moment of inertia of the uniform disc.) What is her answer for L with its uncertainty? (Consider the three original uncertainties independent and remember that the fractional uncertainty in R^2 is twice that in R.) For this and any calculations in the lab involving products proceed in the same way as you did for 4). A very useful step is to pause and explain if any of the terms are obviously negligible before performing the final calculation of the fractional uncertainty. Also, explain why the fractional uncertainty of R^2 is twice that of R.

6) (a) According to theory, the period T of a simple pendulum is $T = 2\pi\sqrt{(L/g)}$, where L is the length of the pendulum. If L is measured as $L = 1.40 \pm 0.01$m, what is the predicted length of T? (b) Would you say that a measured value of $T = 2.39 \pm 0.01$ s is consistent with the theoretical prediction of part (a)? Again, show algebra as well as numbers.

1. Introduction

The main purpose of this experiment is to introduce you to methods of dealing with the uncertainties of the experiment. The basic procedures to correctly estimate the uncertainty in the knowledge of the measured value (*the error of the measurement*) include:

- Estimation of the uncertainty in the values directly measured by, or read from, the measurement device (*directly measured quantities, Taylor, Chapter 1*);

- calculation of the errors of the quantities which are not measured directly (*the propagation of errors, Chapter 3*);
- rounding off the insignificant digits in the directly measured and calculated quantities (Chapter 2 and the Reference Guide).

2. Goals

1. Perform simple measurements as accurately as possible (with a goal of less than .5% for the most accurate ones), and to estimate uncertainties in these measurements.
2. Understand the strengths and limitations of different length measuring instruments.
3. Practice computing errors for quantities derived from several measurements.
4. Use calculated uncertainties in drawing conclusions.

3. Preliminary discussion (10-15 minutes).

Before the lab, you are asked to read and understand the theoretical material for this lab. Before the experiment starts, your group needs to decide which information will be relevant to your experiment. Discuss what you will do in the lab and what preliminary knowledge is required for successful completion of each step.

Think hard about organizing your work in an efficient way. What measurements will you need to make? Go through your lab manual with a highlighter, then make checklist of the needed measurements. What tables or spreadsheets will you need to make to organize the calculations data? How should you use *Kgraph* to expedite your calculations and unit conversions (when necessary)? What tables will you need to summarize your analysis and conclusions from the data? This lab will have more explicit reminders about tables than future labs, but you should be thinking about this organization of data taking, data reduction, and summarization in every lab.

3.1 Questions for the preliminary discussion

You should write your own answer to each question in your lab book, but leave space to change it after discussion. If you do change your answer, say why.

3.1.1 Suppose during each of several measurements we find a value, which lies in the same interval of the scale of the measuring device. For example, each time we measure the length to be between 176 and 177 mm, with the length between the ticks on the ruler equal to 1 mm. How do we estimate an uncertainty in the measured length in this case?

3.1.2 Now suppose we use a much more precise device (say a super duper laser meter). Due to a higher precision, this device can resolve the miniscule changes in the length due to the random mechanical deformations of the object, and in each measurement we will see the slight unsystematic changes in the observed length. From these data, how can we find the most likely value of the length? How can we characterize numerically the typical variation in the measurements? Which statistical measures and formulas will we use for these two points? (*Hint*: see Chapter 4 of Taylor).

3.1.3 Suppose we ask if the two independent measurements from 3.1.1 and 3.1.2 are consistent with each other. Is there a quantitative method to find out if two measured lengths are in agreement? Is there a quantitative method to estimate how certain our conclusion about the agreement or the disagreement of these measured values is?

3.1.4 If we are going to use the results of our measurements to calculate some other quantities (e.g., calculate the density of the rod using the measurements of its dimensions and the mass), which formulas will we use to calculate the mean values and the uncertainties of these quantities?

3.1.5 In our calculation, the calculator (computer) will typically return the results with as many digits as possible, including digits well beyond our measurement uncertainty. What procedure will you follow to systematically get rid of these insignificant digits?

3.1.6 From the homework problems, it was evident that much labor can be saved by judicious simplification of the uncertainty calculations. If you choose to do so, how will you justify approximations to the uncertainty calculations?

3.1.7 The measurement of the pipe poses special problems, which only begin with obtaining a mathematically correct formula for the volume. Discuss which instrument(s) would be best for this measurement. Record your choices and your reasons. As part of the discussion, consider how your answer would change if the diameter of the pipe were much larger, or much smaller; and if the wall was much thicker or thinner.

Density Measurements

4. Introduction

Your text (Sec. 1.3, p. 5) describes how Archimedes was able to determine the composition of a king's crown by measuring its density. We will attempt to perform a similar exercise, but in an effort to limit tuition increases, we shall use copper instead of gold. Copper has a density of 8.91 g/cm^{3}at 20 degrees Celsius (C). We will consider later what to do if the temperature is not exactly 20 degrees C. Your task today is to measure the density, calculate the appropriate uncertainties and decide whether your measurement agrees with the given value. Then you will measure the density of some more complex objects. You will attempt to identify the materials of which these objects are made by comparing a list of known densities for various metals with their measured density and uncertainty.

Report both % and absolute uncertainties for your final values; using % uncertainty in your uncertainty calculation tables will usually make things much clearer, both for you and the grader.

In our lab, we will use rulers, vernier calipers and micrometers. Discuss in your group the following questions:

4.1 Which of these instruments is the most precise; the least precise? How do you know? In particular, is the caliper more precise than the ruler? If you don't see how to use the caliper, refer to the RG (Reference Guide).

4.2 Refer to the appendix on Alloy Densities. If your goal is to distinguish whether a sample is made of copper, or an alloy nearest to copper in density, with what accuracy will you need to measure the density?

4.3 What tables will you need for the measurements below? For the uncertainty calculations? Be aware that you will be expected to make your own tables to organize calculations and summary in the future, so think hard about this.

Now begin your measurements:

4.4 Write down in your lab notebook the **sample code** for the unknowns you are measuring, and the **number of the table** at which you are working (**do for EACH experiment**).

4.5 Use the ruler to measure the three dimensions of the block. Repeat these measurements with the vernier caliper and the micrometer. For each instrument, measure the length with the highest precision possible. Write down the uncertainties of the length measurements with each of these instruments. **Each time you assign an uncertainty of a measurement, you must justify your choice in your lab notebook.**

4.6 Measure the mass of your block and estimate its uncertainty.

4.7 For the ruler measurement, compute the volume of the block and its uncertainty. **To retain your sanity, we recommend you use *Excel* to carry out the calculations.** To organize your calculations, refer to the suggested spreadsheet at the end of the writeup. IF you do the calculations by hand, then consider the following hint for calculating fractional uncertainties with small error bars. Say you measure 1.0 cm ± .001mm. That's a fractional uncertainty of 10^{-4}m or .01%. If all your uncertainties are this small, record the uncertainties as $\delta x = .04\%$ $\delta y = .03\%$ etc, but to do a hand calculation, take out a big factor like this before doing the square roots. Effectively it's a unit change in the uncertainties, by factoring out the scale of .01%:

$$Q = x\,y$$

$$\delta Q/Q = \sqrt{[(.01\%)^2 \{4^2 + 3^2\}]} = .01\% \sqrt{\{4^2 + 3^2\}} = .01\% \sqrt{25} = .05\%$$

Pay attention to the units. In the calculation, follow the rules for rounding off the insignificant figures, particularly for the final density result. If the calculation is done correctly, the smallest significant figure in the final value will be of the same order as the final uncertainty. In order to eventually calculate a t value to 2 s.f., you'd need to record the uncertainty to 2 s.f. as well. That might look like 87.03172 cm/s ± .050% = 87.03 ± .44 cm/s when matching recorded value significant figures to significant figures of uncertainty.

4.8 Make two estimates of your volume uncertainty using alternatively Eqs. 3.18 and 3.19 on p. 61 of the text.

4.9 Which uncertainty is more appropriate for this calculation? Why?

4.10 Compute the ruler data density and uncertainty. Use Eq. 3.18 on p. 61 of Taylor.

4.11 Calculate the density and its uncertainty using the data obtained with the help of the caliper and micrometer. Compare the three measurements (and their inputs and uncertainties) in a table!

Answer the following questions:

4.12 Based on your best measurement, is your value consistent with the density of pure copper? Could the sample be one of the copper alloys listed in the appendix on Alloy Densities? Justify your conclusion quantitatively. (*Hint*: compare the discrepancy between the theoretical and experimental values with some other number; see Taylor §1.3, and the Uncertainties handout: the calculations should be done for Copper, and the nearest alloy).

4.13 In the computation of density using the micrometer, what were the greatest sources of uncertainty? Which were the smallest?

4.14 Based on only your measurements made with the ruler, would you arrive to the same answer for the Question 4.12? Why?

4.15 What systematic errors might we be overlooking? Are any of these big enough to affect your estimate of uncertainty? Consider, one at a time, the temperature dependence of the density of metal (see below); irregularities in the shape of the block; and any other measurement biases you can think of. Try to give an estimate of the size of each effect. Based on this estimate, could it be an important source of error in your density calculation?

Thermal Expansion If a metal is heated, its length increases by an amount ΔL given by $\Delta L = \alpha \cdot L \cdot \Delta T$ where L is the original length, ΔT is the increase in temperature, and α is the thermal coefficient of (fractional) linear expansion. Appendix 1 contains α values.

5. Density Measurements of Other Objects

5.1 Perform a density measurement to determine the material of two other unknown objects. You should measure only the solid cylinder and the silvery pipe. Be sure to record what dimension you measured, the instrument used, the uncertainty, and the justification for the uncertainty. You should choose the appropriate instrument for each measurement to get the most accurate result. State why you chose the instrument you did for each measurement. Again, organize in a table, and use uncertainties (and propagation of errors) to present the numeric arguments supporting your conclusions. **For the pipe, you will need the methods of Taylor §3.8 to calculate the uncertainties. HW 4** will cover this material.

5.2 Should the fractional error on the density will be larger for the pipe, or the cylinder? Explain why. Do your results agree with your claim?

5.3 In your report, include 3 measured densities of the rectangular block and the densities of the 2 unknown objects, as well as the relevant uncertainties, and the material or materials you deduce them to be made of.

5.4 What was the muddiest point of this lab? Point out where specifically the lab materials should be improved.

Appendix 1: Commercial Metal and Alloy Densities

Table of density (specific gravity) of alloys.

SG = Specific Gravity; the units are either g cm^{-3} or kg m^{-3}
CE = Coefficient of fractional linear Expansion (10^{-6} / °F)

Common name and classification		SG	CE
Aluminum alloy 380	ASTM SC84B	2.7	11.6
Aluminum alloy 3003, rolled	ASTM B221	2.73	12.9
Aluminum alloy 2017, annealed	ASTM B22	2.8	12.7
Hastelloy C		3.94	6.3
Cast gray iron	ASTM A48-48. Class 25	7.2	6.7
Ductile cast iron	ASTM A339, A395	7.2	7.5
Ni-resist cast iron	type 2	7.3	9.6
Malleable iron	ASTM A47	7.32	6.6
Cast 28-7 alloy (IID)	ASTM A297-63T	7.6	9.2
Aluminum bronze	ASTM B169, alloy A;ASTM B124, B150	7.8	9.2
Ingot iron	(included for comparison)	7.86	6.8
Plain carbon sheet	AISI-SAE 1020	7.86	6.7
Stainless steel	type 304	8.02	9.6
Beryllium copper 25	ASTM B194	8.25	9.3
Inconel X, annealed		8.25	6.7
Yellow brass (high brass)	ASTM B36, B134, B135	8.47	10.5
Copper	ASTM B152, B124, B133, B1, B2, B3	8.91	9.3
Haynes Stellite alloy 25 (L605)		9.15	7.61

Recommended Tables for Experiment 3

Density Measurements

d is used for uncertainty δ for simplicity

Inputs for Block (lengths in cm)

Object	method	l	dl	dl(%)	w	Dw	dw(%)	h	dh	dh(%)
Block	ruler									
Block	caliper									
Block	micrometer									

Density of Block volume in cm^3, mass in grams

Object	method	v	dv	dv(%)	m	Dm	dm(%)	density	ddensity	ddensity(%)
Block	ruler									
Block	caliper									
Block	micrometer									

Inputs, cylinder & pipe (lengths in cm)

Object	method	Do	dDo	dDo(%)	Di	dDi	dDi(%)	h	dh	dh(%)
solid cyl										
silver										
pipe										

		Area	dA	dA(%)
solid cyl				
silver				
tube				

Density, cylinder & pipe volume in cm^3, mass in grams

Object	method	v	dv	dv(%)	m	Dm	dm(%)	density	ddensity	ddensity(%)
solid cyl										
silver										
pipe										

Summary		density in g/cm^3			expected (Cu), or best fit alloy					
Object	method	density	ddensity	ddens(%)	density best alloy	best alloy name	t best	density 2nd best	2nd best name	t 2ndbest
Block	ruler					Copper				
Block	caliper					Copper				
Block	micrometer					Copper				
Solid cyl										
silver										
pipe										