Name: ______________________________

Student # ______________________________

You may redo one of problems 1-4: My redo problem is:

PHY215, fall 2006

Modern Physics and Thermodynamics

Exam #3. November 30, 2006: 40 points

Work the first 4 and the last two are extra credit...do none, do one, or do two.

Please show all of your work. If you need more space, use the back and indicate clearly what problem is being continued. If you still need more space...ask for another sheet and clearly include your name and what problem is begin continued.

Constants

1 calorie = 4.186 J
1 atmosphere = 1.01×10^5 Pa
Gas Constant: $R = 8.3145$ J/mol·K
Bolzmann's Constant: $k = 1.38 \times 10^{-23}$ J/K = 8.617×10^{-5} eV/K
Sefan-Boltzmann's constant: $\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4
Avagadro's Number: $N_A = 6.023 \times 10^{23}$ mol^{-1}
Speed of Light: $c = 3 \times 10^8$ m/s
Charge of the electron: $-e = -1.6 \times 10^{-19}$ C
1 atomic mass unit: $u = 1.66054 \times 10^{-27}$ kg = 931.494043 MeV/c^2
Mass of the electron: $m_e = 9.1094 \times 10^{-31}$ kg = 511 keV/c^2
Mass of the proton: $m_p = 1.6726 \times 10^{-27}$ kg = 938.3 MeV/c^2
Mass of the proton: $m_p = 1.007277\ u$
Mass of the neutron: $m_n = 1.6749 \times 10^{-27}$ kg = 939.6 keV/c^2
Mass of the neutron: $m_n = 1.008665\ u$
Mass of the alpha particle: $m_\alpha = 3727.4$ MeV/c^2
Mass of the alpha particle: $m_\alpha = 4.00151\ u$
Planck's Constant: $h = 6.63 \times 10^{-34}$J-s = 4.14×10^{-15} eV-s
...times c: $hc = 1.9864 \times 10^{-25}$J-m = 1239.8 eV-nm
Reduced h: $h/2\pi = \hbar = 1.0546 \times 10^{-34}$J-s = 6.5821×10^{-16} ev-s
...times c: $\hbar c = 3.162 \times 10^{-28}$J-m = 197.33 eV-nm
Electrostatic constant: $\dfrac{1}{4\pi\epsilon_0} = 8.9876 \times 10^9$ N-m^2-C^{-2}
...times e^2: $\dfrac{e^2}{4\pi\epsilon_0} = 2.3071 \times 10^{-28}$ J-m = 1.4400×10^{-9}eV-m
Bohr radius: $a_0 = \dfrac{\hbar}{m_e c\alpha} = 0.5292 \times 10^{-10}$ m
Fine structure constant: $\alpha = \dfrac{e^2}{4\pi\epsilon_0\hbar c} = 1/137.036$
Radioactive activity: 1 Curie = 1 Ci = 3.7×10^{10} decays/s

Formulae and Integrals

$$\text{reduced mass: } \mu = \frac{mM}{m+M}$$

$$\text{mean velocity for an ideal gas: } < v > = \frac{4}{\sqrt{2\pi}}\sqrt{\frac{kT}{m}}$$

$$\int \sin x dx = -\cos x$$

$$\int \cos x dx = \sin x$$

$$\int \sin^2 x dx = \frac{1}{2}x - \frac{1}{2}\sin 2x$$

$$\int x \sin^2 x dx = \frac{x^2}{4} - \frac{x \sin 2x}{4} - \frac{\cos 2x}{8}$$

$$\int x^2 \sin^2 x dx = \frac{x^3}{6} - \left(\frac{x^2}{4} - \frac{1}{8}\right)\sin 2x - \frac{x \cos 2x}{4}$$

$$\int e^{-ax} dx = -\frac{1}{a}e^{-ax}$$

1. (total for the problem: 10 pts) The element scandium has $Z = 21$.

a. (3 pts) I want you to construct the ground state electronic configuration (you know, the $1s^2 2s^2 \ldots$ designation.). The Noble gas that's just before Sc in the Periodic Table is Ar ($1s^2 2s^2 2p^6 3s^2 3p^6$). This is one of those elements for which a high n, S state has lower energy than a lower n, but higher L state. So, the outer shells for Sc, above those of Ar, are $4s^2 3x^n$ What are x and n?

b. (2 pts) Those values of x and n define the overall angular momentum and spin quantum numbers for Sc (that is, the inner shells contribute 0 to the overall L and S.) Using your values of x and n from a., what are L and S for the Sc ground state?

1., cont.

c. (3 pts) There is a spin-orbit interaction. Using your values of x and n from a., what are the possible values of J for Sc?

d. (2 pts) Using your values of x and n from a., which is the lowest energy J state for Sc?

2. (10 pts) A gas of atomic hydrogen is at room temperature ($T = 293\ K$). Remember that the ground state energy is $E_G = -13.6$ eV.

a. (2 pts) In the ground state, $n = 1$, the degeneracy factor is $g(E_G) = 2$. Why is that?

b. (3 pts) In the first excited state state, $n = 2$, the degeneracy factor is $g(E_1) = 8$. Why is that? Your explanation might be in terms of n, ℓ, and/or s.

c. (5 pts) What is the relative population of the first excited state for which $E_1 = -3.4$ eV? (Hint: Boltzmann statistics apply.) The answer is a number.

3. (total for the problem 10 pts) Consider a system of fermions.

a. (5 pts) Draw the Fermi-Dirac distribution function as a function of energy for a temperature not too far above T=0. Label the vertical and horizontal axes and point out the location of the Fermi Energy, E_F.

b. (3 pts) Draw the FD distribution as a function of energy for a temperature, T=0. Label the vertical and horizontal axes and point out the location of the Fermi Energy, E_F.

c. (2 pts) For this last situation, what is the occupation level of states with energies $E > E_F$?

4. (total for problem: 10 pts) Plutonium ${}^{236}_{94}\mathrm{Pu}$ is unstable and decays into ${}^{232}_{92}\mathrm{U}$. $M\left({}^{236}_{94}\mathrm{Pu}\right) = 236.046071\ u$ and $M\left({}^{232}_{92}\mathrm{U}\right) = 232.037168\ u$

a. (3 pts) Is this an alpha decay? beta decay? or gamma decay? and why? Justify your choice by explicitly counting the numbers of protons and neutrons in each element of the initial and final states of the decay and by conserving electric charge.

b. (3 pts) Using the choice you made in a., show that this isotope of plutonium is indeed unstable by calculating its Q value in MeV.

c. (4 pts) Calculate the binding energy per nucleon in MeV for the plutonium isotope.