September, 3, 2006

PHY 215 Derivation: D1

Due: September 12 (with other homework)

In class I proposed that Maxwell's Equations are not consistent with the Galilean Transformations. From the material in the slides and what's provided below, please show this.

I took the approach that we had already shown to expect that a force would respect the G.T. and so from applying that expectation to the Lorentz Force, that the electric and magnetic fields transform differently under a G.T. Specifically, this causes

$$E = E' + vB'$$
$$B = B'$$

Remember, I took a particular arrangement for the fields, namely that they were propagating in the x direction, with $\mathbf{E} = E(x,t)\,\hat{\mathbf{j}}$ and $\mathbf{B} = B(x,t)\,\hat{\mathbf{k}}$, so that the "curl" equation for the magnetic field becomes:

$$\frac{\partial B}{\partial x} = -\frac{1}{c^2}\frac{\partial E}{\partial t}$$

Show that by differentiating the G.T. and using the chain rule that the derivatives of the two sides of this equation result in:

$$\frac{\partial B}{\partial x'} = \frac{1}{c^2}\left(\frac{\partial E}{\partial t'} - v\frac{\partial E}{\partial x'}\right)$$

That was for transforming the space and time coordinates. Now, the fields themselves must transform as above in order to preserve the form of the force. This results in the fully-transformed Maxwell's Equation:

$$\frac{\partial B'}{\partial x'} = -\frac{1}{c^2}\frac{\partial E}{\partial t'} + \frac{1}{c^2}\left(v\frac{\partial E'}{\partial x'} - v\frac{\partial B'}{\partial t'} + v^2\frac{\partial B'}{\partial x'}\right)$$

which is clearly not of the same form as the unprimed equation. This can be done for the other curl equation as well. Or, a different approach can be taken that does not rely on the equality of the force relationship. Then, what results is that the G.T. does not lead to consistency among the Maxwell Equations. Any way you slice it, the Maxwell Equations have serious issues with Galilean Transformations and that gave Lorentz headaches.