

Elementary Particles ~ 1932:

Proton	$m \approx 1.7 \times 10^{-27} \text{ kg}$	$Q = +e$	${}^1_1\text{P}$
neutron	$m \approx 1.7 \times 10^{-27} \text{ kg}$	$Q = 0$	${}^1_0\text{n}$
electron	$m = 9.1 \times 10^{-31} \text{ kg}$	$Q = -e$	${}^0_{-1}\text{e}$

Convention:

unified mass unit, u or "Atomic Mass Unit"

${}^{12}_6\text{C}$ is exactly $12u$

So $1u = 1.660559 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV}/c^2$

$$m_p = 1.007276 u$$

$$m_n = 1.008665 u$$

$$m_e = 0.000549 u$$

↑ bigger

and, as you know:

$$m_p c^2 = 938.28 \text{ MeV}$$

$$m_n c^2 = 939.57 \text{ MeV}$$

$$m_e c^2 = 0.511 \text{ MeV}$$

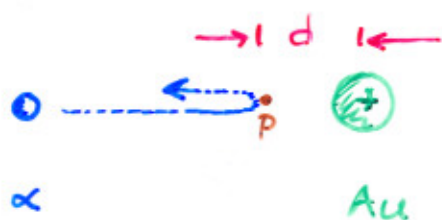
So, neutrons and protons... how many?

element	A	Z	N
${}^1_1\text{H}$	1	1	0
${}^4_2\text{He}$	4	2	2
${}^7_3\text{Li}$	7	3	4
${}^9_4\text{Be}$	9	4	5
${}^{12}_6\text{C}$	12	6	6
${}^{16}_8\text{O}$	16	8	8
${}^{23}_{11}\text{Na}$	23	11	12
${}^{27}_{13}\text{Al}$	27	13	14
${}^{35}_{17}\text{Cl}$	35	17	18
${}^{56}_{26}\text{Fe}$	56	26	30
${}^{63}_{29}\text{Cu}$	63	29	34
${}^{108}_{47}\text{Ag}$	108	47	61
${}^{197}_{79}\text{Au}$	197	79	118
${}^{208}_{82}\text{Pb}$	208	82	126
${}^{238}_{92}\text{U}$	238	92	146

Sizes and shapes of Nuclei -

Hard to not think of them as little spheres - or bundles of spheres -

Remember, Rutherford's scattering early on estimated the "size" \Rightarrow appreciable extent of + charge repulsion of Gold.



at the turning point P, the kinetic energy of the α would be instantaneously all electrostatic repulsive potential

$$\begin{aligned}\frac{1}{2}mv^2 &= \frac{1}{4\pi\epsilon_0} \frac{q_\alpha q_{Au}}{r} \\ &= \frac{1}{4\pi\epsilon_0} \frac{(2e)(Ze)}{d}\end{aligned}$$

$$d = \frac{4}{4\pi\epsilon_0} \frac{Ze^2}{mv^2} \sim 3.2 \times 10^{-14} \text{ m}$$

\Rightarrow Au nucleus is smaller than that

For Ag... he found 2×10^{-14} m.

Concluded that the "nucleus" was $\sim 10^{-14}$ m

New Unit:

femtometer (fm)

$$1 \text{ fm} = 10^{-15} \text{ m}$$

also called the "Fermi"

Many experiments since show roughly:

$$R = r_0 A^{1/3}$$

where $r_0 = 1.2 \times 10^{-15}$ m.

Density...

Assume nucleus is spherical, what's the density of nuclear "matter"?

$$V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi r_0^3 A \quad V \propto A \quad \checkmark$$

$$\rho = \frac{M}{V} = \frac{Am}{\frac{4}{3} \pi r_0^3 A} = \frac{3m}{4\pi r_0^3} \quad m \approx m_p \approx m_n \sim 1.67 \times 10^{-27} \text{ kg}$$

$$\rho = 2.3 \times 10^{17} \text{ kg/m}^3$$

$\sim 2 \times 10^{14}$ more dense than water.

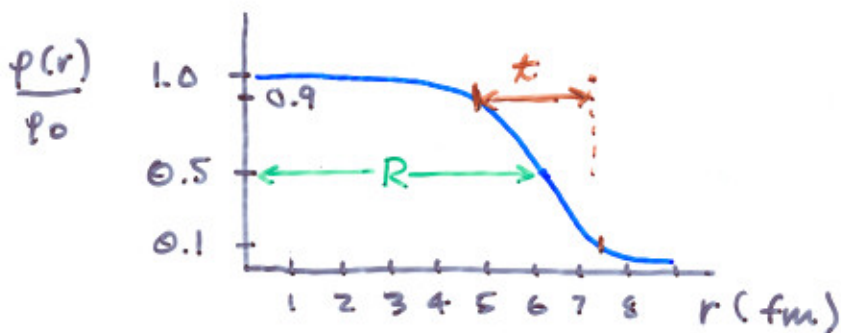
the density of a neutron star

prob 9.35: $\sim 10^{18} \text{ kg/m}^3$

Refinements in beam projectors led to a long program of electron scattering at the Stanford Linear Accelerator Center -- SLAC

The charge distribution --

for a representative nucleus, $Z \sim 50$



ρ_0 : central charge density

t : a definition of the "surface"

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R)/a}}$$

$$t = 4.4a$$

reasonable for $Z \gtrsim 10$

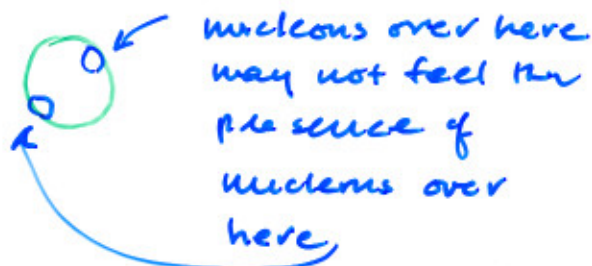
looks like F_{FD} and is indeed called the "Fermi Distribution"

Did you notice ...

$$R \propto A^{1/3} \Rightarrow \rho \text{ is independent of } A.$$

This suggests that as A goes up, the ~~size~~ density does not.

And this suggests that the "nuclear force" is of very short range



\Rightarrow the forces of attraction between

n-p

n-n

p-p

are all about the same ...

Keeps nuclei very compact, but still growing with A .

Implies the force to remove a p or n is very strong.

Nuclear Spin

Nuclei have a quantum mechanical spin: " I "

can be half-integer or integer.

recall that ${}^4\text{He}$ acts like a boson
and condenses...

${}^3\text{He}$ in fact is a fermion...
and does not condense.

As usual:

$$|\vec{I}| = \hbar \sqrt{I(I+1)}$$

So... nuclei have magnetic moments

new unit: "nuclear magneton"

$$\mu_N \equiv \frac{e\hbar}{2m_p} = 5.05 \times 10^{-27} \text{ J/T}$$

$\ll \mu_B$

$$\mu_p = 2.7928 \mu_N$$

BUT:

$$\mu_n = -1.9135 \mu_N !$$

neutron is neutral...
but has \neq magnetic
moment \Rightarrow structure.

So, apply a field to a nucleus...

get a level splitting and μ precession

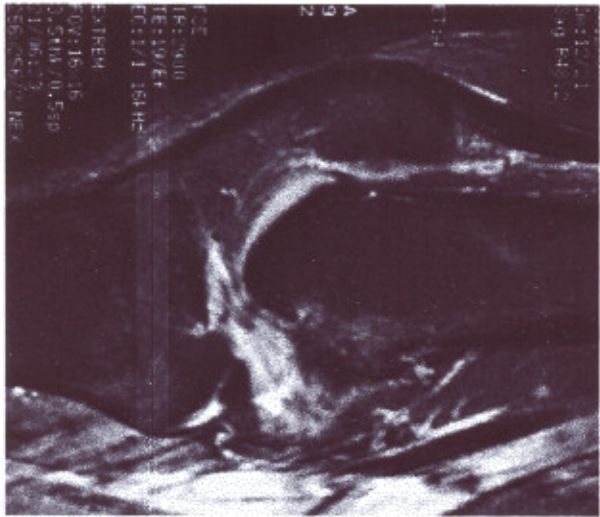
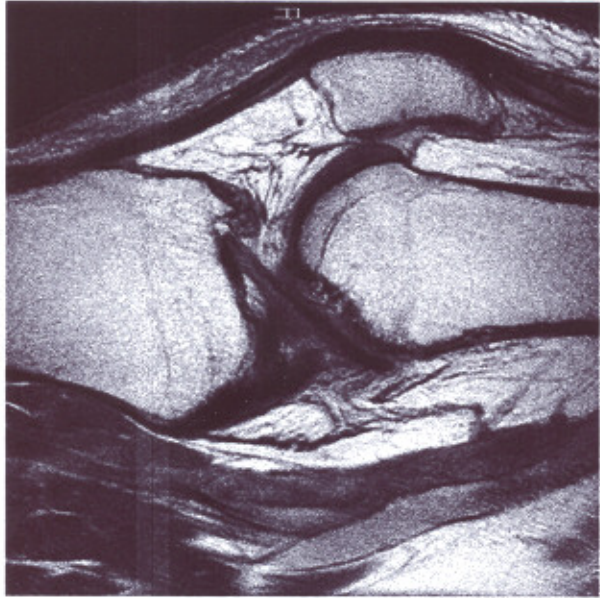


for $I = \frac{1}{2}$
nucleus.

Boltzmann tells us:

$$\frac{N_{up}}{N_{dn}} = e^{-\frac{(E_{up} - E_{dn})}{kT}}$$

so this population difference can be exploited.



"Nuclear Magnetic Resonance", NMR

... because softened to "Magnetic Resonance Imaging",
MRI

Nuclear Forces and Binding

Referred to this many times---

That 2 protons can come together and bind, overcoming their electrostatic repulsion suggest a **STRONG** force -- called:

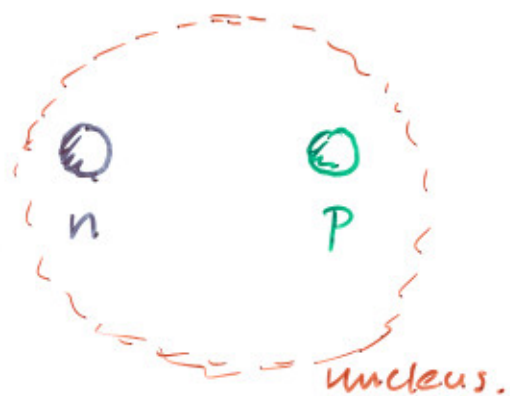
the strong force.

(or historically, the nuclear force)

In fact, nuclei are not just protons

neutrons are also contributors to the total attractive force of the nucleus.

A quantum "explanation" -- or at least "picture"



Suppose the neutron, at rest energy $m_n c^2$

emits another quantum of ΔE

How \ddot{c} still be a neutron & conserve energy?

Uncertainty

$$\Delta E \Delta t \sim \hbar$$



too short to be observed.

Energy violations over such a time scale? OK.

Consistent with our quantum mechanical worldview...



this game of catch is attractive - "Exchange Force"

Interpret

$$\Delta E = m_{\pi} c^2$$

$$\Delta E \Delta t = \hbar$$

$$\Delta t = \frac{\hbar}{m_{\pi} c^2} \quad \Rightarrow \text{"shortest" time}$$

how short? ... furthest it could travel is for $v_{\pi} \sim c$

$$x = c \Delta t$$

$$x = \frac{\hbar c}{m_{\pi} c^2}$$

So: $m_{\pi} c^2 = \frac{\hbar c}{x}$

What's a good x ? ... confine it to be inside the nucleus before it disappears into an adjacent p (or n).

$$x \sim 1 \text{ fm} = 10^{-15} \text{ m}$$

$$m_{\pi} c^2 = \frac{\hbar c}{10^{-15} \text{ m}} = \frac{197.3 \text{ eV} \cdot \text{nm}}{10^{-6} \text{ nm}} \approx 200 \text{ MeV}$$

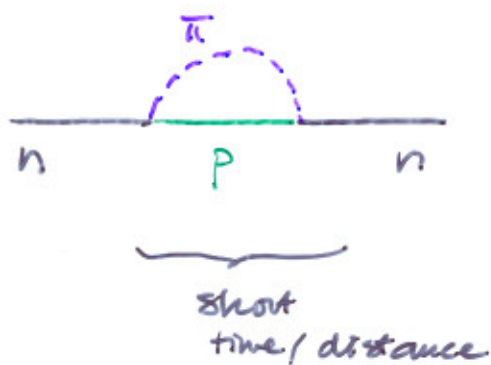
Predicted in 1935 by Hideki Yukawa

"Yukawa particle", "Y" ... now pion, π



Hidetsu Uehara

1949



called "virtual particles"

more on this later...

Simplest nucleus \rightarrow ${}^1_1\text{H}$

Next simplest \dots ${}^2_1\text{H}$ Deuterium

Remember the "E=mc² discussion" of H?

$$m_e c^2 + m_p c^2 = m_H c^2 + 13.6 \text{ eV}$$

↑
a mass deficit

$$m_e c^2 + m_p c^2 - m_H c^2 = B$$

called by chemists ;)

"Binding Energy"

to liberate e & p to freedom... must supply B.

same for nuclear binding... but much bigger deal.