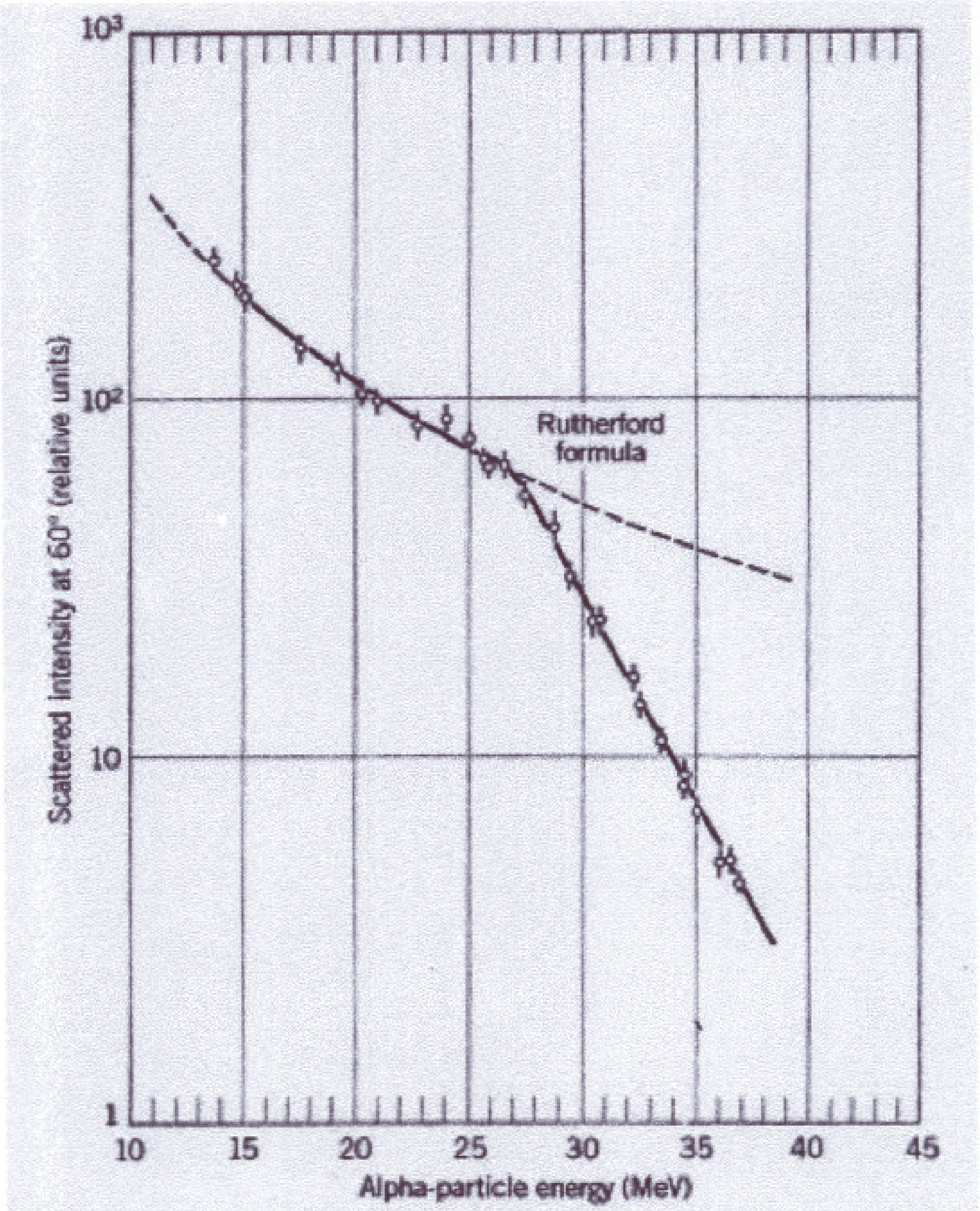
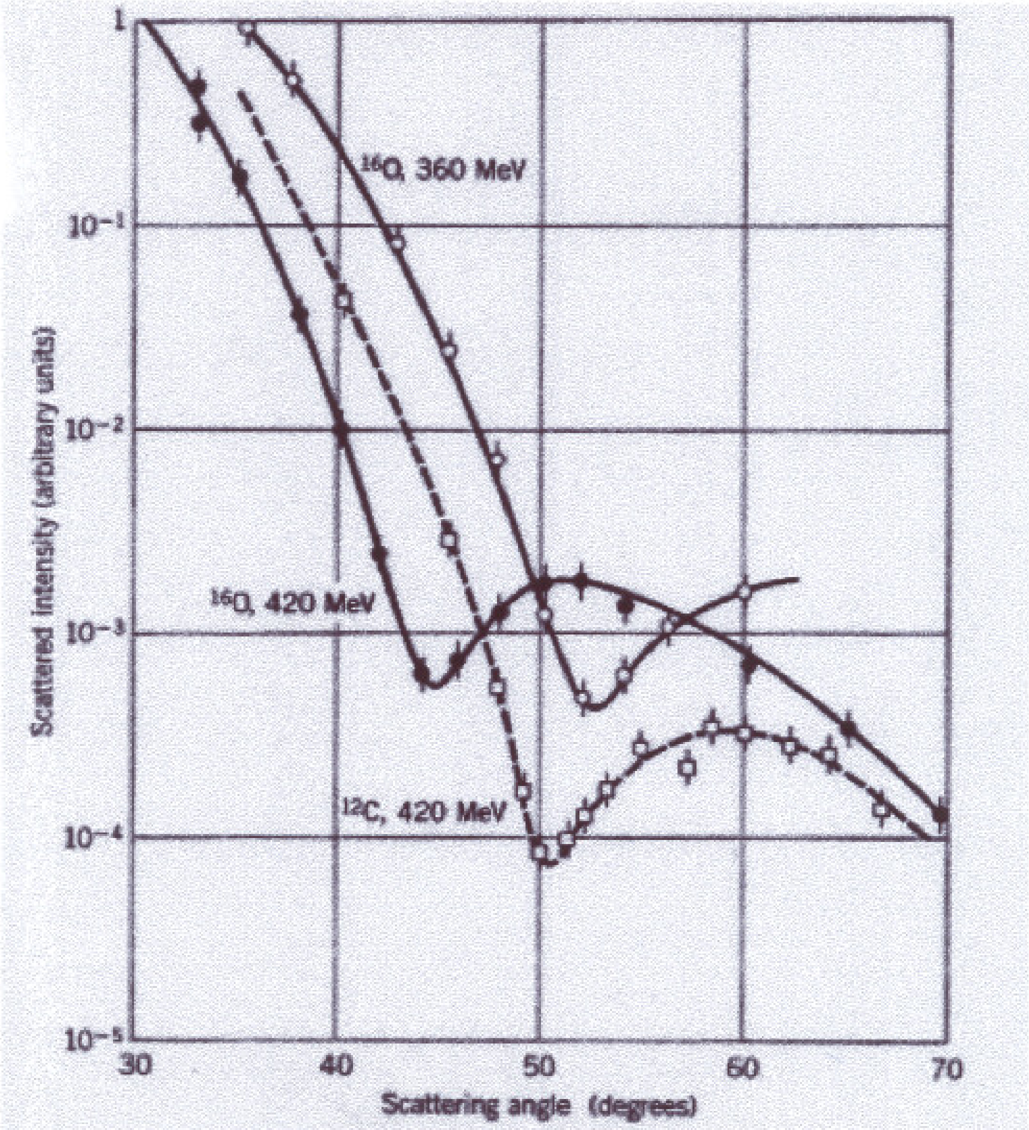


10^3
10^2
10
1
Rutherford formula
Scattered intensity at 60° (relative units)
10
15
20
25
30
35
40
45
Alpha-particle energy (MeV)


1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
Scattered intensity (arbitrary units)
30
40
50
60
70
Scattering angle (degrees)
^{16}O, 360 MeV
^{16}O, 420 MeV
^{12}C, 420 MeV

$m_n = 1.008665\ u$

$M(^1H) = 1.007825\ u$

$M(^2H) = 2.014102\ u$

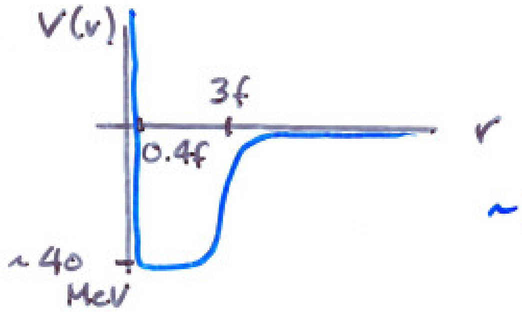
$$\frac{B}{c^2} = m_n + M(^1H) - M(^2H) = 0.002388\ u$$

converting $\quad B/c^2 = (0.002388\ u)\left(931.5\ \frac{MeV/c^2}{u}\right)$

$$B = 2.224\ MeV$$

which is a very loosely bound nucleus....

How to make a Deuteron?



~ nuclear binding energy

⇒ get 'em close and they stick.

... they can play catch with pions.

How tightly can a neutron & proton bind?

$$B = m_n c^2 + m_p c^2 - m_D c^2$$

aside: $m_{atom} c^2 = m_{nucleus} c^2 + Z m_e c^2 +$ electron binding

$m_{nucleus} c^2 \rightarrow \sim 10^9 - 10^{11}$ eV, $Z m_e c^2 \rightarrow 10^6 - 10^8$ eV, electron binding $\rightarrow 10 - 10^5$ eV

nuclear mass of Hydrogen = ~~m_p~~ $m_H - m_e$

nuclear mass of Deuterium = $m_D - m_e$

$$B = m_n c^2 + [m({}^1H)c^2 - \underline{m_e}]c^2 - [m({}^2H) - \underline{m_e}]c^2$$

cancel

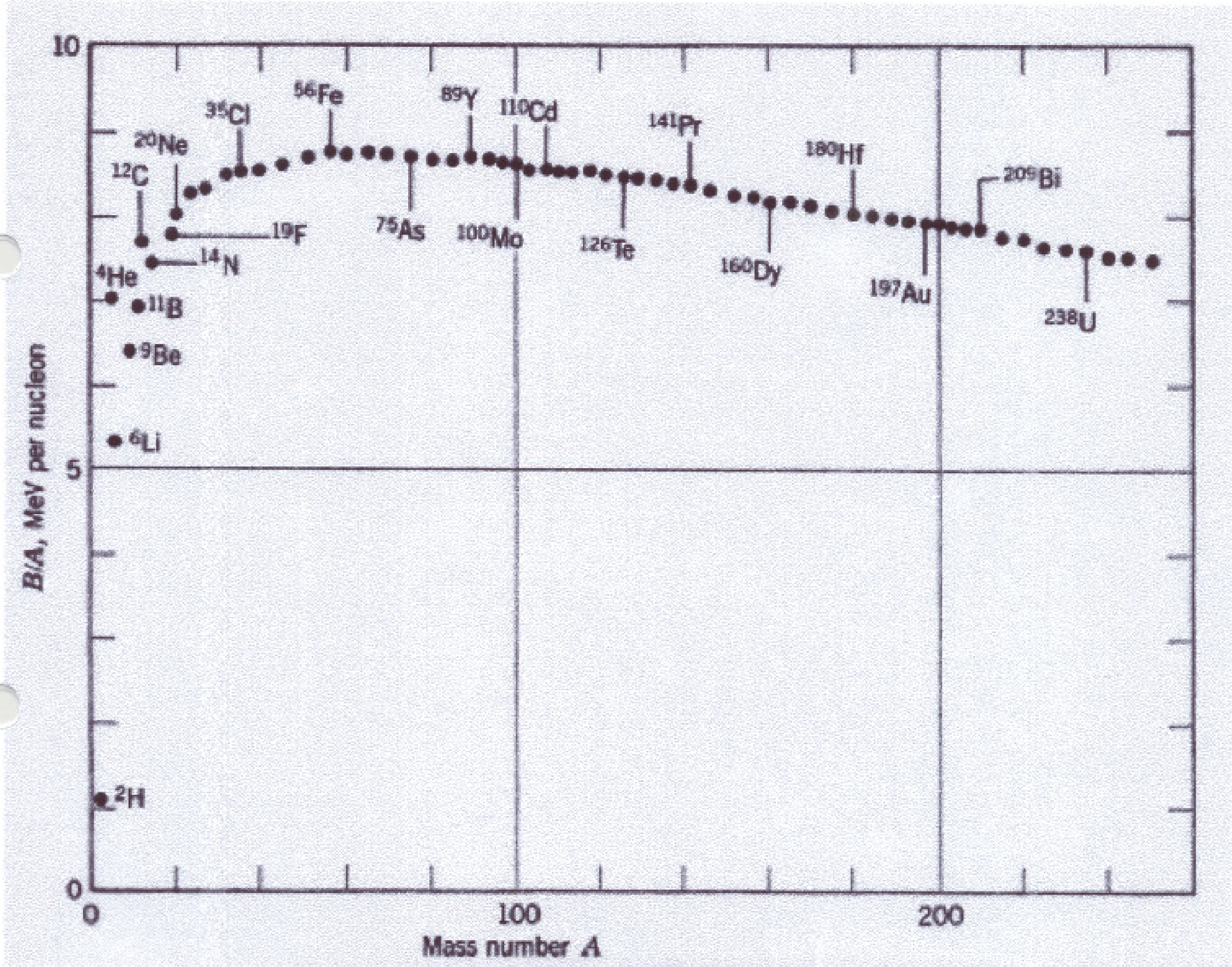
$$B = [m_n + m({}^1H) - m({}^2H)]c^2$$

cancellation of m_e's always happens ... so for ${}^A_Z X_N$:

$$B = [N m_n + Z m({}^1_1H_0) - m({}^A_Z X_N)]c^2$$

using atomic masses

your book: $B = [N m_n + Z M({}^1_1H_0) - M({}^A_Z X_N)]c^2$



10
5
0
100
200
Mass number A
B/A, MeV per nucleon
^{2}H
^{6}Li
^{9}Be
^{11}B
^{4}He
^{14}N
^{12}C
^{19}F
^{20}Ne
^{35}Cl
^{56}Fe
^{75}As
^{89}Y
^{100}Mo
^{110}Cd
^{126}Te
^{141}Pr
^{160}Dy
^{180}Hf
^{197}Au
^{209}Bi
^{238}U

Binding energy per nucleon?

$^{9}_{4}Be_{5}$ $\quad$ $^{56}_{26}Fe_{30}$ $\quad$ $^{238}_{92}U_{146}$

$\downarrow$

$M(^9Be) = 9.0121\ u$

$$B = \left[(5)(1.008665\,u) + (4)(1.007276\,u) - 9.0121\,u\right] 931.5\,\frac{MeV}{u}$$

$$= (5.043325 + 4.029104 - 9.0121)(931.5\ MeV)$$

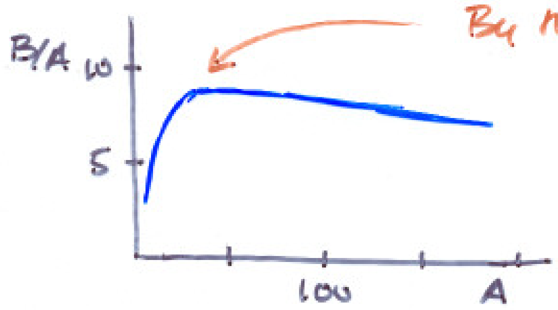
$$= (0.060329)(931.5\ MeV)$$

$B(^9Be) = 56.196\ MeV.$

per nucleon: $\frac{B}{A} = B(^9Be)/\text{nucleon} = \frac{56.196\ MeV}{9} = 6.24\ MeV$

likewise $\quad \frac{B}{A} = 492.3\ MeV \quad ^{56}_{26}Fe \Rightarrow B/A = 8.791$

$\frac{B}{A} = 7.571\ MeV \quad ^{238}_{92}U$



By the time all nucleons are ~ surrounded by others -- there is a saturation

$\leftarrow$ ~ 8 MeV/nucleon

Nuclear Models...

very much a "phenomenological" exercise.

Two broad models, historically.

Liquid Drop Model – Bohr 1936

nucleons act like molecules confined to a drop of fluid... Bohr had fission in mind

Three influences:

1. Volume effect.

   since $B/A \sim$ constant, $B \propto A$
   $\propto V$

2. Surface effect.

nuclei on surface will reduce the overall binding energy.

3. Coulomb repulsion.

   total Coulomb energy ~ work required to assemble Z protons from ∞ to the volume of nucleus.

$$\propto \frac{Z(Z-1)}{A^{1/3}}$$

led to the "semi empirical binding energy formula"

$$B({}^A_Z X_N) = a_V A - a_A A^{2/3} - \frac{3}{5} \frac{Z(Z-1)e^2}{4\pi\epsilon_0 r} - a_s \frac{(N-Z)^2}{A} + \delta$$

fitting to data -- for $A \gtrsim 15$:

$a_V = 14$ MeV "volume"

$a_A = 13$ MeV "surface"

$a_s = 19$ MeV "symmetry"

$$\delta = \begin{cases} +\Delta & \text{even-even} \\ 0 & \text{even-odd} \\ -\Delta & \text{odd-odd.} \end{cases}$$

$\Delta = 33\ \text{MeV} \cdot A^{-3/4}$
"pairing"

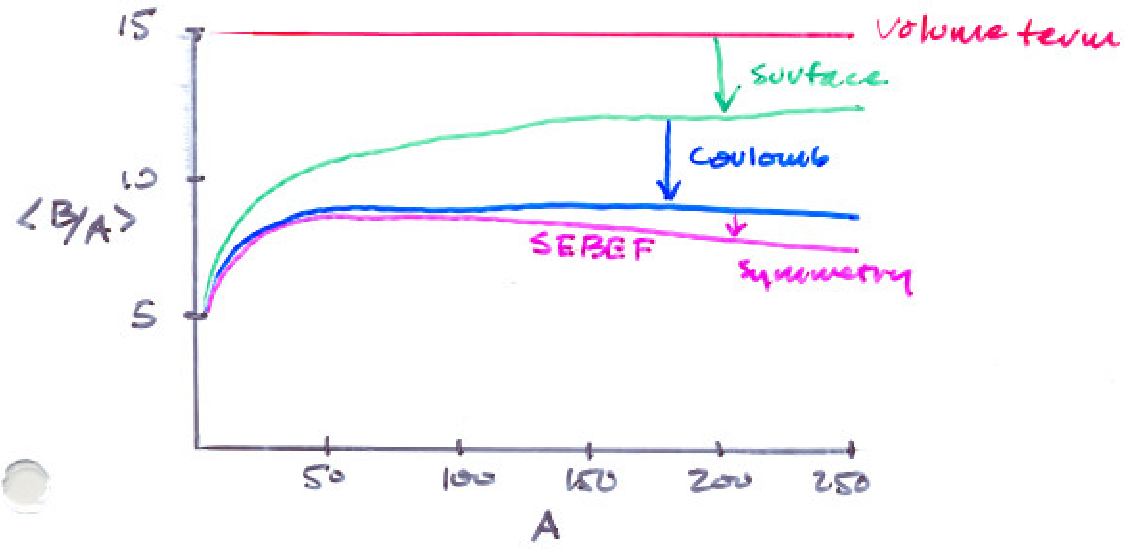
Independent Particle Model ("Shell Model")

each nucleon moves in a well-defined

quantum mechanical orbit $\Rightarrow$ quantized orbit.

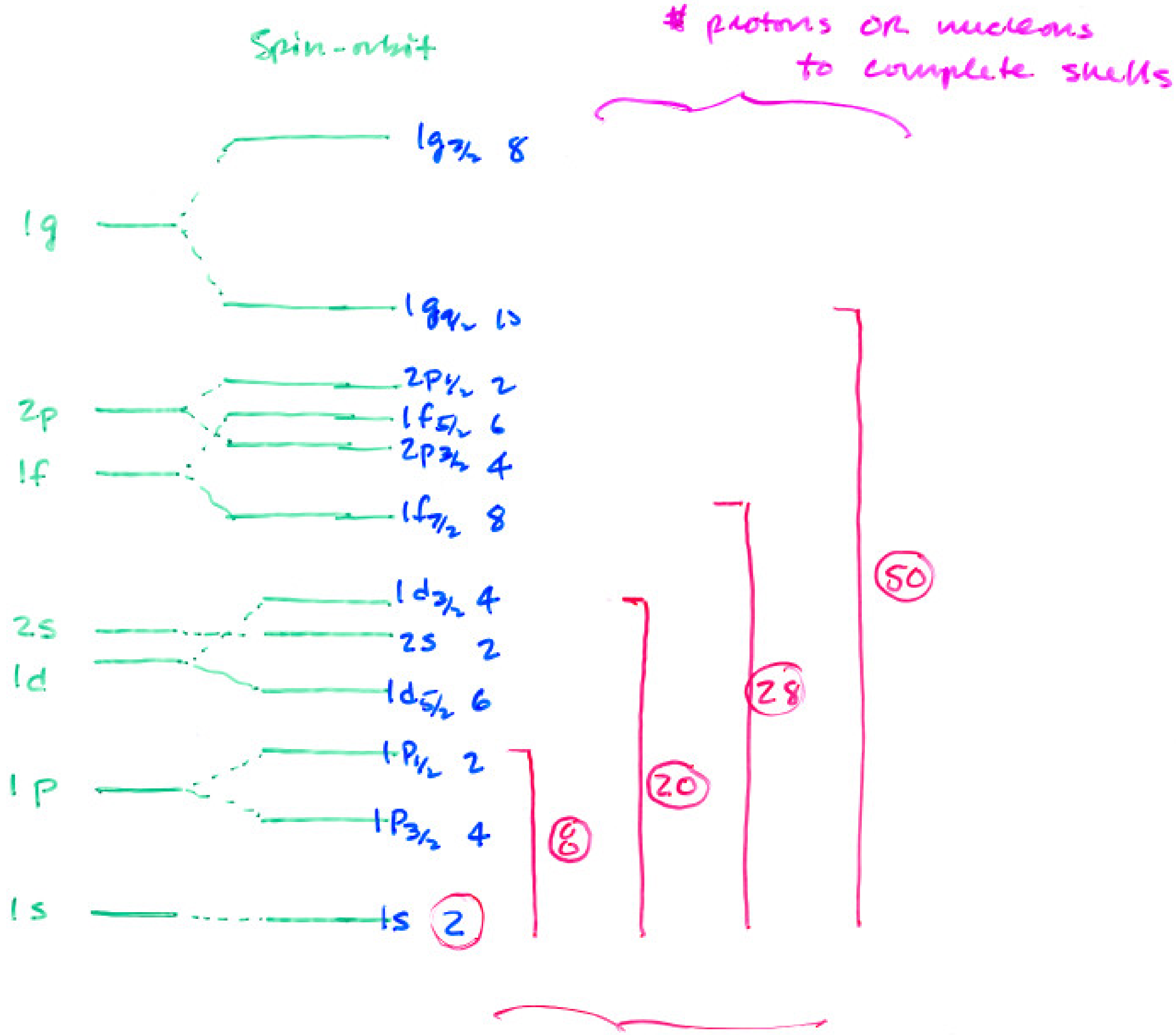
- in the average potential created by all of the other nucleons.

... like a Fermi Gas in some ways.



15
10
5
⟨B/A⟩
Volume term
surface
Coulomb
SEBEF
symmetry
50
100
150
200
250
A

For a square-well-like nuclear potential, we found



nuclear closed shells – like Noble gases

– very stable

Maria Goeppert Mayer

1963

d number of identical nucleons in a state j › give a total spin j and a magnetic moment t of a single particle in that state.

given nucleus the "pairing energy" of the the same orbit is greater for orbits with

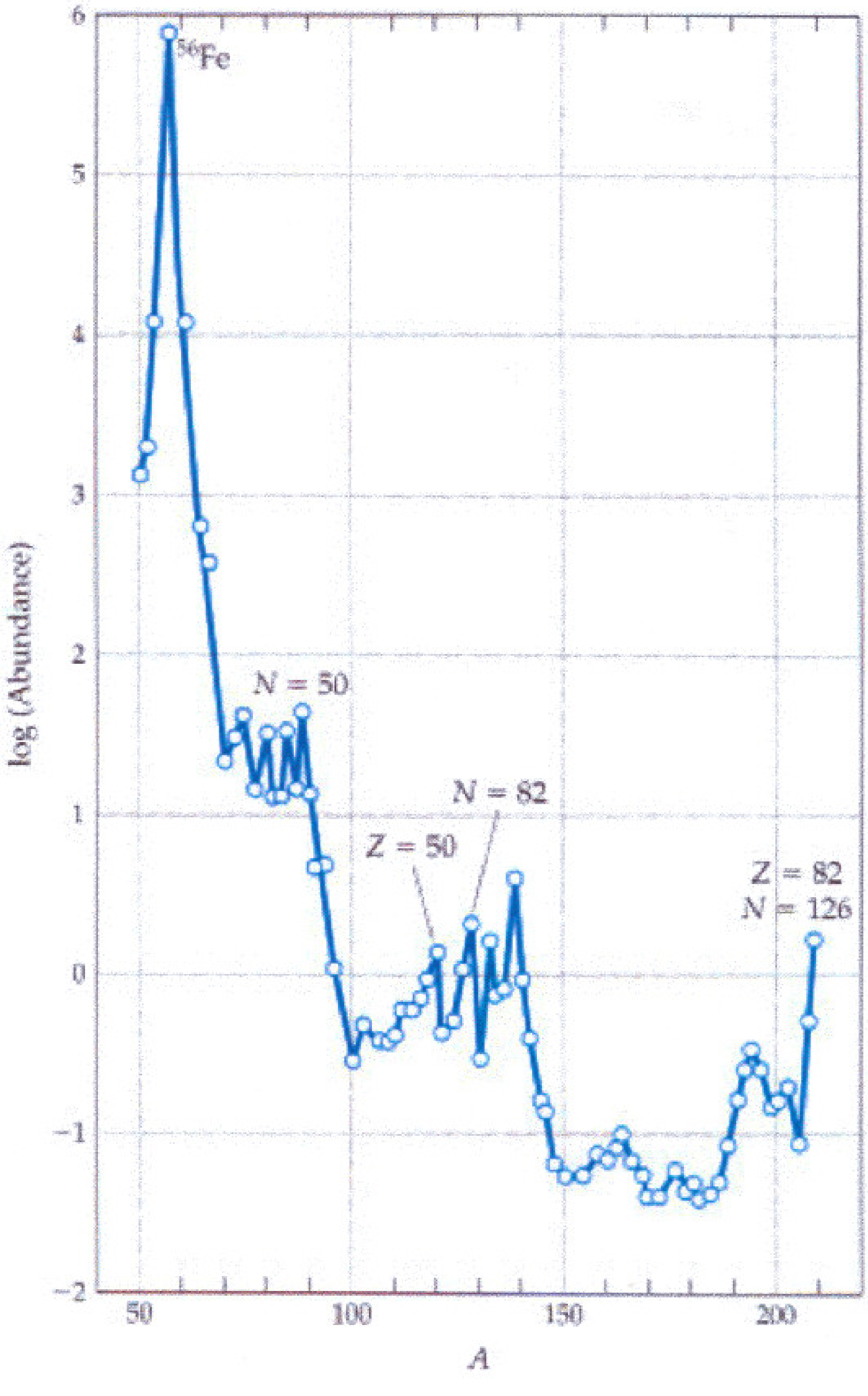
ssumption leads to the prediction that the lue appears less often as the spin of odd the energy order of Table II predicts. For he $3s_{1/2}$ level has slightly lower energy than he pairing energy of $h_{11/2}$ exceeds that of $s_{1/2}$ in this difference, the spin 11/2 would not lei, but 1/2 would be observed instead. ome theoretical justification for assumptions and this will be discussed in the next paper. on 2 has the consequence that all even-even spin zero. The main testing ground for the nent consists then in the spins and mag- its of the nuclei of odd A. According to the we will adopt for these nuclei the extreme picture, ascribing both spin and magnetic the last odd proton or neutron.

NETIC MOMENTS OF ODD A NUCLEI

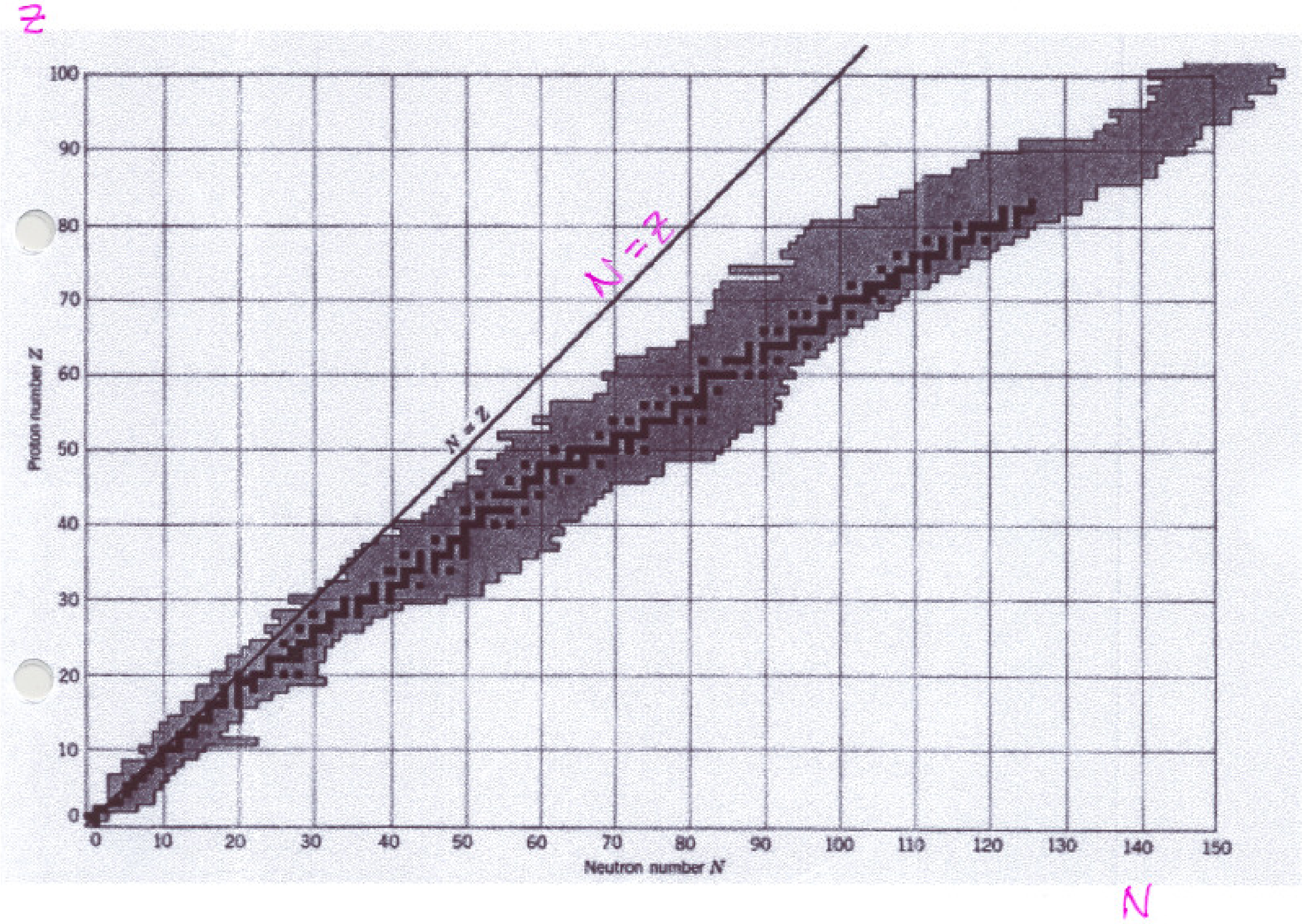
ition 3 were exactly correct, the magnetic all odd nuclei could be computed by the om the known gyromagnetic ratios of neutron. The two possible cases, $l=j-\frac{1}{2}$ for given j value lead to two computed lot of magnetic moment μ against j for odd neutron number and two (different) clei with odd proton number. These theo- will be referred to as "Schmidt lines."[5] The l values lie in between the Schmidt lines, coincide with them. For each j value the oments seem to fall into two groups, one close to the line corresponding to $l=j+\frac{1}{2}$, attered from near the line corresponding to bout halfway. It turns out that the assign- ls made attributes to the first group an odd state $l=j+\frac{1}{2}$, to the second one $l=j-\frac{1}{2}$. In scussion l-values as derived from magnetic ll be quoted only if the magnetic moment of

TABLE II. Order of energy levels obtained from those of a square well potential by spin-orbit coupling.

Osc. no.	Square well	Spin term	No. of states	Shells	Total no.
0	1s	$1s_{1/2}$	2	2	2
1	1p	$1p_{3/2}$	4		
		$1p_{1/2}$	2	6	8
2	1d	$1d_{5/2}$	6		
		$1d_{3/2}$	4	12	
	2s	$2s_{1/2}$	2		20
		$1f_{7/2}$	8	8	28
3	1f	$1f_{5/2}$	6		
	2p	$2p_{3/2}$	4	22	
		$2p_{1/2}$	2		
		$1g_{9/2}$	10		50
4	1g	$1g_{7/2}$	8		
	2d	$2d_{5/2}$	6		
		$2d_{3/2}$	4	32	
	3s	$3s_{1/2}$	2		
		$1h_{11/2}$	12		82
5	1h	$1h_{9/2}$	10		
	2	$2f_{7/2}$	8		
		$2f_{5/2}$	6	44	
	3p	$3p_{3/2}$	4		
		$3p_{1/2}$	2		126
		$1i_{13/2}$	14		
6	1i, 2g, 3d, 4s	$1i_{11/2}$			



^{56}Fe
N = 50
N = 82
Z = 50
Z = 82
N = 126
log (Abundance)
A
6
5
4
3
2
1
0
−1
−2
50
100
150
200



Z
N = Z
N = Z
Proton number Z
Neutron number N
N
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150

Shell Model and stability -- makes some sense.

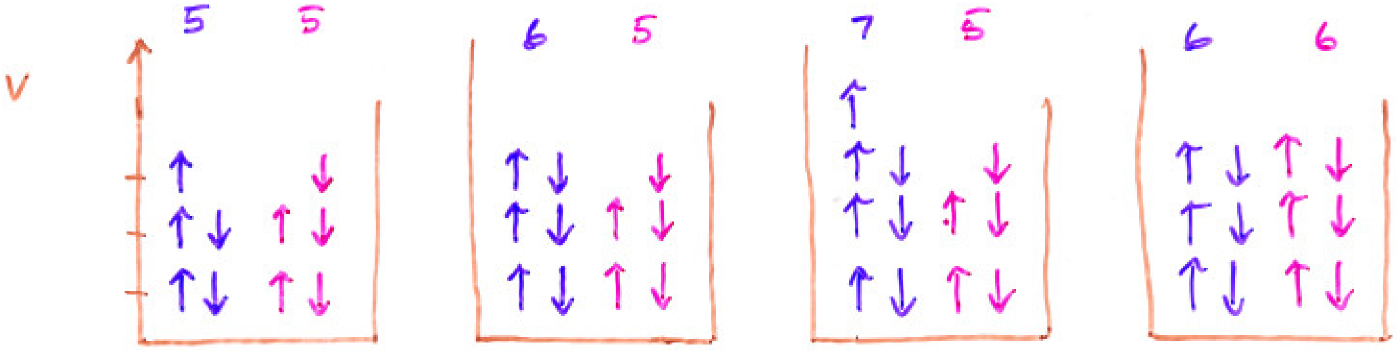
Nucleus Z - N	# stable nuclei	# very long lived nuclei
even - even	155	11
even - odd	53	3
odd - even	50	3
odd - odd	4	5

Want to be <u>really</u> stable?

Be a nucleus with an even # protons <u>and</u> even # neutrons.

Why? Pauli Exclusion.

↑ neutron ↑ proton



$^{10}_{5}B$	$^{11}_{5}B$	$^{12}_{5}B$	$^{12}_{6}C$
~ stable	~ stable	unstable	stable

Radioactivity.

Conserved:
1. nucleon number, A
2. electric charge, net.
3. energy
4. momentum.
5. angular momentum.

3 kinds of nuclear radioactive decay:

1. alpha decay. ^4He
2. beta decay. e^- or e^+ (β^- or β^+)
3. gamma decay. γ

Generic description -

N = # nuclei present

The rate at which they decay

$$-\frac{\Delta N}{\Delta t} = \text{Activity}, R \propto N$$

So,

$$\frac{dN}{dt} = -\lambda N$$

λ - "decay constant"

or

$$\frac{dN}{N} = -\lambda\, dt$$

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

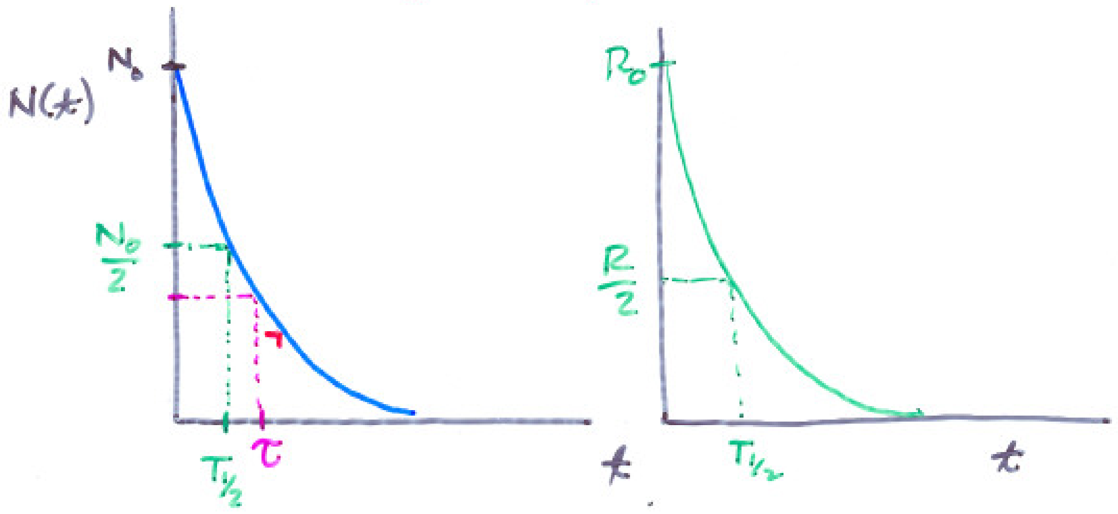
$$N = N_0 e^{-\lambda t}$$

obviously N_0 = # at $t = 0$.

Decay Rate

$$R \equiv \left|\frac{dN}{dt}\right| = N_0 \lambda e^{-\lambda t} = R_0 e^{-\lambda t}$$

both useful



Two standard "measures" of the particular decay.

Half-life: time it takes for a sample to decay ½ of its original number, $T_{1/2}$

$$N = \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$e^{\lambda T_{1/2}} = 2$$

$$\lambda T_{1/2} = \ln 2$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Mean lifetime: the time equal to $1/\lambda \equiv \tau$

$$N(t) = N_0 e^{-t/\tau}$$

$$N(\tau) = N_0 e^{-1} = \frac{N_0}{e} = \frac{N_0}{2.718} = 0.368\, N_0$$

Units: $[T_{1/2}] = s$

$[\tau] = s$

$[R] =$ decays per s

$1\ Ci = 3.7\times10^{10}$ decays/s "Curie"

$1\ Bq = 1$ decay/s "Becquerel" SI

often mCi & μCi are practical

Example $T_{1/2}$ of ${}^{226}_{88}Ra$ is 1.6×10^{3} years.

What is the activity at this time? $N_0 = 3\times10^{16}$ nuclei.

$$T_{1/2} = 1.6\times10^{3}\,y\left(\frac{3.15\times10^{7}\,s}{y}\right) = 5.0\times10^{10}\,s$$

from $T_{1/2} = \frac{0.693}{\lambda}$, $\lambda = 1.4\times10^{-11}\,s^{-1}$

$$R_0 = \lambda N_0 = (1.4\times10^{-11}\,s^{-1})(3\times10^{16}) = 4.2\times10^{5}\ \text{decays/s}$$

$$(4.2\times10^{5}\ \text{decays/s})\left(\frac{1\ Ci}{3.7\times10^{10}\ \text{decays/s}}\right) = 11.3\,\mu Ci$$

Disintegration Energy.

Suppose we have the following generic decay chain:

$$ {}^{A}_{Z}X_N \longrightarrow D + y + Q $$

daughter nucleus

light product

energy

Then the "Q" of the decay

$$ Q = \left[M({}^{A}_{Z}X_N) - M(D) - M(y) \right] c^2 \qquad J $$

or in u's :

$$ Q = \left(M({}^{A}_{Z}X_N) - M(D) - M(y) \right] \cdot 931.5 \ \underline{\text{MeV}}/u $$

Notice from before $\qquad Q = -B$

if $\quad B > 0$, $Q < 0 \quad \Rightarrow$ nucleus <u>is</u> stable

if $\quad B < 0$, $Q > 0 \quad \Rightarrow$ nucleus is unstable and MIGHT decay.

Other factors may inhibit decay.

$\boxed{\alpha \text{ decay}}$

$$^{A}_{Z}X_{N} \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

parent nucleus

daughter nucleus

$\boxed{\beta \text{ decay}}$

$$^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + \beta^{-}$$

$$^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + \beta^{+}$$

notice, you can think of this as.

$$n \rightarrow p + \beta^{-}$$

within the nucleus

$$p \rightarrow n + \beta^{+}$$

$\boxed{\gamma \text{ decay}}$

$$^{A}_{Z}X^{*} \longrightarrow {}^{A}_{Z}X + \gamma$$

literally, the radiative transition from a nucleus in an excited state (shell) to a lower one

α Decay.

not all nuclei are alpha-emitters

Radium and Uranium isotopes are famous α emitters

Radon 222 is also...

$${}^{A}_{Z}X_N \rightarrow {}^{A-4}_{Z-2}D_{N-2} + {}^{4}_{2}He_2$$

$$Q = \left[M({}^{A}_{Z}X_N) - M({}^{A-4}_{Z-2}D_{N-2}) - M({}^{4}_{2}He_2)\right]c^2$$

Example Radium 226 $\longrightarrow$ Radon 222 + α

$M({}^{226}Ra) = 226.025406\ u$

$M({}^{222}Rn) = 222.017574\ u$

$M({}^{4}He) = 4.002603\ u$

$$Q = (226.025406 - 222.017574 - 4.002603)u \cdot 931.5 \frac{MeV}{u}$$

$$= (0.005229\ u)(931.50\ MeV/u)$$

$$Q = 4.87\ MeV \quad > 0 \Rightarrow$$ unstable and decay might be allowed.