so, $$P_\alpha = P_D$$

and $$Q = M(X) - M(D) - M(\alpha) = K_D + K_\alpha$$

assume both α and D move at non-relativistic speeds:

$$Q = \frac{P_D^2}{2M_D} + \frac{P_\alpha^2}{2M_\alpha} = \frac{P_\alpha^2}{2M_\alpha} + \frac{P_\alpha^2}{2M_D}$$

$$Q = \frac{P_\alpha^2}{2}\left(\frac{1}{M_\alpha} + \frac{1}{M_D}\right)$$

$$= \frac{2K_\alpha M_\alpha}{2}\left(\quad\right) = K_\alpha M_\alpha\left(\frac{1}{M_\alpha} + \frac{1}{M_D}\right)$$

$$Q = K_\alpha\left(1 + \frac{M_\alpha}{M_D}\right)$$

or.

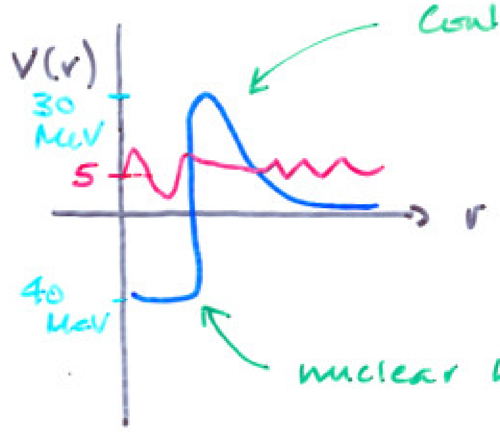
$$K_\alpha = \frac{M_D}{M_\alpha + M_D}\,Q$$

Here $$K_\alpha = \left(\frac{222}{226}\right)(4.87 \text{ MeV}) = 4.8 \text{ MeV}$$

so, probably non-relativistic assumption okay.

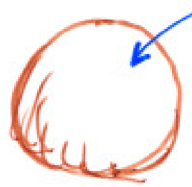
What's α decay?

Quantum mechanical tunneling.



We just saw the K_α for ^{226}Ra decay was $\sim \boxed{+5\ MeV}$

and it tunnels through the Coulomb barrier potential of the ^{226}Ra nucleus.



lots of protons and neutrons

sometimes 2p and 2n stick together inside of ^{226}Ra

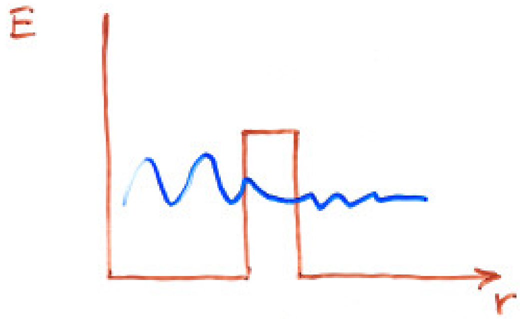
^{226}Ra

with $K_\alpha = 5\ MeV \Rightarrow v_\alpha \sim 2\times10^{7}\ m/s$

So, it goes back and forth, banging on the Coulomb potential in time intervals of $\Delta t = \frac{2R}{v_\alpha} \sim 7\times10^{-22}\ s$

$\Rightarrow$ at frequency of $1.4\times10^{21}\ Hz$

go back to the barrier penetration models & the Schrödinger equation for a model.



remember the transmission coefficient, $T(E)$?

→ that's the probability of escape.

$$T(E) = e^{-(\text{complicated integral of } r)}$$

very sensitive to K_α

for our ^{226}Ra example of $K_\alpha = 4.7$ MeV

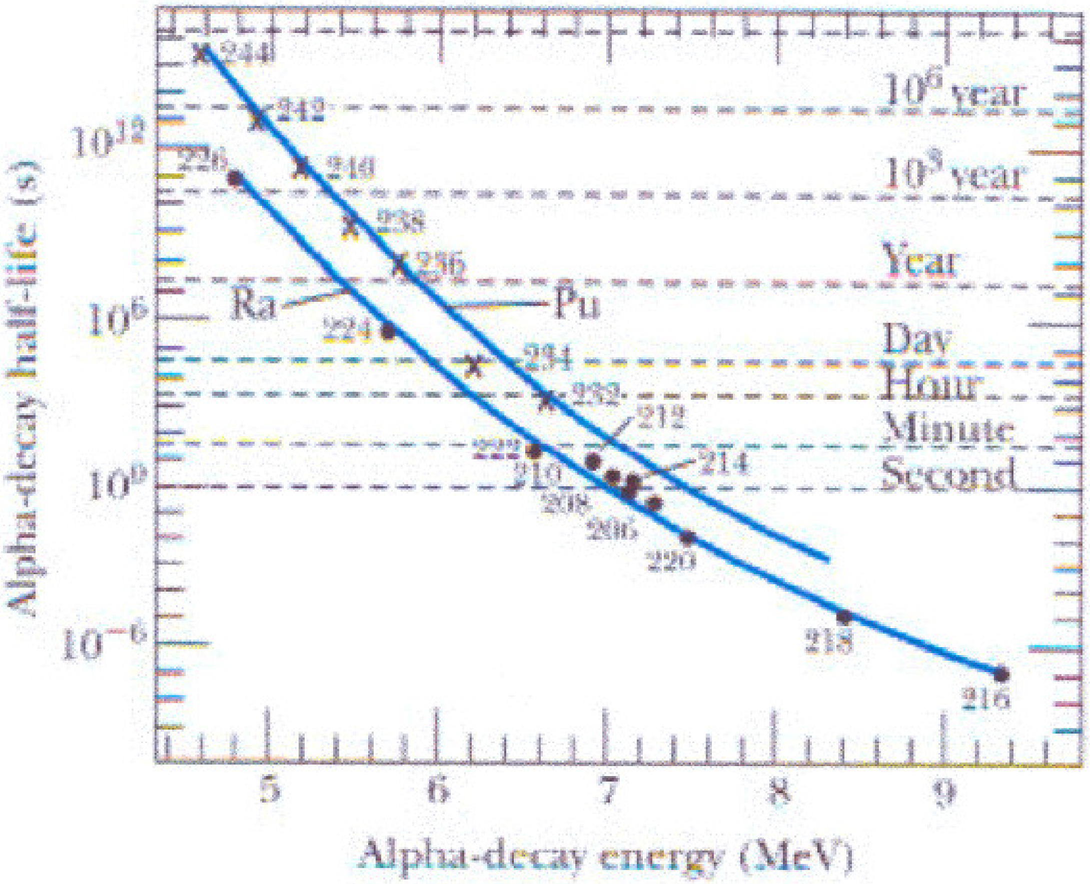
$$T(4.7\,\text{MeV}) = 10^{-34}$$

But there are 10^{21} tries per second $10^{-34} \cdot 10^{+21} = 10^{-13}\,s^{-1}$

That's λ. From $\lambda = \frac{\ln 2}{T_{1/2}}$, $T_{1/2} = \frac{0.693}{10^{-13}} = 7 \times 10^{12}\,s$
$= 200{,}000\,y$

The actual $T_{1/2}$ is $1600\,y$.

Had $K_\alpha = 5.3$ MeV instead of 4.7 MeV? ✓ (!)



Alpha-decay half-life (s)
Alpha-decay energy (MeV)
10^{12}
10^{6}
10^{0}
10^{-6}
5
6
7
8
9
10^6 year
10^3 year
Year
Day
Hour
Minute
Second
Ra
Pu
244
242
240
238
236
234
232
226
224
222
212
214
210
208
206
220
218
216

β Decay — now here's a story.

Beta decay's had been studied for decades

19-teens, people were carefully measuring $E(\beta)$

for example

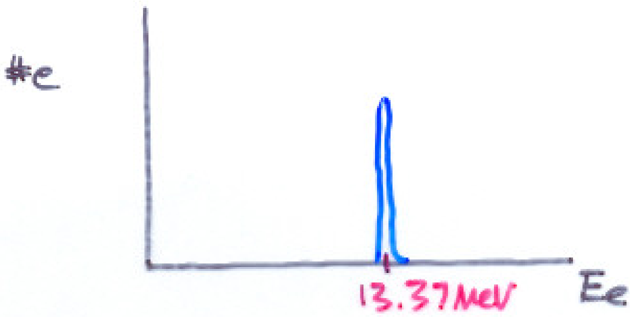
$^{12}_{5}B \rightarrow {}^{12}_{6}C + {}^{0}_{-1}e$

Not good. ⇒ a violation of conservation of energy/mom.

Bohr actually suggested this.

$$Q = [M({}^{12}_{5}B) - M({}^{12}_{6}C)]c^2 = 13.37 \text{ MeV} = K_{max}$$ above.

But.. there is a spectrum of $E(e)$, when it should have been:



if C ⟵ B ⟶ e

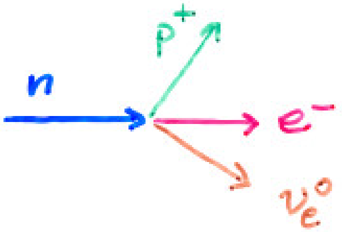
!!

In 1930 Pauli had an idea which he hated.

"I have done a terrible thing today. I have postulated a particle that cannot be detected."

He called it the "neutron" -- but his suggestion was so timid that nobody paid much attention and Chadwick used the name "neutron"

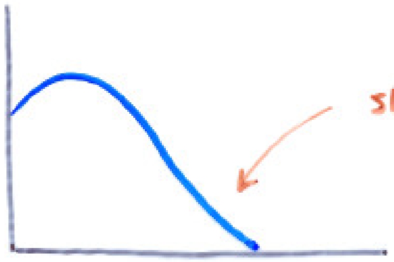
1934 Enrico Fermi wrote down the nearly complete theory of -- what he renamed "little neutron" = neutrino



inside of the nucleus.

share the Q

What's the mass of the ν ? zero we thought



shape of β spectrum is sensitive to $m(\nu)$

now we know $m(\nu) \sim 10^{-3}$ eV
ish

going on inside nucleus & other β decays happen –

$p^+ +$ (energy from nucleus) $\rightarrow n + e^+ + \nu_e$

positron –– anti-electron

So, decays like

$^{14}_{6}C \rightarrow {}^{14}_{7}N + \beta^- + \nu$ beta decay

$^{12}_{7}N \rightarrow {}^{12}_{6}C + \beta^+ + \nu$ positron decay

$n \rightarrow p + \beta^- + \nu$ (free neutron)

and a nucleus can eat its own electron ...

"electron capture"

$$^{A}_{Z}X_N + {}^{0}_{-1}e \rightarrow {}^{A}_{Z-1}X + \nu$$

competes with positron decay

Gamma Decay.

Often, when $X \to D^* + \alpha$ or $e + \nu$

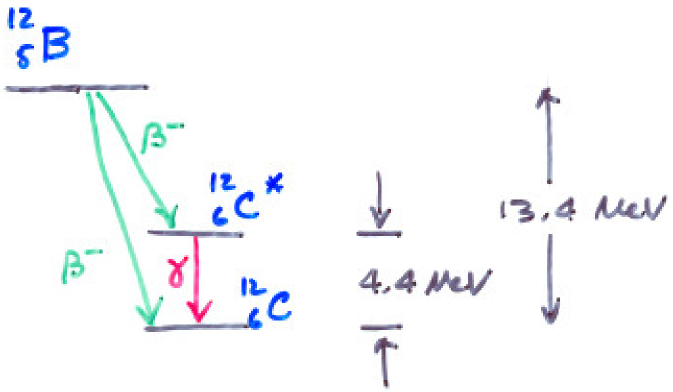
left in an excited nuclear state

$D^* \to D + \gamma$

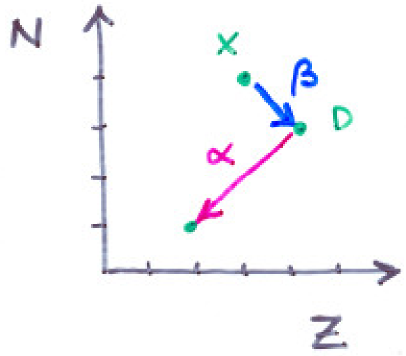
taking away angular momentum

Also... collisions can excite a nucleus which then γ-decays.

for example:



Cascades



and so-on and so-on... and so-on!

Radioactivity can be:

- natural → found in nature, created by stars.
- artificial → created in the lab, nuclear reactors, nuclear explosives.

From decay chains... transmutation happens by -

changing A by α decay $A \to A-4$

change Z by α decay $Z \to Z-2$

Z by β decay $Z \to Z+1$

There are 3 naturally occurring radioactive chains:

$^{238}_{92}U$ $T_{1/2} = 4.47 \times 10^9$ y $\longrightarrow$ $^{206}_{82}Pb$ "Uranium"

$^{235}_{92}U$ $T_{1/2} = 7.04 \times 10^8$ y $\longrightarrow$ $^{207}_{82}Pb$ "Actinium"

$^{232}_{90}Th$ $T_{1/2} = 1.41 \times 10^{10}$ y $\longrightarrow$ $^{208}_{82}Pb$ "Thorium"

and one artificial series

$^{237}_{93}Np$ $T_{1/2} = 2.14 \times 10^6$ y $\longrightarrow$ $^{209}_{83}Bi$ "Neptunium"

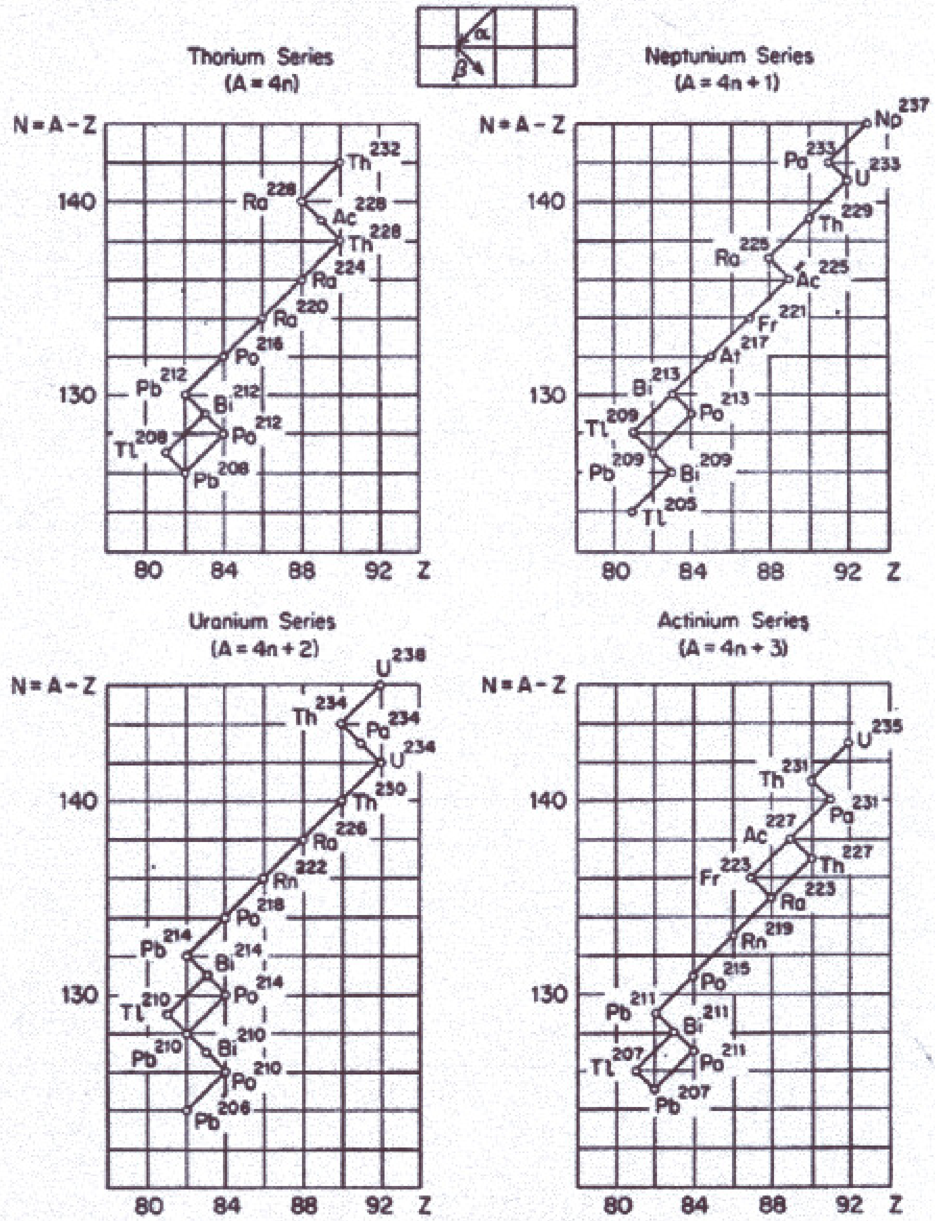
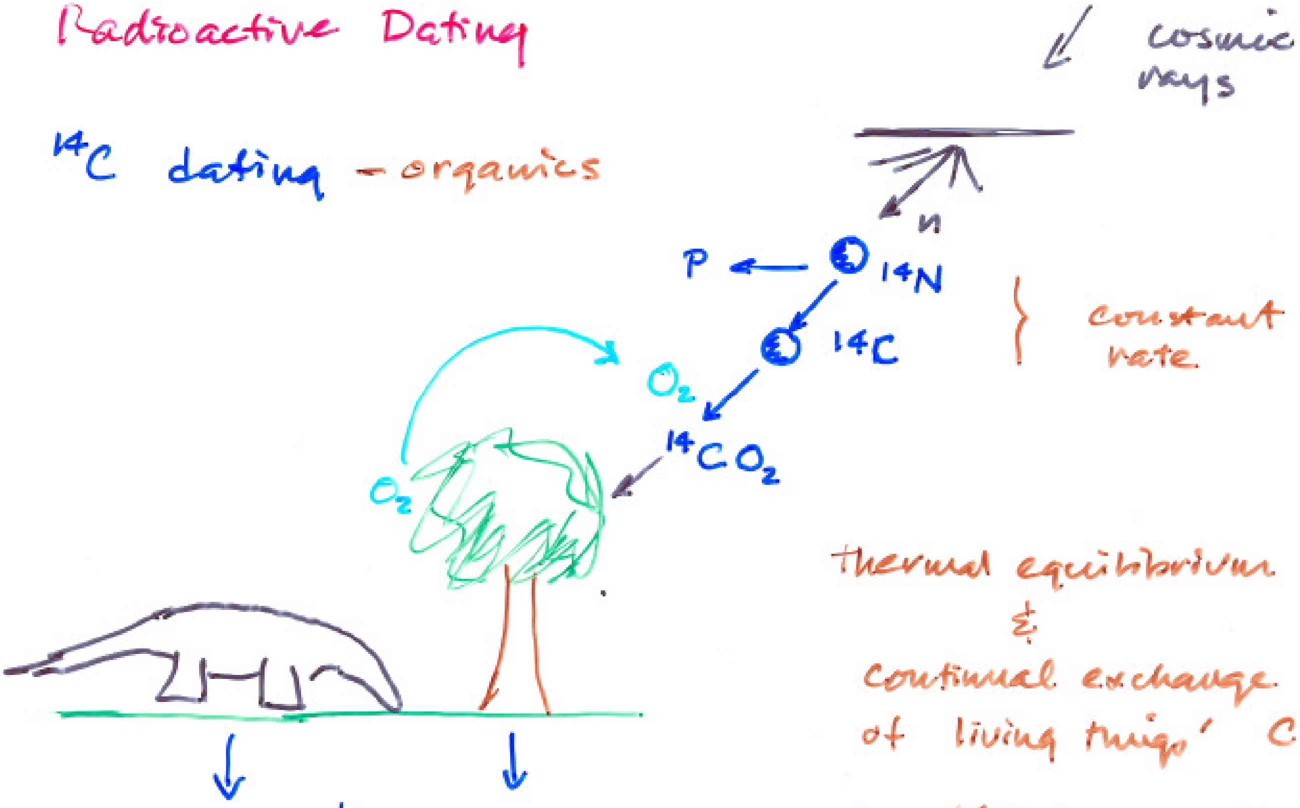


Fig. 11. Decay schemes of the four families of natural radioactivity.

Radioactive Dating

^{14}C dating – organics



constant rate

die, stops eating new C

thermal equilibrium & continual exchange of living things' CO_2

$\Rightarrow$ $^{14}C/^{12}C$ ratio same in biological creatures/plants as in atmosphere

the $\frac{^{14}C}{^{12}C}$ ratio at death changes as ^{14}C decays

$$\frac{^{14}_{6}C}{^{12}_{6}C} \simeq 1.3 \times 10^{-12}$$

$$T_{1/2}(^{14}C) = 5730\ y$$

$$^{14}_{6}C \longrightarrow {}^{14}_{7}N + \beta + \bar{\nu}$$

~ 0.016 MeV peak.

Measure **ACTIVITY**

Example – capture a sample of CO_2 from the atmosphere into $V = 200\ cm^3$

$P = 2.0 \times 10^4\ Pa$

$T = 295\ K$

Presume your ability to count ^{14}C is perfect – after a week, how many counts would you record?

$$n = \frac{PV}{RT} = \frac{[2.0 \times 10^4\ N/m^2][2 \times 10^{-4}\ m^3]}{(8.314)(J/mol\ K)(295K)}$$

$$= 1.63 \times 10^{-3}\ mol$$

$$N = N_A n = (6.02 \times 10^{23}\ molecules/mol)(1.63 \times 10^{-3}\ mol)$$

$$= 9.82 \times 10^{20}\ molecules\ CO_2$$

$$= \#\ atoms\ of\ carbon$$

$$\#\ ^{14}C \cong (1.3 \times 10^{-12})(9.82 \times 10^{20}) = 1.28 \times 10^9\ atoms.$$

$$R_0 = N_0 \lambda = (1.28 \times 10^9)\left(\frac{\ln 2}{T_{1/2}}\right)$$

$$= (1.28 \times 10^9)\frac{(0.693)}{(5730\ y)}\left(\frac{1\ y}{3.15 \times 10^7\ s}\right)$$

$$R_0 = 4.91 \times 10^{-3}\ decays/s$$

$$R_0 = 2969\ decays/wk.$$

now a piece of old wood is put into same detection device... after a week, 1420 counts are recorded. How old is the wood?

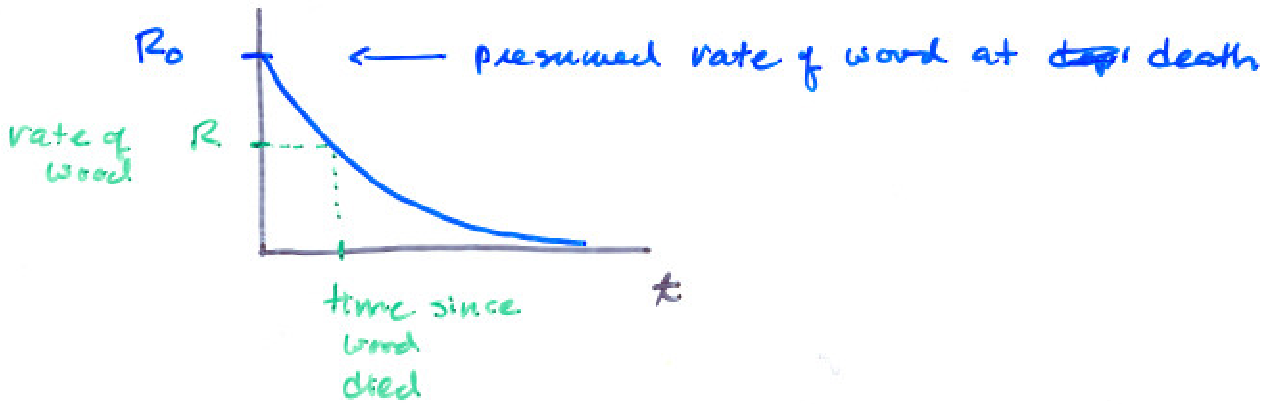
$$R = R_0 e^{-\lambda t}$$

$R_0 = 2969 \ s^{-1}$
$R = 1420 \ s^{-1}$

$$t = \frac{1}{\lambda} \ln \left(\frac{2969}{1420} \right)$$

$$= \frac{5730}{0.693} \ln \left(\frac{2969}{1420} \right)$$

$$t = 6099 \ y$$



Pb dating — geological objects

^{204}Pb : STABLE
- nothing decays to it
- it does not decay

$\Rightarrow$ abundance is CONSTANT

BUT:

$^{238}U \quad T_{1/2} = 4.47 \times 10^9 y \longrightarrow {}^{206}Pb$

$^{235}U \quad T_{1/2} = 0.7 \times 10^9 y \longrightarrow {}^{207}Pb$

earth about $4.5 \times 10^9 y$ $\rightarrow$ 6 half-lives of ^{235}U

$\Rightarrow$ most of it is gone.

But ^{238}U is still decaying from its original abundance.

$$\frac{^{207}Pb}{^{204}Pb} = \text{constant} \quad \& \quad \frac{^{206}Pb}{^{204}Pb} \text{ still increasing}$$

abundance of

compared with

can sensitively date lead ore

meteorites ~ 4.5 By

oldest rock formations ~ 4 By