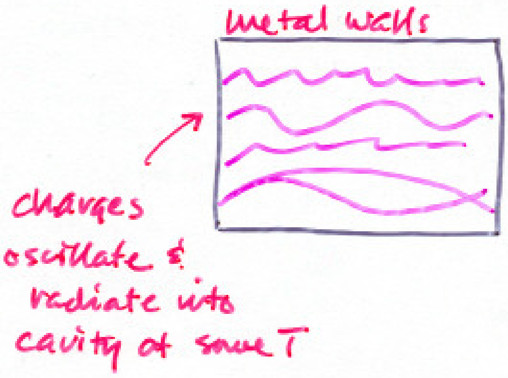
The definitive classical model — Lord Raleigh and James Jeans - Raleigh-Jeans formula

1900 ~ 1905 !

A theory of the RADIATION itself.



all normal modes of the radiation

Long wavelengths ... not many

short wavelengths ... lots

$$u(f,T) = N(f)\langle E\rangle$$

N.B.
experiments measure λ...
I'm talking f...
because it's easier here

For radiation

$$N(f) = \frac{8\pi f^2}{c^3}$$

$$u(f,T) = \frac{8\pi f^2}{c^3}\langle E\rangle$$

$$\langle E\rangle_{E\&M} = (\#\,\mathrm{dof})\cdot\tfrac{1}{2}kT = kT$$

so,

$$u(f,T)\,df = \frac{8\pi f^2}{c^3} kT\,df$$

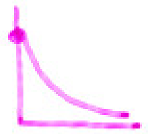
↖ !! rises as f^2! "ULTRAVIOLET catastrophie"

$\int u\,df = \infty$

Interlude... probability alá Boltzmann

The probability of finding a "system"... like an oscillator... with energy E above some minimum, E_0 is

$$P(E) = P_0 e^{-(E-E_0)/kT}$$



called the "Boltzmann Distribution" (ch 9)

For some probability distribution $p(x)$, the mean value of x is

$$\langle x\rangle = \frac{\int x\,p(x)\,dx}{\int p(x)\,dx}$$

OKAY... w/ Boltzmann Dist^n what's $\langle E\rangle$?

$$\langle E\rangle_{classical} = \frac{\int_0^\infty E\,P(E)\,dE}{\int_0^\infty P(E)\,dE}$$

$\int_0^\infty \Rightarrow$ any E available + classical assumption

$$= \frac{\int_0^\infty E P_0 e^{-E/kT}\,dE}{\int_0^\infty e^{-E/kT}\,dE}$$

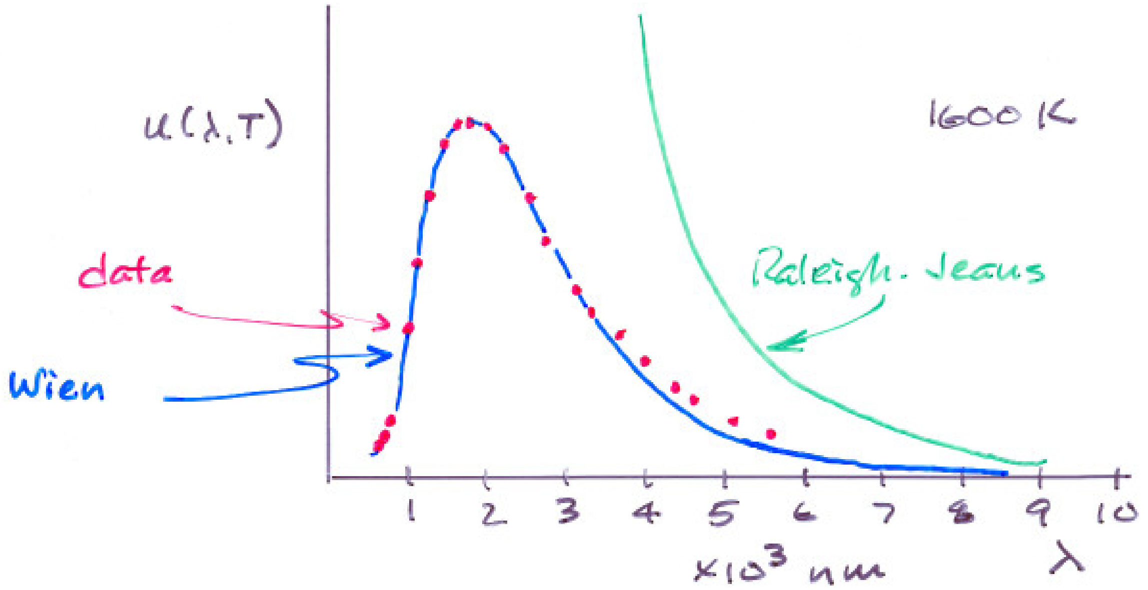
$$\langle E\rangle_{classical} = kT$$

bingo... Equipartition Rules!

How we doing?

well experiments about 1900 got better...

in the infrared wavelength regime



Wien undershoots at long λ, good at short λ

Raleigh-Jeans good at long λ... notsomuch at short λ

not so well.

Max Planck decides to attach the problem ~1890

he attacked

and attacked

and attacked... then in an act of "sheer desperation..."

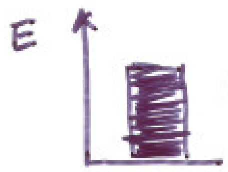
His model...

Based on Boltzmann's method of compartmentalizing energy.. to do the calculation

BUT.. the "box" remains in Planck's approach

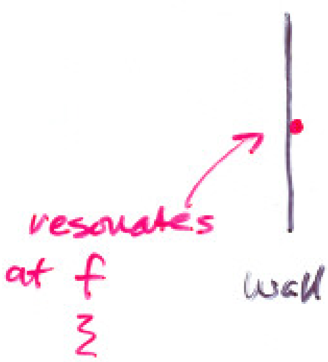
A model of the WALLS

R.J made the perfectly valid assumption



← the wall oscillators could resonate – and radiate – at any energy.

Planck found that if he restricted them to PARTICULAR frequencies (easier for him to work with frequency).. that he got good agreement



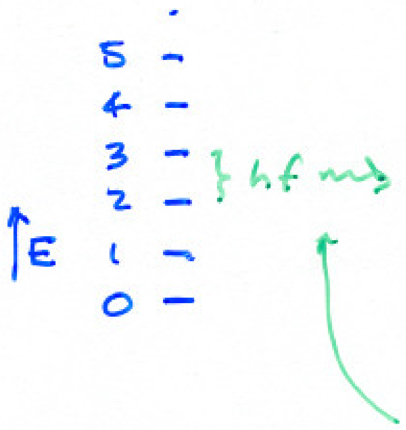
← radiation must have

$E = \text{constant} \cdot f$

or it will not vibrate...

no vibration, no radiation, thermal equilibrium, after all

He decided that the wall oscillators



can have total energy $E = nhf$

n is an integer

h is a constant → fit from data

oscillator can radiate or absorb only

$$\Delta E = hf$$

NOW, the average energy depends on ~~temp~~ frequency..

He too counted the number of states

$$N(\nu)\,df = \frac{8\pi f^2}{c^2}\,df$$

w/ frequency between f and $f + df$

and calculated

$$u(f,T) = N(f)\langle E \rangle$$

But... $\langle E \rangle$ is critically different

since $E = hf$

His use of Boltzmann's Distribution is DISCRETE

$$P(E) = P_0 \, e^{-nhf/kT}$$

But

$$\langle E \rangle = \frac{\sum_{n=0}^{\infty} (nhf)\, e^{-nhf/kT}}{\sum_{n=0}^{\infty} e^{-nhf/kT}}$$

aside...

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$

so, denominator says--

$$\sum_{n=0}^{\infty} e^{-nhf/kT} = \frac{1}{1-e^{hf/kT}}$$

numerator: let $\eta = hf/kT$

$$\sum n e^{-n\eta} = -\frac{d}{d\eta}\left(\sum e^{-n\eta} \right)$$

$$= -\frac{d}{d\eta}\left(\frac{1}{1-e^{-\eta}} \right) = \frac{e^{-\eta}}{(1-e^{-\eta})^2}$$

$$\sum_{n=0}^{\infty} n e^{-nhf/kT} = \frac{e^{-hf/kT}}{\left(1-e^{-hf/kT}\right)^2}$$

put it together...

$$\langle E \rangle = hf\,(1-e^{-hf/kT})\,\frac{e^{-hf/kT}}{(1-e^{-hf/kT})^2}$$

$$= \frac{hf}{e^{hf/kT}-1} \qquad \neq kT$$

So...

$$u(f,T)\,df = N(f)\,\langle E \rangle$$

$$u(f,T)\,df = \frac{8\pi f^2}{c^3}\,\frac{hf}{e^{hf/kT}-1}\,df$$

"Planck Distribution"

h comes from fitting...

$$h \simeq 6.626 \times 10^{-34}\ \text{J}\cdot\text{s} \qquad \text{Planck's Constant}$$

note let $\frac{hf}{kT} \ll 1$

- f small, λ large
- T large
- $h \to 0$

$$u(f,T) \longrightarrow \frac{8\pi f^2}{c^3}\,\frac{hf}{1+\frac{hf}{kT}-1} = \frac{8\pi f^2}{c^3}\cdot\frac{hf}{hf}\cdot kT$$

$$= \frac{8\pi f^2}{c^3}\,kT$$

Planck kept h finite...

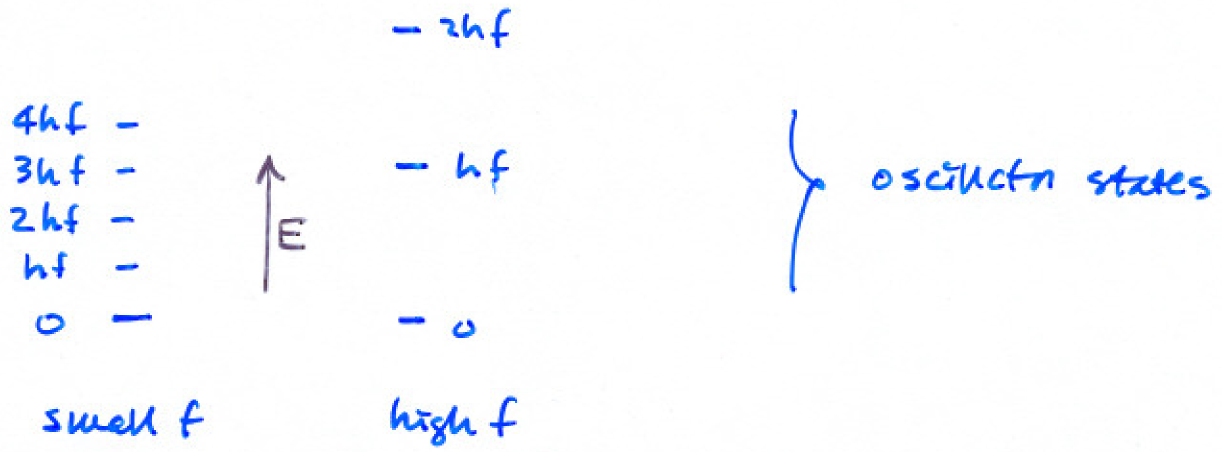
Raleigh-Jeans!

how about $\frac{hf}{kT} \gg 1$

$$u(f,T) \rightarrow \frac{8\pi f^2}{c^3}\,\frac{hf}{e^{hf/kT}} = \frac{8\pi f^3}{c^3}\, e^{-hf/kT}$$

which is Wien's law
with a and α found.

Why a suppression at high f ?



the characteristic energy is still kT and if $hf \gg kT$, then there is almost ∅ probability that even the first state is occupied

Max Planck

"

But even if the radiation formula proved to be perfectly correct, it would after all have been only an interpolation formula found by lucky guess-work and thus would have left us rather unsatisfied. I therefore strived from the day of its discovery to give it real physical interpretation…After some weeks of the most intense work of my life, light began to appear to me and unexpected views revealed themselves in the distance. Nobel Address, 1920

It was an act of desperation. For six years I had struggled with the blackbody theory. I knew the problem was fundamental and I knew the answer. I had to find a theoretical explanation at any price... correspondence, 1931

it reproduced the data over the entire range

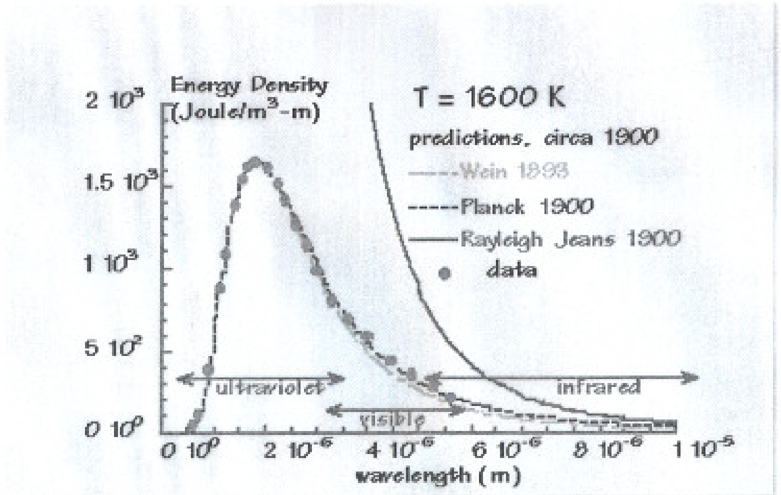
His result is the Planck Distribution function:

$$u(T,\lambda) = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\left(\frac{hc}{\lambda kT}\right)} - 1}$$

for very high frequencies (low wavelengths) it reproduces Wien's law.

and, he could predict:

Boltzmann's constant, $k = c_1 / c_2$, the electron charge and Avagadros' number.



the arbitrary constants are gone in favor of now fundamental constants: c, k and now h

at the expense of an unwelcome interpretation

Planck suspected that this was important, and admitted as much to a young son…one who died in WWI. Two of his three daughters died in childbirth and he lost his wife in 1909.

Planck behaved with dignity during WWII - persistently, but unsuccessfully, tried to persuade Hitler into different directions.

He lost his eldest son to execution as a part of an assassination attempt on Hitler.

He was barely rescued by the Allies as the Russians approached Berlin as a frail, homeless elderly man.

WHAT DID EINSTEIN DO? "QUANTA" inventor

He proposed – not that the walls resonated/radiated at only discrete energies...

BUT THE LIGHT ONLY COMES AS DISCRETE PACKETS!

He calculated the internal energy density

of a GAS of quantized light particles

→ TOTALLY GET PLANCK'S FORMULA $u(f,T)$!

now this is a little strange.

COMPLETELY REJECTED

for years...

Planck, Nernst, Rubens, and Warburg in their proposal that Einstein be accepted for membership in the Prussian Academy of Sciences, 1913

"In sum, one can say that there is hardly one among the great problems in which modern physics is so rich to which Einstein has not made a remarkable contribution. That he may sometimes have missed the target in his speculations, as, for example, in his hypothesis of light-quanta, cannot really be held too much against him, for it is not possible to introduce really new ideas even in the most exact sciences without sometimes taking a risk.

Light behaving as quanta – particles – defies a common-sense "feeling" ...

Classically – intensity of E&M wave -- (look at classical notes)

$$I = S_{average} = \frac{1}{c\mu_0} |\mathcal{E}^2|_{ave} \quad \left(\frac{\text{power}}{\text{area}}\right)$$

$$= \frac{1}{c\mu_0} \langle \mathcal{E}_m^2 \sin^2(kx - \omega t) \rangle$$

and get naturally rms: $\mathcal{E}_{rms} = \frac{\mathcal{E}_m}{\sqrt{2}}$

$$I = \frac{1}{c\mu_0} \mathcal{E}_{rms}^2 = \epsilon_0 c \, \mathcal{E}_{rms}^2$$

For Einstein ... intensity of light is proportional to the number of photons (named in 1926...)

$$I = Nhf$$

So,

$$\epsilon_0 c \, \mathcal{E}_{rms}^2 = Nhf$$

and we get a classical – quantum connection

$$N \propto \mathcal{E}^2$$

stay tuned.

100 Watt bulb.

how many photons on fingertip one meter away?

say it's mostly yellow -- not true, mostly infrared, right?

$\lambda = 580\ nm = 5.8 \times 10^{-7} m$

$f = 5 \times 10^{14}\ s^{-1}$

Energy per photon is $E = hf$

HOLD IT. CAN'T AVOID LIFELONG USE OF "γ" TO BE A PHOTON SYMBOL. OKAY? LIVE WITH IT!

ahem... sorry.

$$E_\gamma = hf$$
$$= (6.34 \times 10^{-34} J \cdot s)(5 \times 10^{14} s^{-1})$$
$$E_\gamma = 3.17 \times 10^{-19} J$$

$$I = \frac{P_s}{4\pi r^2} = \frac{100\ W}{4\pi (1)^2} = 7.96\ Watt/m^2 = 7.96\ J/m^2 \cdot s$$

$$N = \frac{I}{E_\gamma} = \frac{7.96}{3.17 \times 10^{-19}} = 2.5 \times 10^{19} \frac{\gamma}{m^2 \cdot s} \left(\frac{m}{100\ cm}\right)^2$$

$$N = 2.5 \times 10^{15} \frac{\gamma}{cm^2 \cdot s} \qquad N(finger) \simeq 2.5 \times 10^{15} \frac{\gamma}{s}$$

" to the moon, Alice ! "

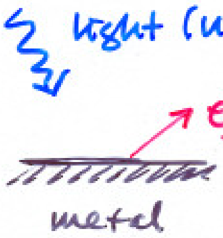
I get ~ 25 $\frac{\gamma}{s}$ at the surface of the moon

(ignoring the atmosphere :-))

Einstein made his proposal in his first 1905 paper:

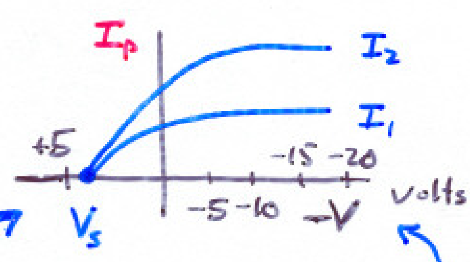
"A Heuristic Point of View About the Generation and Transformation of Light"

→ trying to understand the photoelectric effect.

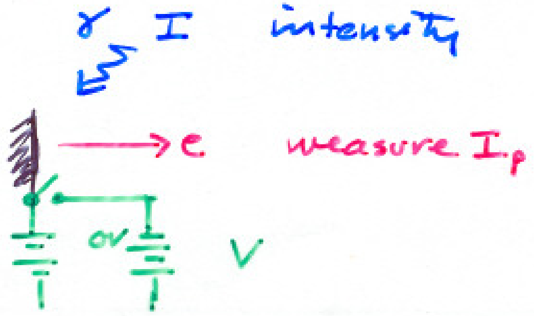


e's pop out → a photocurrent

Phillip Lenard studied this carefully. He found



$I_2 > I_1$



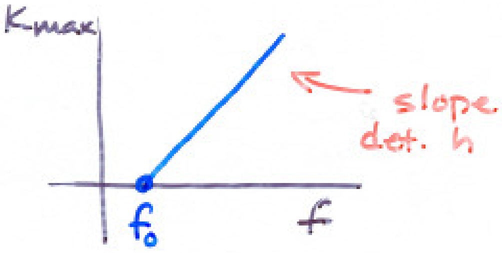
- when V is negative current flows, more intensity, more current
- when V is positive, there is a maximum $V = V_s$ where current stops

the retarding voltage V_s overcomes the fastest electrons .. with K_{max}

$$K_{max} = \frac{1}{2} m_e v_e^2 = e V_s = hf - hf_0$$

↑ W... the work function

Further... there is frequency dependence:



more f, faster are the fastest electrons

↑ and a cut-off - threshold below which no current flows.

Einstein's solution - billiard balls $\gamma + e \rightarrow e + \text{recoil}$

γ act as momentum-carrying particles & knock electrons out of metal. one-by-one

Remember

$$E^2 = p^2c^2 - m^2c^4 \qquad ?$$

he hypothesized γ.. light quanta .. are massless objects, so

$$E^2 = p^2c^2$$

$$E = pc$$

↑ momentum of γ

A DIRECT PHOTOELECTRIC DETERMINATION OF PLANCK'S "h."[1]

By R. A. Millikan.

1. Introductory.

QUANTUM theory was not originally developed for the sake of interpreting photoelectric phenomena. It was solely a theory as to the mechanism of absorption and emission of electromagnetic waves by resonators of atomic or subatomic dimensions. It had nothing whatever to say about the energy of an escaping electron or about the conditions under which such an electron could make its escape, and up to this day the form of the theory developed by its author has not been able to account satisfactorily for the photoelectric facts presented herewith. We are confronted, however, by the astonishing situation that these facts were correctly and exactly predicted nine years ago by a form of quantum theory which has now been pretty generally abandoned.

no!

It was in 1905 that Einstein[2] made the first coupling of photo effects and with any form of quantum theory by bringing forward the bold, not to say the reckless, hypothesis of an electro-magnetic light corpuscle of energy $h\nu$, which energy was transferred upon absorption to an electron. This hypothesis may well be called reckless first because an electromagnetic disturbance which remains localized in space seems a violation of the very conception of an electromagnetic disturbance, and second because it flies in the face of the thoroughly established facts of interference. The hypothesis was apparently made solely because it furnished a ready explanation of one of the most remarkable facts brought to light by recent investigations, viz., that the energy with which an electron is thrown out of a metal by ultra-violet light or X-rays is independent of the intensity of the light while it depends on its frequency. This fact alone seems to demand some modification of classical theory or, at any rate, it has not yet been interpreted satisfactorily in terms of classical theory.

While this was the main if not the only basis of Einstein's assumption, this assumption enabled him at once to predict that the maximum energy

[1] An abstract of this paper was presented before the Am. Phys. Soc. in April, 1914. (Phys. Rev., IV., 73, '14.) The data on lithium were however first reported at the meeting of the Am. Phys. Soc. in April, 1915. (Phys. Rev., VI., 55, '15.)

[2] Ann. d. Phys. (4), 17, 132, 1905, and (4), 20, 199, 1906.

of emission of corpuscles under the influence of light would be governed by the equation

$$\tfrac{1}{2}mv^2 = V.e = h\nu - p, \qquad (1)$$

in which $h\nu$ is the energy absorbed by the electron from the light wave, which according to Planck contained just the energy $h\nu$, p is the work necessary to get the electron out of the metal and $\frac{1}{2}mv^2$ is the energy with which it leaves the surface, an energy evidently measured by the product of its charge e by the P.D. against which it is just able to drive itself before being brought to rest.

At the time at which it was made this prediction was as bold as the hypothesis which suggested it, for at that time there were available no experiments whatever for determining anything about how P.D. varies with ν, or whether the hypothetical h of equation (1) was anything more than a number of the same general magnitude as Planck's h. Nevertheless, the following results seem to show that at least five of the experimentally verifiable relationships which are actually contained in equation (1) are rigorously correct. These relationships are embodied in the following assertions:

1. *That there exists for each exciting frequency* ν, *above a certain critical value, a definitely determinable maximum velocity of emission of corpuscles.*
2. *That there is a linear relation between* V *and* ν.
3. *That* $\frac{dV}{d\nu}$ *or the slope of the* $V\,\nu$ *line is numerically equal to* h/e.
4. *That at the critical frequency* ν_0 *at which* $v = 0$, $p = h\nu_0$, *i. e., that the intercept of the* $V\,\nu$ *line on the* ν *axis is the lowest frequency at which the metal in question can be photoelectrically active.*
5. *That the contact E.M.F. between any two conductors is given by the equation*

$$\text{Contact E.M.F.} = h/e(\nu_0 - \nu_0') - (V_0 - V_0').$$

No one of these points except the first[1] had been tested even roughly when Einstein made his prediction and the correctness of this one has recently been vigorously denied by Ramsauer.[2] As regards the fourth Elster and Geitel[3] had indeed concluded as early as 1891, from a study of the alkali metals, that the more electro-positive the metal the smaller is the value of ν at which it becomes photo-sensitive, a conclusion however which later researches on the non-alkaline metals seemed for years to contradict.

[1] Lenard (Ann. d. Phys. (4), 8, 149, 1902) discovered a maximum velocity though he did not find the effect of frequency upon it.

[2] Ann. d. Phys., 45, 1120, 1914 and 45, p. 961.

[3] Wied. Ann., 43, 225, 1891.

Na

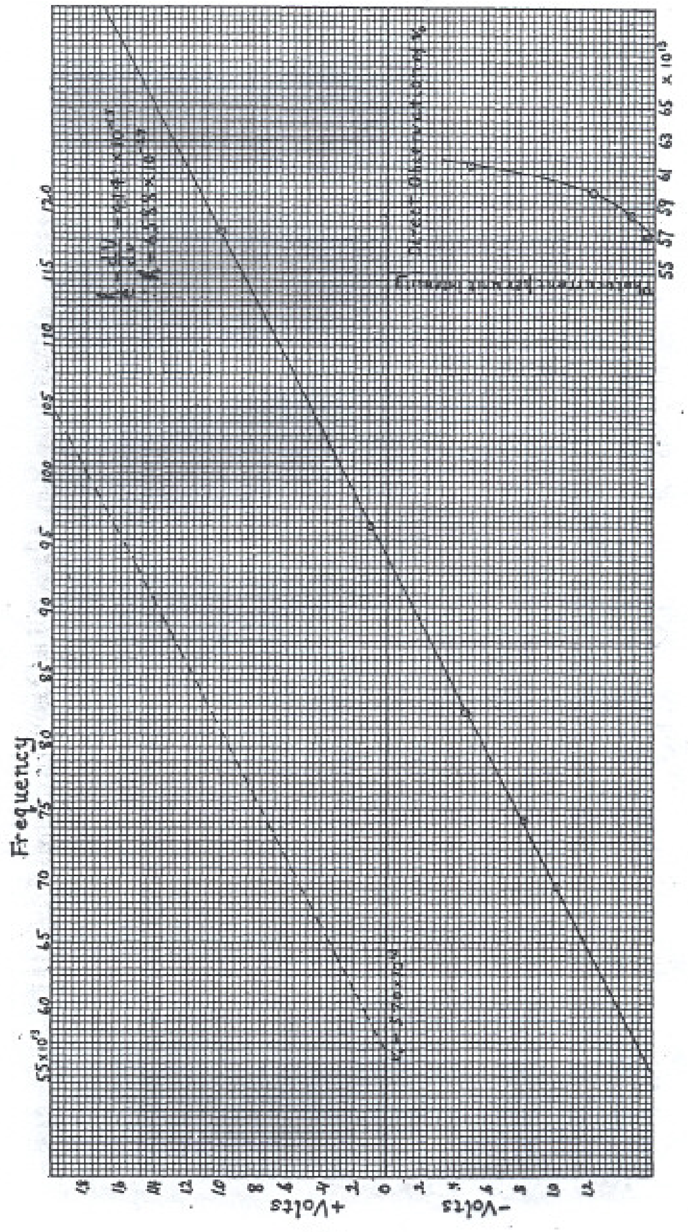


Fig. 8.

sumed to travel along the ether strings are proportional to the impressed frequency and (2) that they are transferred upon absorption as wholes to an electron. This being the case, the objections to an ether-string theory, that is, to any theory in which the energy remains localized in space instead of spreading over the entire wave-front, must hold for the Einstein theory. Lorenz[1] and Planck[2] have pointed out some of these. Despite these objections, however, Sir J. J. Thomson[3] and Norman Campbell[4] still adhere to it. I wish to call attention to one more difficulty which in itself seems to me to be very serious.

If a static electrical field has a fibrous structure, as postulated by any form of ether-string theory "each unit of positive electricity being the origin and each unit of negative electricity the termination of a Faraday tube,"[5] then the force acting on one single electron between the plates of an air condenser cannot possibly vary *continuously* with the potential difference between the plates. Now in the oil-drop experiments[6] we actually study the behavior in such an electric field of one single, isolated electron and we find, over the widest limits, exact proportionality between the field strength and the force acting on the electron as measured by the velocity with which the oil drop to which it is attached is dragged through the air.

When we maintain the field constant and vary the charge on the drop, the granular structure of electricity is proved by the discontinuous changes in the velocity, but when we maintain the charge constant and vary the field the lack of discontinuous change in the velocity disproves the contention of a fibrous structure in the field unless the assumption be made that there are an enormous number of ether strings ending in one electron. Such an assumption takes all the virtue out of an ether string theory.

Despite then the apparently complete success of the Einstein equation, the physical theory of which it was designed to be the symbolic expression is found so untenable that Einstein himself, I believe, no longer holds to it. But how else can the equation be obtained?

no!

Before attempting to answer this question, let us consider the energy relations which it imposes. It requires the absorption at some time or other by the escaping electron of at least the energy $h\nu$ from incident waves of frequency ν. The total luminous energy falling per second from

1 Phys. Zeit., 11349, 1910.

2 Ann. der. Phys., 39, 1912. Berliner Ber., 723, 1911.

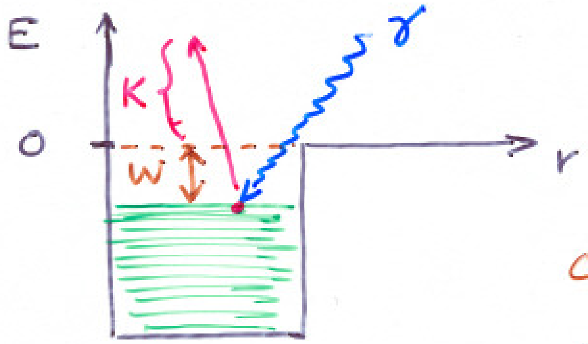
3 Proc. Phys. Soc. London, XXVII., 105, December 15, 1914.

4 Modern Electrical Theory, Cambridge Press, 1913, p. 248.

5 J. J. Thomson's Electricity and Matter, p. 9.

6 PHYS. REV., 2, 109, 1913.

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$



Costs energy to free electrons from the metal — "WORK FUNCTION"

$$hf = W + \frac{1}{2} m_e v_e^2$$

If $hf < W$ ... electron not liberated

when $hf = hf_0 = W$ -- bingo, free.

$hf > W \rightarrow$ excess goes to K

- K of electrons depends only on f.
  - independent of I -- that's just # γ
- # photoelectrons — I_p — does depend on I
  - more γ → more e
- no delay