CHAPTER 13 NUCLEAR REACTIONS

These are all processes involving <u>scattering</u> of a projectile (beam) against a target

sometimes changing only the directions

sometimes completely changing the particles.

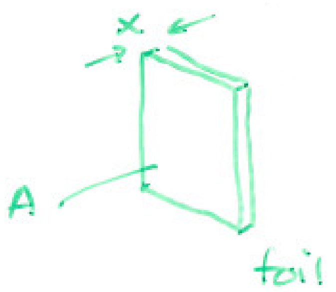
Rutherford's discovery of the proton was the latter.

$$\alpha + {}^{14}_{7}N \rightarrow p + {}^{17}_{8}O$$

nuclear physicists' shorthand: ${}^{14}N(\alpha, p){}^{17}O$

Useful concept: the "cross section"

each target nucleus presents an effective area — cross section — to the beam particles.



R_0 – rate at which beam particles strike the target [particles – s^{-1}]

R – rate at which collisions occur. [collisions – s^{-1}]

n – # target nuclei per unit volume [particles – m^{-3}]

having cross section "σ"

Total number of target nuclei in the foil:

$$n A x$$

Total "area" presented to the beam:

$$\sigma n A x$$

$$\frac{\text{collisions}}{\text{beam particles}} = \frac{R}{R_0} = \frac{\sigma n A x}{A} = \sigma n x$$

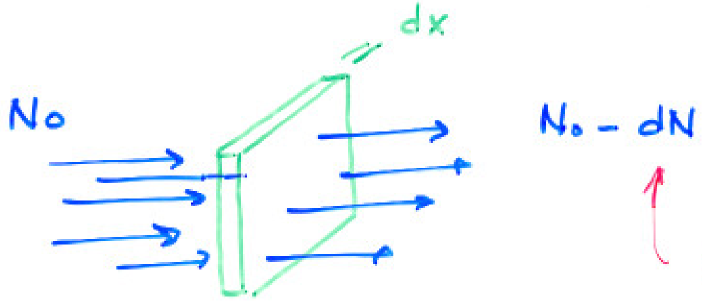
A physics theory of what happens when a hits x

can often calculate σ.

experiments' jobs are to measure σ and

check the theory.

Most particles go right on through — penetrate



$$-\frac{dN}{N} = \frac{nA\sigma dx}{A} = n\sigma dx$$

$$\int_{N_0}^{N} \frac{dN}{N} = -n\sigma \int_0^x dx$$

$$\ln(N/N_0) = -n\sigma x$$

$$N = N_0 e^{-n\sigma x}$$

Nuclear cross sections have dimensions of AREA.

standard unit: $1 \text{ bn} = 10^{-28} \text{ m}^2$ "barn"

literally, as in "broad side of a barn"

NOTE: not literally a geometrical concept —

σ really functions like a probability —

dynamics and fundamental forces at work.

Low energy reaction kinematics.

Consider $\quad a + B \rightarrow C + D$

in a frame where B is at rest - "LAB frame"

$$E_0 = E_f$$

$$M_a c^2 + K_a + M_B c^2 + \underbrace{K_B}_{=0 \text{ in LAB}} = M_C c^2 + K_C + M_D c^2 + K_D$$

rearrange:

mass energies = kinetic energies

$$M_a c^2 + M_B c^2 - (M_C c^2 - M_D c^2) = K_C + K_D - K_a$$

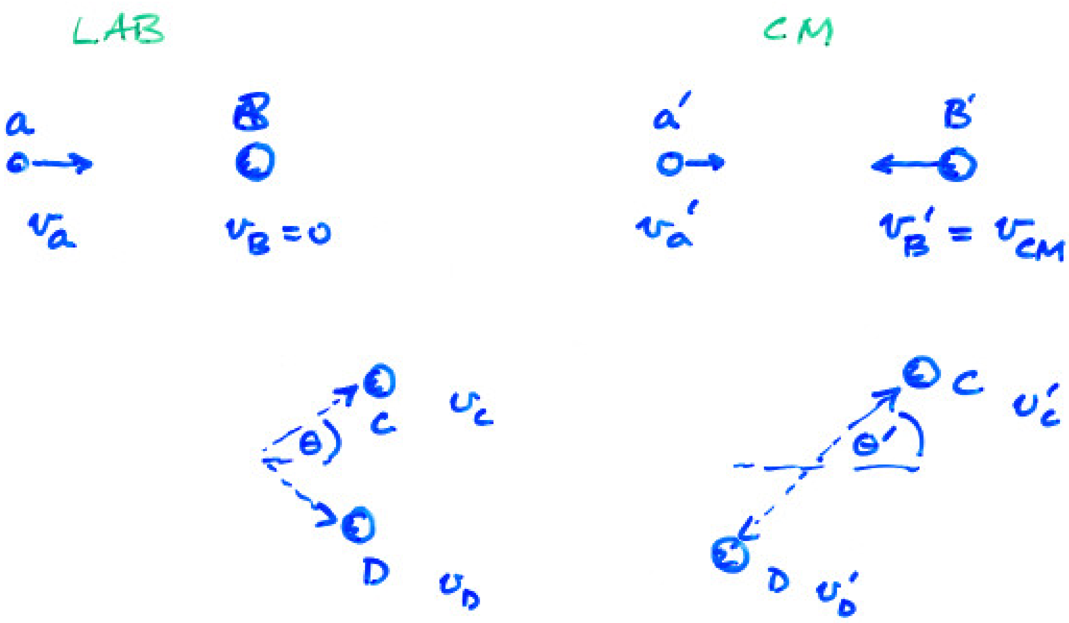
$\equiv$ "Q"

when $Q > 0 \Rightarrow$ final KE's > initial KE's
(exoergic, exothermic)
due to a mass deficit

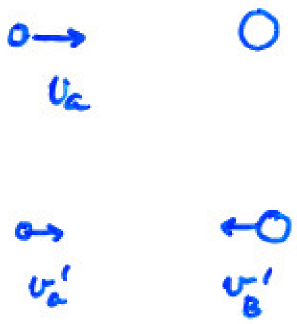
$Q < 0 \Rightarrow$ kinetic energy has been converted
(endoergic, endothermic)
into mass

to make the process happen requires a "threshold"

K_a -- found from a CM frame analysis



transformation:



$$\vec{P}_a' = \vec{P}_B'$$

$$M_a v_a' = M_B v_B'$$

$$v_B' = v_{CM}$$

$\vdots$

taken "from") v_a to get v_a'

so: $v_a' = v_a - v_{CM}$

$v_B' = v_{CM}$

$$v_{CM} = v_a \left(\frac{M_a}{M_a + M_B} \right)$$

so, now in CM frame:

$$E_0 = E_f$$

$$\tfrac{1}{2} M_a (v_a - v_{CM})^2 + M_a c^2 + \tfrac{1}{2} M_B c^2 v_{CM}^2 + M_B c^2$$

$$= M_c c^2 + \tfrac{1}{2} M_c v_c'^2 + M_D c^2 + \tfrac{1}{2} M_D v_D'^2$$

The threshold condition is when C & D are produced .. sitting still $v_C' = v_D' = 0$

$$\tfrac{1}{2} M_a \left(v_a - \frac{M_a v_a}{M_a + M_B} \right)^2 + \tfrac{1}{2} M_A \frac{M_a^2 v_a^2}{(M_a + M_B)^2}$$

$$= M_c c^2 + M_D c^2 - M_a c^2 - M_B c^2 = -Q$$

$$\underline{\tfrac{1}{2} v_a^2} \left(\frac{M_a M_B}{M_a + M_B} \right) = -Q$$

$$K_a^{\text{threshold}} = -Q \left(\frac{M_a + M_B}{M_B} \right)$$

Example

Calculate Q for $\quad {}^{16}_{8}O(\gamma, p){}^{15}_{7}N$

$$Q = [M_a + M_B - (M_C + M_D)]c^2$$

$$= [15.994915\,u - 0\,u - (15.000108\,u + 1.007825\,u)](931.5\,\tfrac{MeV}{u})$$

$$= -12.13 \text{ MeV.}$$

What is the threshold kinetic energy?

$$K_{th} = -Q\left(\frac{M_a + M_B}{M_B}\right)$$

$$= (12.13)\left(\frac{15.994915 + 0}{15.994915}\right) = 12.13 \text{ MeV}$$

didn't need to make any mass energy for γ.

Example   Calculate Q for $^{16}_{8}O(p,d)^{15}_{8}O$

$$Q = (15.994915u + 1.007825u - 15.003070u - 2.014102u)\left(931.5\ \frac{MeV}{u}\right)$$

$$Q = -13.44\ MeV$$

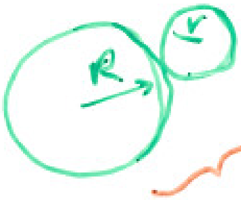
Calculate K_p^{thresh}

$$K_p^{thresh.} = -Q\left(\frac{16u + 1u}{16u}\right) = (13.44\ MeV)\left(\frac{17}{16}\right)$$

$$K_p^{th.} = 14.28\ MeV$$

There's more -- the proton has to overcome the nuclear electrostatic repulsion -

estimate.



$R = r_0 A^{1/3}$

$$\delta = R + r$$

$$\delta = r_0 (A^{1/3} + 1)$$

$$E_c = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)e}{\delta} = \frac{1.44\ MeV \cdot fm}{1.4\ fm}\ \frac{Z}{A^{1/3}+1}$$

for $^{16}_{8}O(p,d)^{15}_{8}O$ $E_c = 2.34\ MeV$

So, $K_p^{thresh} > E_c$ & the reaction will happen.

Example – not

Show Q for $^{209}_{83}\mathrm{Bi}(p,d)^{208}_{83}\mathrm{Bi}$ is -5.23 MeV

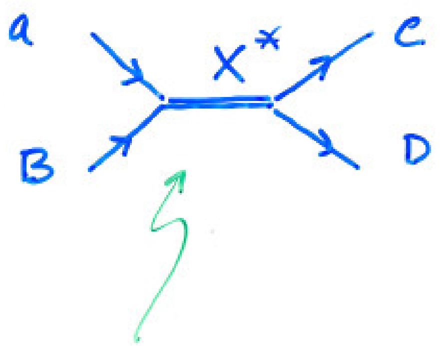
$K_p^{th} = 5.26$ MeV

$E_c = 12.33$ MeV

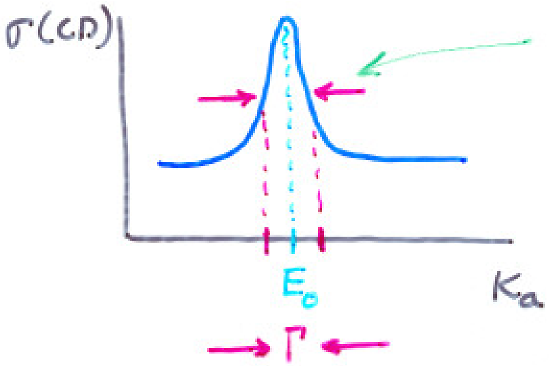
Will this reaction be likely to happen?

Intermediate states –

Often something like this happens:



some short-lived, excited state



probability (σ) goes way up as K_a is changed

"Resonance"

the resonant energy is thought of as $M_x c^2$

$\rightarrow$ producing a quantum X which quickly decays $X \rightarrow C + D$

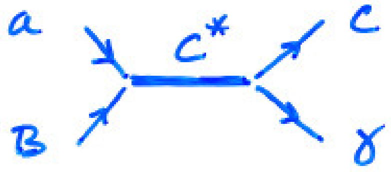
doesn't "remember" how it was made.

From uncertainty, can calculate the lifetime

$$\Delta E \, \Delta t \geq \frac{\hbar}{2}$$

$$\Gamma \tau \geq \frac{\hbar}{2}$$

In nuclear physics, often



and E_0 would correspond to the C^* excited state energy

Neutrons are special.

Often, the following can happen:

$$ {}^{1}_{0}n + {}^{A}_{Z}X \longrightarrow {}^{A+1}_{Z}X^{*} \longrightarrow {}^{A+1}_{Z}X + \gamma $$

$$ \hookrightarrow {}^{A+1}_{Z+1}Y + \beta + \nu $$

"Neutron Capture"

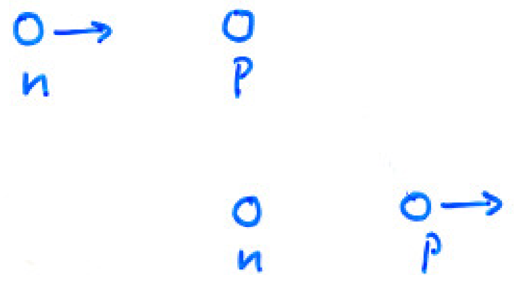
How often?

Depends on the nucleus... but gobbling neutrons is useful

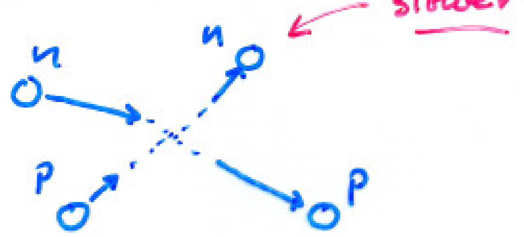
neutrons are hard to stop / absorb except for some special substances. Cd.

The other thing neutrons can do...

just slow down like billiards



or more likely



Substances rich in protons or light nuclei:

good "moderators"

Water, H, paraffin

Hence... the "neutron bomb"

enhanced radiation weapon

We're water... so neutrons freely scatter

from our protons

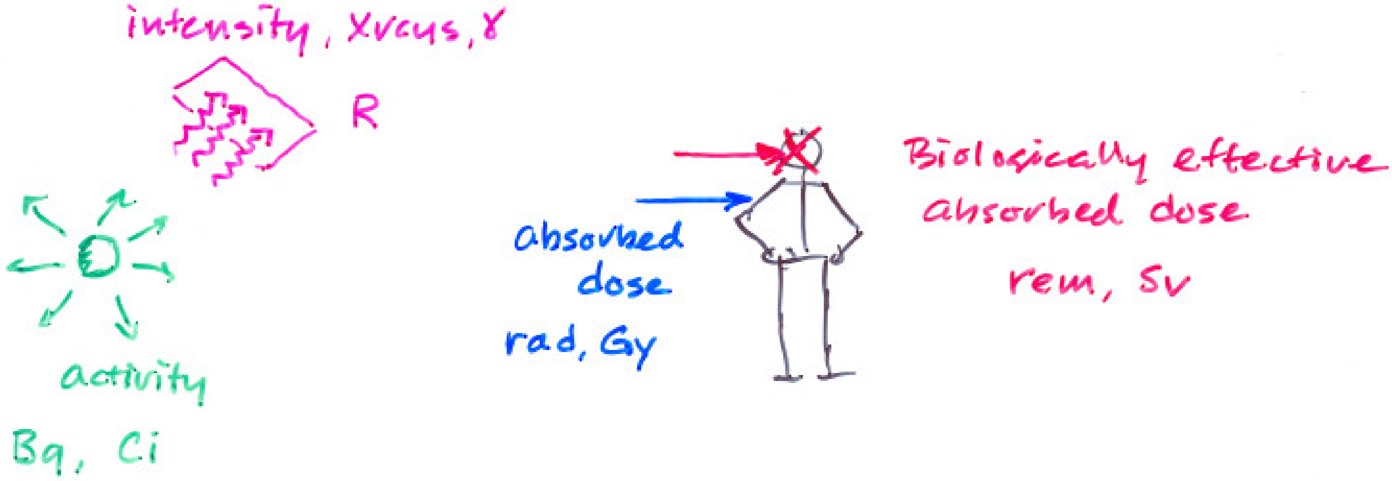
creating charged fast particles...

which ionize and damage DNA,

cells, etc.

Quick Primer on radiation units...

✓ = S.I. unit



ACTIVITY: ✓ Bequerrel — 1 Bq = material producing 1 decay/s
Curie — 1 Ci = " 3.7×10^{10} d/s

INTENSITY: Roentgen — 1 R = 0.000258 C/kg air ionization

ABSORBED DOSE: 1 rad = 0.01 J/kg of tissue

✓ 1 Gray = 1 Gy = 1.0 J/kg of tissue

BIO. EFFECTIVE DOSE: 1 rem = 1 rad × Q

✓ 1 sievert = 1 Sv = 100 rem

Q ~ 10-20 neutrons, energy dependent.
20 α
1 γ
1 β

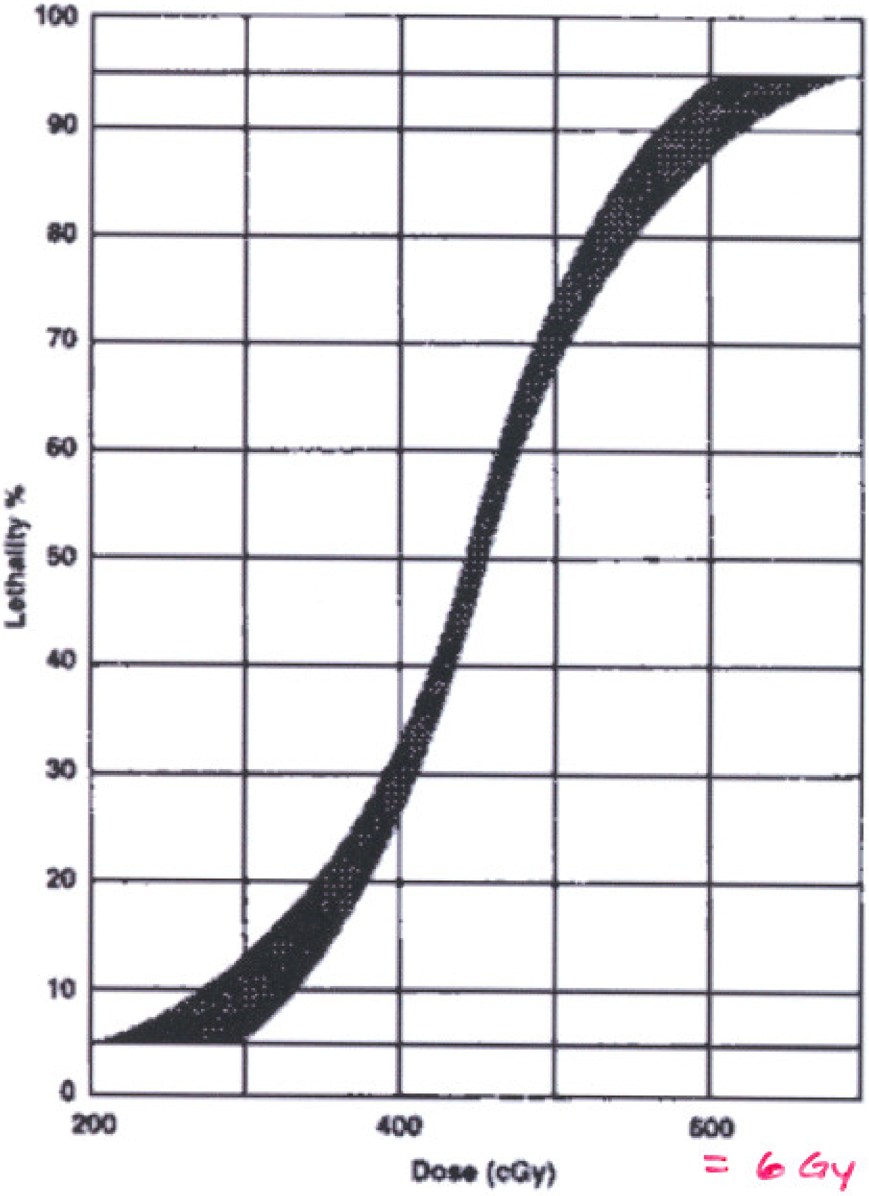


Figure 5-1. Typical Lethality as a Function of Dose

Fission

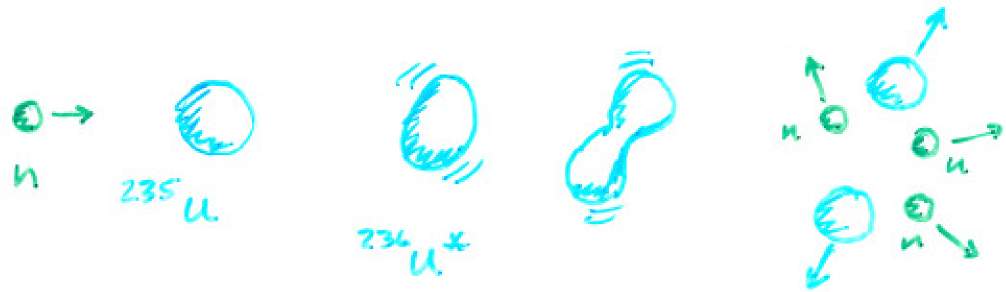
The splitting of a heavy nucleus due to Coulomb repulsion.

natural

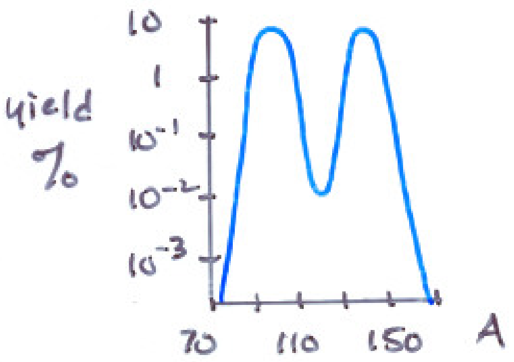
induced, or artificial

First discovered in 1939 by Otto Hahn & Fritz Strassman — explained by Lisa Meitner and Otto Frisch. $n + U \rightarrow Ba + La \ldots$

Bohr's model



typically 2-3 neutrons produced

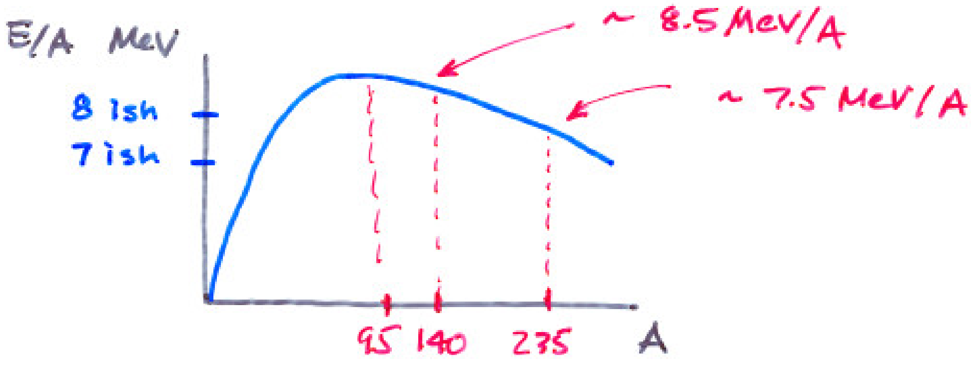


⇒ usually heavy + light fission products

~ 95 ish + 140 ish

Rough calculation...

remember the "curve of Binding Energy"



energy released ~

$$Q \simeq (235 \text{ nucleons})\left(8.5 \frac{\text{MeV}}{\text{nucleon}} - 7.5 \frac{\text{MeV}}{\text{nucleon}}\right)$$

$$\sim 235 \text{ MeV}$$

an enormous amount of energy.

Typical fissions of ^{235}U ..

$$^{1}_{0}n + ^{235}_{92}U \longrightarrow \underbrace{^{141}_{56}Ba + ^{92}_{36}Kr}_{} + 3\,^{1}_{0}n$$

$$\longrightarrow \underbrace{^{140}_{54}Xe + ^{94}_{38}Sr}_{} + 2\,^{1}_{0}n$$

$$\longrightarrow \underbrace{^{132}_{50}Sn + ^{101}_{42}Mo}_{} + 3\,^{1}_{0}n$$

$$\longrightarrow \underbrace{^{99}_{40}Zr + ^{134}_{52}Te}_{\text{these guys are unstable as well}} + 3\,^{1}_{0}n$$

these guys are unstable as well

So, for $^{238}U \rightarrow {}^{119}Pd + {}^{119}Pd$ — very rare —

$$2B_{119,46} - B_{238,92} = [-180 + 360] \text{ MeV}$$
$$= 180 \text{ MeV}$$

... so the energy released generally is ~ 200 MeV.

Example. Total energy of $\boxed{1 \text{ kg}}$ of ^{235}U via induced fission. Take $Q = 208$ MeV.

need # nuclei: $N = \dfrac{6.02 \times 10^{23} \text{ molecules}}{235 \text{ g/mole}} (10^3 \text{ g})$

$$N = 2.56 \times 10^{24} \text{ nuclei}$$

So, $E = N \cdot Q$

$$= (2.56 \times 10^{24})\left(208 \frac{\text{MeV}}{\text{nucleus}}\right)$$

$$E = 5.32 \times 10^{26} \text{ MeV}$$

1 ton of TNT releases 10^9 cal $= 4.2 \times 10^9$ J

$$E = (5.32 \times 10^{26} \times 10^6 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})\left(\frac{\text{ton TNT}}{4.2 \times 10^9 \text{ J}}\right)$$

$$E = 20{,}000 \text{ tons TNT}$$

for the complete fission of 1 kg of pure ^{235}U.

that's a tragic problem.