PHY 215 Fall 2006 $\boxed{\text{MODERN PHYSICS}}$

Thermodynamics, Relativity, quantum mechanics, atomic physics, nuclear physics, elementary particle physics, cosmology!

Prerequisites – 184 or equivalent
MTH 234

Generally, "modern physics" means the physics of the 20th century (and after!) which forced a significant change to how we must think about SPACE, TIME, DETERMINISM, MEASUREMENT & WHAT IT MEANS TO KNOW. BIG stuff.

Everything previous to this ~~period~~ period (to Galileo) is called "classical physics" – which glosses over just how much a change Maxwell's electromagnetic theory and Boltzmann's statistical theory were!

Let's review the BIG IDEAS of "classical physics" – some of which will crop up and get changed completely.

Mechanics.

Almost completely associated with a (small) fraction of the life work of Isaac Newton: his 3 "laws" of motion plus gravitation.

His second law describes the motion of objects with mass, on which forces act:

$$\sum_{i}^{\text{all forces}} \vec{F}_i = m\vec{a} \qquad i1$$

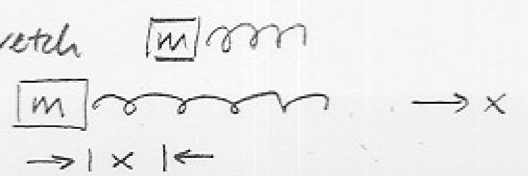
But, actually, Euler said that, Newton never did. Rather, he wrote the equivalent of

$$\sum_{i}^{\text{all forces}} \vec{F}_i = \frac{d\vec{p}}{dt} \qquad i2$$

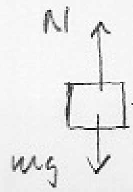
where $\vec{p} = m\vec{v}$ is the momentum. i3

For the important circumstance of elasticity (springiness), Hooke found that experimentally the force that a spring exerts on a mass when it's stretched or compressed is proportional to the distance of the stretch or compression.

no stretch



empirical spring constant



$\to F_s \propto kx$

$= -kx$

because always a restoring force

So, in the 2^{nd} law...

$$-kx = m\frac{d^2x}{dt^2} \qquad i4$$

which has solutions which are sinusoidal. For example

$$x(t) = A\sin(\omega t + \phi) \qquad i5$$

phase - initial conditions

angular frequency

by solving i4 we find

$$\omega = \sqrt{k/m} \qquad i6$$

Also useful, the frequency $f = \frac{1}{T} = \frac{\omega}{2\pi}$ i7

period, or repeating time

Of course, if there is a complete balance among all forces on a mass, then

$$\sum_i \vec{F}_i = 0 \qquad i8$$

and there is no net acceleration. This is Newton's First Law.

Newton's third law was his singular genius:

$$\vec{F}_{21} = -\vec{F}_{12}$$ for 2 objects in contact, exerting forces between them. i9

This directly results in the Conservation of Linear Momentum for collisions or general interactions among multiple bodies. A quite general statement:

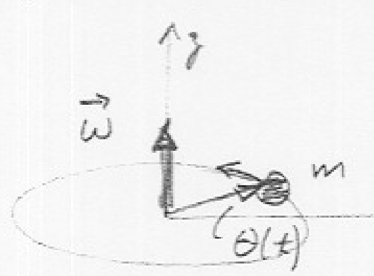
$$\frac{d\vec{p}_1}{dt} = -\frac{d\vec{p}_2}{dt}$$ i10

or $$\frac{d}{dt}\left(\vec{p}_1 + \vec{p}_2\right) = 0$$

so is constant in time.

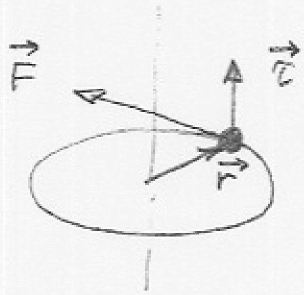
So, $$\sum_i^{\text{all objects}} \vec{p}_i = 0$$ i11

The concept of force and momentum have rotational counterparts.



$\vec{\omega}$, the angular velocity comes from the RH rule and, should it change, an angular acceleration, $\vec{\alpha}$, results.

Should this be the case, it is as a result of some applied torque, $\vec{\tau}$



$$\vec{\tau} = I\vec{\alpha} = \frac{d\vec{L}}{dt} \qquad i12$$

where $\vec{L} = \vec{r} \times \vec{p}$, the angular momentum i13
and I is the rotational or moment of inertia.
In general, for multiple masses the rotational inertia is

$$I \equiv \sum_i (m_i) r_i^2 \qquad i14$$

where r_i connects the i^{th} mass to the axis of rotation.

The torque is calculated from an applied force as

$$\vec{\tau} \equiv \vec{r} \times \vec{F} \qquad i15$$

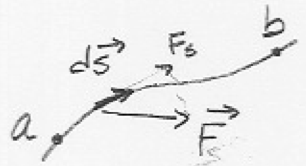
Energy.
very difficult concept! Due to Leibnitz, who didn't really understand it, and to a whole string of people, right through the present.

For just mechanical energy, imagine a constant, net force on a mass acting over a distance Δx. The velocity changes and we say (now) that work is done.

$$W = \sum_s F_s \, \Delta s \qquad i16$$

Suppose this is constant over an infinitesimal distance

$$dW = \sum_s F_s \, ds$$



and the total work done from a to b is,

$$W_{ab} = \int_a^b \vec{F} \cdot d\vec{s} = \int_a^b \sum F_s \, ds \qquad i17$$

since $$\sum_s F_s = m \frac{dv}{dt} \qquad i18$$

where now v could change as a function of distance. So,

$$\frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v \frac{dv}{ds} \qquad i19$$

So,

$$W_{ab} = \int_a^b \sum F_s \, ds = \int_a^b m \frac{dv}{dt} \, ds = \int_{v_a}^{v_b} m v \, dv$$

$$W_{ab} = \frac{1}{2} m v_b^2 - \frac{1}{2} m v_a^2 \qquad i20$$

which is called the Work-Energy Theorem

and $\frac{1}{2}mv^2$ is called the Kinetic Energy

so, collecting,

$$\sum_s F_s \Delta s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \Delta KE \qquad i21$$

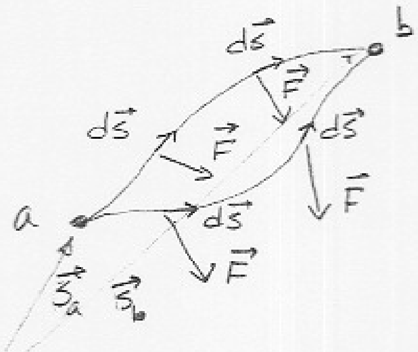
This is analogous to the other conservation relationships: a force acting through a <u>time</u>

$$\sum_s F_s = \frac{dp_s}{dt} \qquad i22$$

$$\sum F_s \,\Delta t = p_f - p_i = \Delta p \qquad i23$$

the impulse

If the work done by a force over different paths <u>doesn't</u> <u>change</u>, then the force is called <u>conservative</u>.



it only depends on the configuration of the endpoints

$$\int_{\vec{s}_a}^{\vec{s}_b} \vec{F}\cdot d\vec{s} = \underbrace{f(\vec{s}_b) - f(\vec{s}_a)}_{\text{some functions}} \qquad i24$$

Usually arranged: $$-\int_{\vec{s}_a}^{\vec{s}_b} \vec{F}\cdot d\vec{s} = U(\vec{s}_b) - U(\vec{s}_a) \qquad i25$$

and the sign facilitates the following statement

$$W_{ab} = \int \Sigma F_s \, ds = + \Delta KE \quad \text{WE theorem} \qquad i26$$
$$= KE_b - KE_a$$

<u>and</u> $= - \Delta U$ if conservative

$$= - U(s_b) + U(s_a) \qquad i27$$

so,

$$KE_b - KE_a = - U(s_b) + U(s_a)$$

$$KE_a + U(s_a) = KE_b + U(s_b) \qquad i28$$

$U(s)$ is the Potential Energy associated with a <u>configuration</u> at point s.

The result is the statement that

$$KE + U = \text{constant} \qquad i29$$

for conservative forces

gravitation,
Another of Newton's genius realizations was that all objects with mass attract one another with a force

$$F_G = G \frac{m_1 m_2}{r_{12}^2} \qquad i30$$

Gravitational and elastic forces are conservative and a potential energy can be associated with each.

$$U(x)_{elastic} = \frac{1}{2} k x^2 \qquad \text{i31}$$

$$U(r)_{gravitational} = -\frac{G m_1 m_2}{r} \qquad \text{i32}$$

where $U_g(\infty) \equiv 0$ and $U_e(0) \equiv 0$. i33

Given a potential energy, one can find the force

$$\vec{F}(\vec{r}) = -\vec{\nabla} U(\vec{r}) \qquad \text{i34}$$

where $\vec{\nabla}$ is the Gradient Operator

$$\vec{\nabla} \equiv \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \qquad \text{i35}$$

in Cartesian Coordinates

For $U(\vec{r}) = U(x)$, then $F(x) = -\frac{dU}{dx}$ i36

Rotational quantities are

$$W_{if} = \int_{\theta_i}^{\theta_f} \tau \, d\theta \qquad \text{i37}$$

and results in a change of kinetic energy

$$KE = \frac{1}{2} I \omega^2 \qquad \text{i38}$$

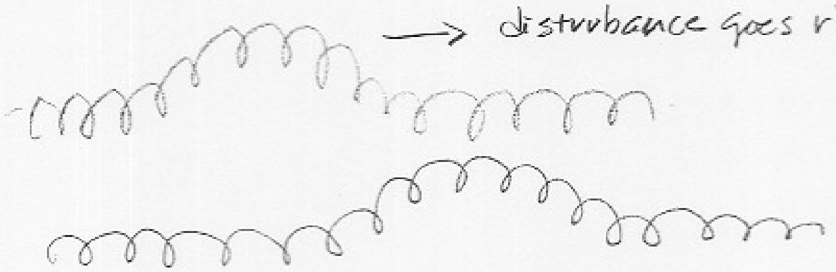
Wave motion

Wave motion can happen for a variety of causes — Hooke Law elastic forces can cause many mechanical waves, for example, a rope.

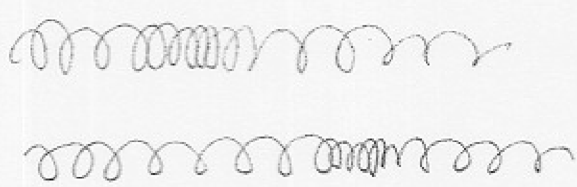
There are 2 kinds of waves

→ disturbance goes right

transverse

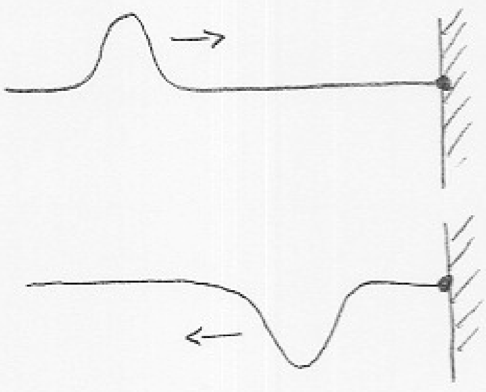


longitudinal

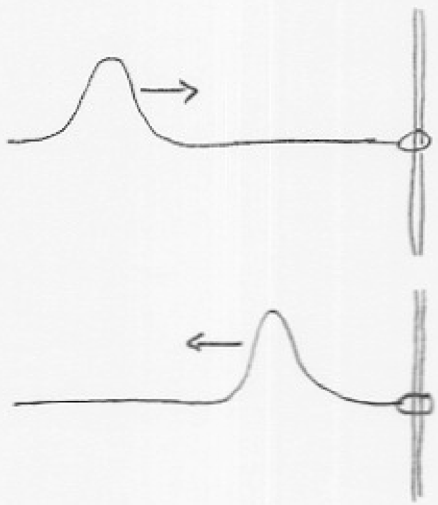


→ disturbance goes right.

Imagine a rope with an harmonic force on an end. A transverse wave propagates. If just a single flick, and a fixed end, then



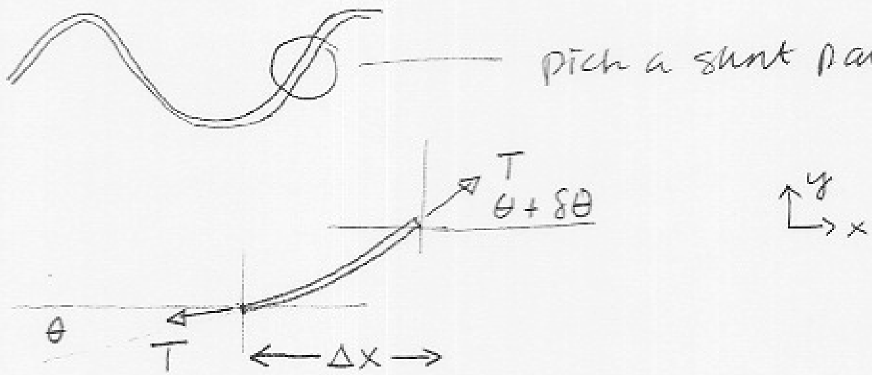
if a free end –



These are reflections. If the medium is flexible, then there will be reflection and refraction into the medium.

For a periodic force, the wave propagates – a traveling wave, and if sines and cosines are involved, then the wave is harmonic, such as springs.

Let's imagine a rope of linear mass density (mass per unit length), μ.



if short enough, the tension, T is constant, but the angles at the ends are slightly different.

Look at the transverse direction

$$(F_{net})_y = T\sin(\theta+\delta\theta) - T\sin\theta \qquad i39$$

the situation is such that θ is very small, so

$$\sin\theta \sim \tan\theta \sim \frac{\partial y}{\partial x} \rightarrow \text{the slope at point } x$$

also, the y coordinates along the rope are going up and down — at any point their transverse velocities are

$$\frac{\partial y}{\partial t}$$

since the tensions are approximately equal

$$(F_{net})_y = T\left[\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_x\right] \qquad i40$$

This is like a derivative: force per unit length

$$\frac{(F_{net})_y}{\Delta x} = \frac{T\left[\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_x\right]}{\Delta x}$$

and in $\lim_{\Delta x \to 0} \frac{T\left[\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_x\right]}{\Delta x}$ i41

$$= \frac{\partial^2 y}{\partial x^2} \equiv \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right) \qquad i42$$

$$(F_{net})_y = T\left(\frac{\partial^2 y}{\partial x^2}\right)\Delta x \qquad i43$$

The mass of this segment is $m = \mu \Delta x$
and from Newton's 2nd law,

$$(F_{net})_y = \sum F_y = m \underset{\uparrow}{a_y}$$

acceleration in y direction

$$= m \frac{\partial^2 y}{\partial t^2} \qquad i44$$

So,

$$T\left(\frac{\partial^2 y}{\partial x^2}\right)\Delta x = m\frac{\partial^2 y}{\partial t^2} = \mu \Delta x \frac{\partial^2 y}{\partial t^2}$$

$$\frac{\partial^2 y}{\partial x^2} = \left(\frac{\mu}{T}\right)\frac{\partial^2 y}{\partial t^2} \qquad i45$$

notice $\left[\frac{\mu}{T}\right] = \frac{\left(\frac{M}{L}\right)}{\left(\frac{ML}{T^2}\right)} = \frac{T^2}{L^2} = \left[\frac{1}{V}\right]$

$\sqrt{\frac{T}{\mu}}$ is the velocity in the longitudinal direction of individual points on the rope, v

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \qquad i46$$

This is called The Wave Equation. y can represent many quantities — height of a disturbance (transverse waves), density of gas (longitudinal waves), electric or magnetic fields, etc.

Solutions are real:
A particular solution is of the form

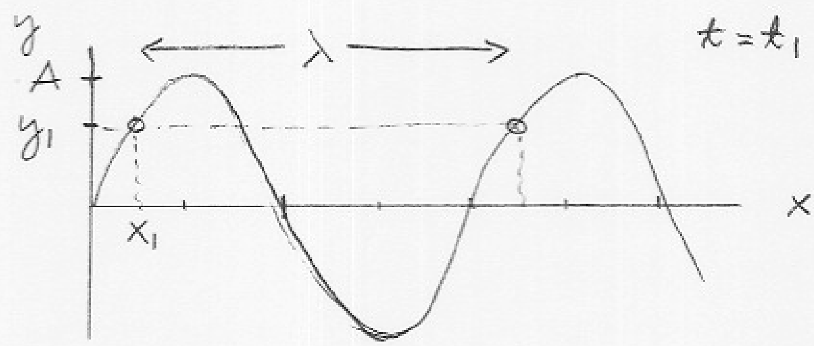
$$y(x,t) = A \sin \frac{2\pi}{\lambda}(x - vt + \delta) \qquad i47$$

phase — generally arbitrary and associated with when $t=0$

so let's forget the phase, $\delta = 0$

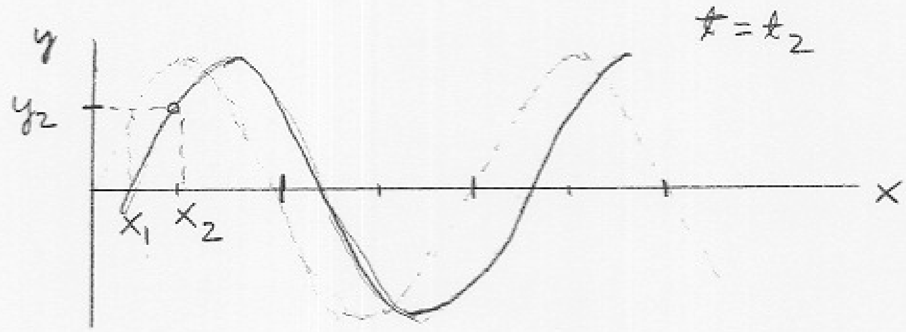
$$y(x,t) = A \sin \frac{2\pi}{\lambda}(x - vt) \qquad i48$$

Note that this is oscillating in both space and time... take slices in time, and can represent



$y = A$ is the Amplitude

points of equal phase are one wavelength, λ, apart. At a later time, the wave has moved:



$$y_1(x_1, t_1) = A \sin \frac{2\pi}{\lambda}(x_1 - v t_1) \qquad \text{and at } t_2 \qquad i49$$

$$y_2(x_2, t_2) = A \sin \frac{2\pi}{\lambda}(x_2 - v t_2) \qquad i50$$

But, these equal-phase points are the same height

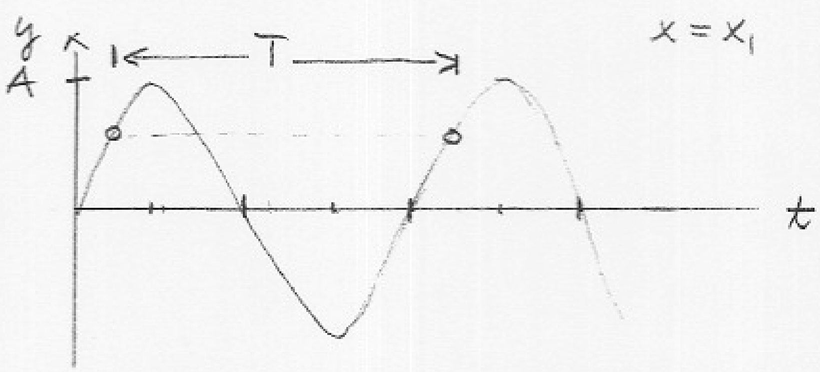
$y_1 = y_2$, so $\quad x_1 - vt_1 = x_2 - vt_2$

$$v = \frac{x_2 - x_1}{t_2 - t_1}$$

showing that $\quad \sin \frac{2\pi}{\lambda}(x - vt) \qquad$ i51

represents a wave moving to the right ($+x$). While $\sin \frac{2\pi}{\lambda}(x + vt)$ is a wave moving to the left.

The wave can also be represented as oscillations in time at a given x. So, at, say the x_1 position, the wave is going up and down as



points of equal phase are T (period) apart.

T is time for one vibration, so $\frac{1}{T}$ is the rate of vibration, the frequency.

$$f = \frac{1}{T} \qquad \text{cycles per time} \qquad \text{i52}$$

The time for the wave to travel one wavelength is T, so

$$\lambda = v T \qquad \text{length} \qquad i53$$

or

$$v = f \lambda \qquad i54$$

$$y(x,t) = A \sin 2\pi \left(\frac{x}{\lambda} - \frac{v}{\lambda} t \right)$$

$$= A \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right) = A \sin 2\pi \left(\frac{x}{\lambda} - f t \right)$$

define

$$k \equiv \frac{2\pi}{\lambda} \qquad \text{the wave number} \qquad L^{-1}$$

$$\omega \equiv \frac{2\pi}{T} \qquad \text{the angular frequency} \qquad T^{-1}$$

$$y(x,t) = A \sin (kx - \omega t) \qquad i55$$

Typical mechanical waves all have speeds of

$$v = \sqrt{\frac{\text{Elastic modulus}}{\text{density}}}$$

For longitudinal (compressional) waves in liquid or solid

$$v = \sqrt{\frac{B}{\rho}}$$

B = "bulk modulus"

transverse waves (only in a solid)

$$v = \sqrt{\frac{S}{\rho}}$$

S = shear modulus

compressional waves in gas

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

γ = ratio of heat capacities at constant pressure and constant volume $\equiv \frac{C_P}{C_V} \simeq 1.4$ for air

P = pressure

Energy of waves.

From i6 $\quad k = m\omega^2$

From i31 $\quad E = \frac{1}{2} k A^2 = \frac{1}{2} m \omega^2 A^2$

Each little element of a string, dm, has kinetic energy

$$dE = \frac{1}{2}\, dm\, \omega^2 A^2$$

$$= \frac{1}{2}\, \mu\, dx\, \omega^2 A^2$$

For wave traveling to the right, this comes from the work done by an element on the left . .

The energy is passed along the string by each element at a rate

$$\frac{dE}{dt} = \frac{1}{2}\mu \frac{dx}{dt}\,\omega^2 A^2 = P \text{, power.}$$

$$P = \underbrace{\left(\tfrac{1}{2}\mu\omega^2 A^2\right)}_{\substack{\text{energy}\\ \text{per}\\ \text{unit length}}} v$$

$\propto$ (frequency)^2

$\propto$ (amplitude)^2

$\propto$ (velocity)

bass speaker has to be big and oscillate much.

Imagine sound in 3 dimensions:

t

Electricity and magnetism.

Maxwell's mathematical treatment of Faraday's "lines of force" was REVOLUTIONARY. All other approaches to electricity and magnetism were Newtonian in character. It's easy to see why folks thought this way:

Coulomb's Electrostatic Force

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$$

1 $$\vec{F}_2 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{21}$$

permittivity of free space

$$\frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \frac{Nm^2}{C^2}$$

(note: there is a whole "thing" about units. Thornton and Rex use MKS units: meter, kilogram, second — and Coulomb, C. In these units factors like ϵ_0 stand out and the unit of charge, Coulomb, is used. A Coulomb is an enormous charge!

let $q_1 = q_2 = 1C$ and $r = 1m$.

$$|\vec{F}| = 8.99 \times 10^9 \text{ N} \rightarrow 1.01 \text{ million tons}$$

this units thing will confuse your physics career more than once! Typically we use "cgs" or "Gaussian" units. Here $F_c = \frac{q_1 q_2}{r^2}$ and charges take

their definitions from the force

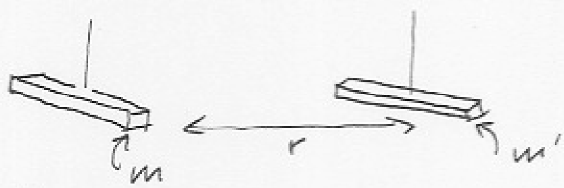
$$1\,C = 2.998 \times 10^9 \text{ esu} \qquad \text{"electrostatic units"}$$

It causes a variety of things to turn out nice in quantum mechanics.)

ahem.

Coulomb also did experiments with magnets – suspended them and carefully measured forces. He found

$$f_{poles} \propto \frac{m m'}{r^2}$$



Here, m and m' are the strengths of the "poles" of the magnets.

SO: two rules for forces, each of the $1/r^2$ variety. Just like gravity which everyone <u>knew</u> acted instantaneously across long distances

2 $$\vec{F}_{G_{12}} = G \frac{m_1 m_2}{r_{12}^2} \hat{r}_{12}$$

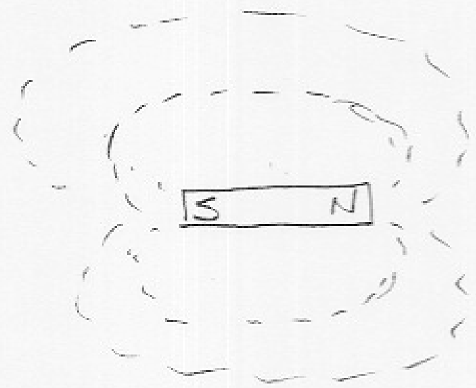
"Action at a Distance"

EVERYONE KNEW THIS

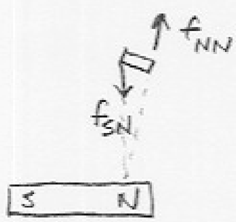
except Michael Faraday.

* Great Story – see course page for history / biography

Faraday is famous for his iron filings around a magnet



Ampere and other Newtonians knew it too. They would calculate the little pole-pole magnetic forces between iron filings as little magnets and the big magnet



etc → instantaneous

Faraday "saw" what he called "lines of force" which curved. Nobody believed him .. except James Clerk Maxwell. Hence, the birth of the FIELDS

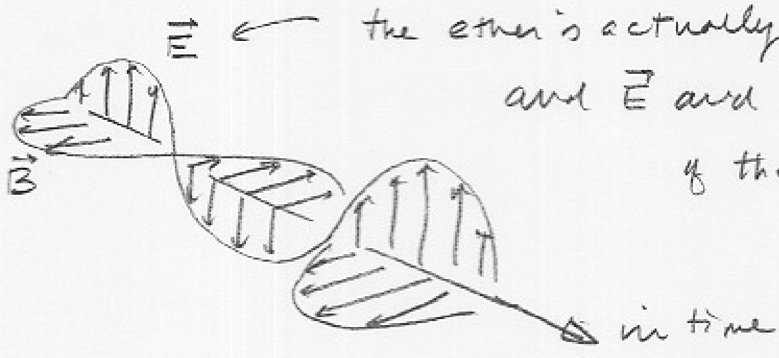
* Maxwell .. another great story – see course page

Okay, after Hertz found Electromagnetic Waves and measured their speed and their polarization in 1887

* Hertz .. a sad story – see course page

It was generally (not universally, for Maxwell's theory had competition) accepted that $\vec{E}$ and $\vec{B}$ fields were real.

How did they manifest themselves? As undulations – waviness – of the ether



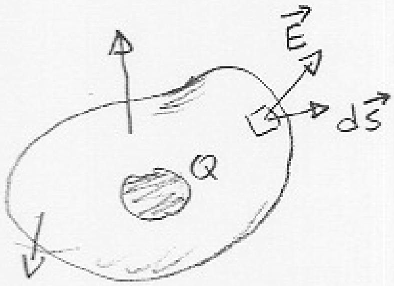
← the ether is actually oscillating and $\vec{E}$ and $\vec{B}$ are substances of the ether.

EVERBODY KNEW THIS

x

Maxwell's Equations in "Integral Form":

1.

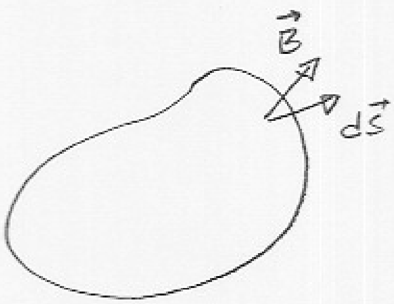


$$\epsilon_0 \oint_{\text{surface}} \vec{E} \cdot d\vec{S} = Q$$

Gauss' Law for Electrostatics

an arbitrary volume containing a net electric charge, Q

2.

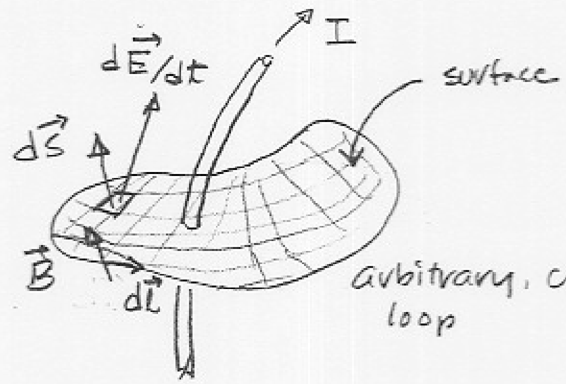


$$\oint_{\text{surface}} \vec{B} \cdot d\vec{S} = 0$$

Gauss' Law for magnetic fields

$\Rightarrow$ no "magnetic monopoles"

3.



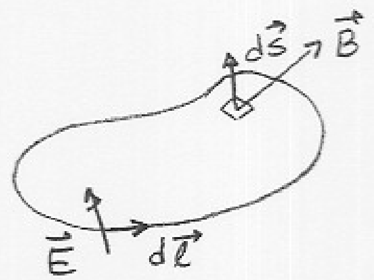
arbitrary, closed mathematical loop

enclosing, generally, a net current, I and a time-varying electric field, $\frac{d\vec{E}}{dt}$

$$\oint_{\text{loop}} \vec{B}\cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d}{dt}\int_{\text{surface}} \vec{E}\cdot d\vec{S} + \mu_0 I$$

Generalized Ampere's Law

4.



$$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\iint_{\text{surface}} \vec{B}\cdot d\vec{S}$$

Faraday's Law

In each of these:

$$\iint \vec{E}\cdot d\vec{S} \equiv \Phi_E \quad \text{electric flux}$$

$$\iint \vec{B}\cdot d\vec{S} \equiv \Phi_B \quad \text{magnetic flux}$$

(5.) Now included with Maxwell's equations of 1863 is the Lorentz force on particulate charges – a model of the ether proposed by him in 1892

$$\vec{F} = \frac{d\vec{p}}{dt} = q\vec{E} + q\vec{v} \times \vec{B}$$

which is, at some level, the seat of all of the trouble.

These versions of Maxwell's Equations are for extended regions of space. More useful are their forms for single points in space, the so-called "Differential Form" of Maxwell's Equations.

$$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0} \qquad 3$$

$$\oint \vec{B} \cdot d\vec{s} = 0 \qquad 4$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i + \epsilon_0 \frac{d\phi_E}{dt} \right) \qquad 5$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad 6$$

These refer to regions of space. More useful are their forms at particular points in space – the so-called differential form of Maxwell's Equations.

One can obtain the electric field as the derivative of the electric potential, $E_l = -\frac{dV}{dl}$ for the l^{th} arbitrary direction.

Since E is a vector, then a complete description of this involves

$$\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k} \quad \text{so} \qquad 7$$

$$\vec{E} = -\left(\frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right) \qquad 8$$

where V is a scalar function of only space $V = V(x, y, z)$

This is a standard vector calculus operator – the GRADIENT

$$\vec{\nabla} \equiv \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \qquad 9$$

so $$\vec{E} = -\vec{\nabla} V$$ 10

which you sometimes see written $\vec{E} = \text{grad } V$ (but not here).

The gradient operator functions in 3 ways: 1) like before, to find the change wrt coordinates of a scalar, and representing the change in each direction as a <u>vector</u>. 2) to find the change of a vector wrt each coordinate, producing a <u>scalar</u> function.

11 $$\vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$ the DIVERGENCE

and

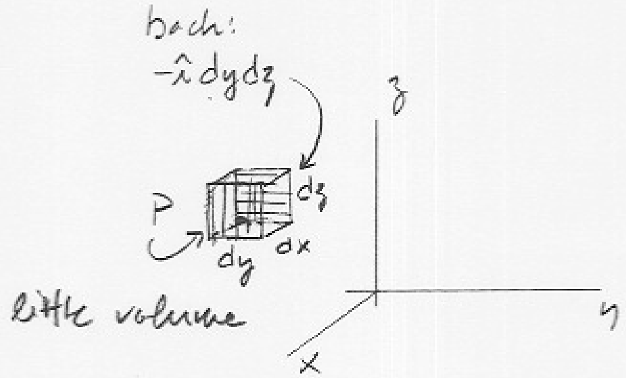
3) to calculate a different kind of vector, involving a cross product

$$\vec{\nabla} \times \vec{A} = \hat{\imath}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\jmath}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$ the CURL 12

(think the twist, or torque applied by a force field on an object by adding up all the contributions around the edge)

Now, let's work on eq. 3 & 4

ac



full of $\vec{E}$ field

P at x, y, z

volume element's <u>back</u> face has (outward) area

$$d\vec{S} = -\hat{\imath}\,dydz \qquad 13$$

front face

$$d\vec{S} = +\hat{\imath}\,dydz \qquad 14$$

The electric field at the back face is $\vec{E}(x,y,z)$ and at the front face it's changed by the amount associated with the distance from x to $x+dx$. So, it's

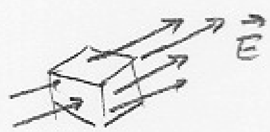
$$\vec{E} + \underbrace{\left(\frac{\partial \vec{E}}{\partial x}\right) dx}_{} \qquad 15$$

value at back

difference from back to front – only in x direction is there change.

The flux through the entire little volume is defined as

$$\oint \vec{E}\cdot d\vec{S}$$



Due to just these two faces,

cancel

$$= \vec{E}\cdot(-\hat{\imath}\,dydz) + \left(\vec{E} + \frac{\partial \vec{E}}{\partial x}dx\right)\cdot(+\hat{\imath}\,dydz)$$

$$= dx\,dy\,dz\left(\frac{\partial \vec{E}}{\partial x}\cdot\hat{\imath}\right) = dx\,dy\,dz\,\frac{\partial}{\partial x}\left(\vec{E}\cdot\hat{\imath}\right) \qquad 16$$

$$= dx\,dy\,dz\ \frac{\partial E_x}{\partial x}$$ 17

the other contributions for all 6 total faces looks the same:

$$\oint \vec{E}\cdot d\vec{S} = dx\,dy\,dz\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right)$$ 18

$$\oint \vec{E}\cdot d\vec{S} = dx\,dy\,dz\ (\ \vec{\nabla}\cdot\vec{E}\ )$$ 19

Go back to 3. q is the charge enclosed by the surface, by definition. So if ρ is the charge density, charge / unit volume, then

$$q = \rho\, dx\, dy\, dz$$ 20

3 + 20:

$$\oint \vec{E}\cdot d\vec{S} = dx\,dy\,dz\ \frac{\rho}{\epsilon_0}$$ 21

and 19

$$= dx\,dy\,dz\ (\ \vec{\nabla}\cdot\vec{E}\ )$$ 22

so,

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$ 23

is Maxwell's Equation for Gauss' Law.

The others follow suit...

Likewise, all four equations can be written as differential equations

charge density

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} \qquad 24$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad 25$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad 26$$

current density

$$\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} + \mu_0\vec{J} \qquad 27$$

In <u>free space</u> – no ρ, no $\vec{J}$ –

$$\vec{\nabla}\cdot\vec{E} = 0 \longrightarrow \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \qquad 28$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad 29$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad 30$$

← a time-changing $\vec{B}$ will produce a perpendicular $\vec{E}$

$$\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} = \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} \qquad 31$$

← a time-changing $\vec{E}$ will produce a perpendicular $\vec{B}$

for x component of $\vec{E}$

$$\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} = \mu_0\epsilon_0\frac{\partial E_x}{\partial t} \qquad 32$$

Back to electromagnetism: When EM waves appeared to confirm Maxwell's approach, one person - besides the young Hertz - took it very seriously *and* had the mathematical skill to investigate the subject completely -

A "textbook" maneuver - from equation 30

$$\underbrace{\vec{\nabla} \times \vec{\nabla} \times}_{} \vec{E} = \vec{\nabla} \times \left(- \frac{\partial \vec{B}}{\partial t} \right) \qquad 39$$

an identity in vector calculus -

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2}$$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \vec{\nabla} \left(\vec{\nabla} \cdot \vec{E} \right) - \nabla^2 E \qquad 40$$

=0 for no charges, eq. 28

$$\nabla^2 \vec{E} = \frac{\partial}{\partial t} \left(\vec{\nabla} \times \vec{B} \right)$$

$$\nabla^2 \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \qquad 41$$

(another for $\vec{B}$, also)

characteristic equation form → double space derivative + double time derivative → "WAVE EQUATION"

generally: $$\frac{\partial^2 \varphi}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2 \varphi}{\partial t^2} = 0$$

That's what Maxwell saw as a consequence of his equations, so EM WAVES a prediction with wave speed $\quad \frac{1}{v^2} = \mu_0 \epsilon_0$

$$v = \sqrt{\frac{1}{\mu_0 \epsilon_0}} \qquad 42$$

$$\mu_0 = 4\pi \times 10^{-7}\ N \cdot C^{-2} \cdot s^2 \qquad 43$$

$$\epsilon_0 = 8.8542 \times 10^{-12}\ C^2 \cdot N^{-1} \cdot m^{-2} \qquad 44$$

so

$$\sqrt{\frac{1}{\left(4\pi \times 10^{-7} \cancel{\frac{N \cdot s^2}{C^2}}\right)\left(8.8542 \times 10^{-12} \cancel{\frac{C^2}{N \cdot m^2}}\right)}}$$

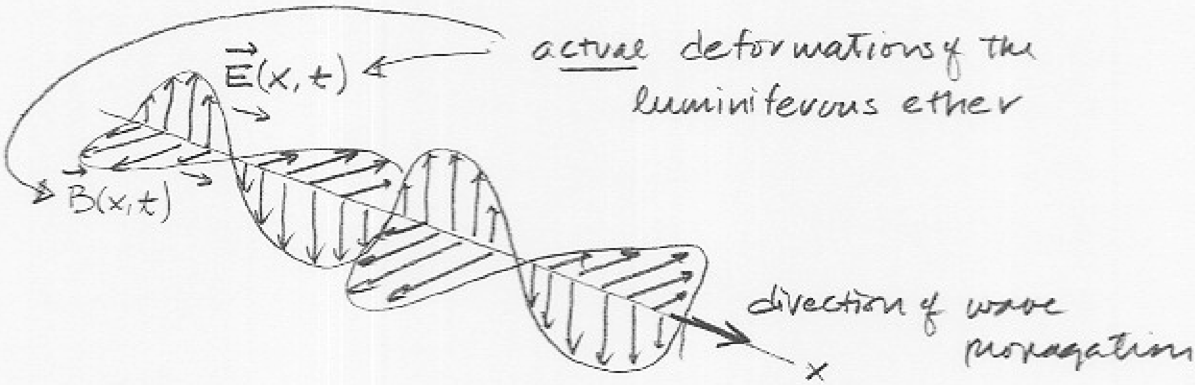
$$= 2.998 \times 10^8\ m/s = c \qquad 45$$

which he recognized as close to the measured speed of light

Notice that solutions to 41 would be of the form

$$\cos k(x - vt) \qquad 46$$

where k is the wave number



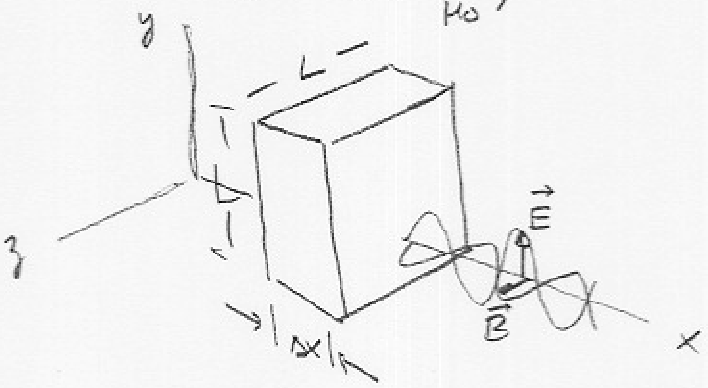
the $\frac{d\vec{E}}{dt}$ creates the $\vec{B}$ and the

$\frac{d\vec{B}}{dt}$ creates the $\vec{E}$: no charges, no currents.

The EM wave transmitts energy associated with the energy density of electric and magnetic fields

$$u_E = \frac{1}{2} \epsilon_0 E^2$$

$$u_B = \frac{1}{2} \left(\frac{B^2}{\mu_0} \right)$$



The total energy in the volume

$$\Delta U = \Delta U_E + \Delta U_B$$

$$= \frac{1}{2} L^2 \Delta x \left(\epsilon_0 E_y^2 + \frac{1}{\mu_0} B_z^2 \right)$$

since $\quad c = (\mu_0 \epsilon_0)^{-1/2}$

and

$$E_y = c B_z$$

$$\Delta u = \frac{1}{2} L^2 \Delta x \left(\epsilon_0 c E_y B_z + \frac{1}{\mu_0 c} E_y B_z \right)$$

$$= \frac{1}{2} L^2 \frac{\Delta x}{c} E_y B_z \left(\epsilon_0 c^2 + \frac{1}{\mu_0} \right)$$

aj

$$\Delta U = L^2 E_y B_z \frac{\Delta x}{c} \left(\frac{1}{\mu_0}\right)$$

The time it takes to pass through the face is

$$\Delta t = \frac{\Delta x}{c}$$

Define the energy per unit time flowing through the unit area

$$S = \frac{E}{(\text{Area})(\text{time})}$$

$$= \frac{\Delta U}{L^2 \Delta t}$$

$$S = \frac{1}{\mu_0} E_y B_z$$ along the direction of motion

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$ the Poynting vector

This is instantaneous. For sinusoidally varying waves

$$S_{ave} = \frac{1}{2\mu_0} E_M B_M$$

It's easy to show that the momentum of a EM wave is

$$p = \frac{U}{c}$$

ah

The intensity is just the magnitude of $\langle \bar{S} \rangle$, often written

$$I = \frac{1}{\mu_0} B_{rms} E_{rms}$$

$$|B| = \frac{|E|}{c}$$

$$I = \frac{E_{rms}^2}{\mu_0 c}$$

$$c^2 = \frac{1}{\mu_0 \epsilon_0}$$

$$c \mu_0 c = \frac{1}{\epsilon_0}$$

$$\mu_0 c = \frac{1}{c \epsilon_0}$$

$$I = \epsilon_0 E_{rms}^2 c$$

An important point.

The whole emphasis in Relativity is INVARIANCE of physics relations. This has old, hallowed origins and is the absolute centerpiece of modern physics today. It's pretty mathematical — and, pretty mathematics! — and Albert Einstein was the first person to make INVARIANCE and SYMMETRY the primary tool of his theoretical work.. and Special Relativity was the first place he did it.

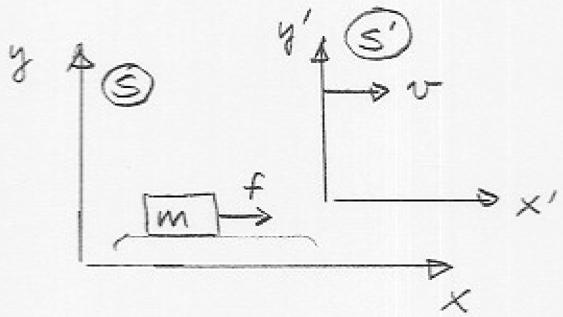
am

What does invariance mean physically?

Galileo described it by imagining doing experiments inside the hold of a ship moving at a constant speed. You could not tell that the ship is moving, he decided.

In modern terms this means that the physical "laws" are the same no matter where you do experiments in 2 such frames of reference IF THEY ARE COMOVING AT CONSTANT VELOCITIES, inertia rest frames.

In practice, this requires that the mathematical — functional — FORMS of the equations of that physics be the same.



The coordinates of S' and S are related by a Galilean Transformation

$x = x' + vt$

$t = t'$ ⇐ crucial aspect of Newton's World

Newton's 2nd law would govern the description for the motion (acceleration) of the block

$$f = ma \qquad (33)$$

and if the physics is the same described by an observer in S'

$$f' = ma' \qquad (34)$$

In one frame

$$f = ma$$

In the other frame

$$f' = ma'$$

frames

connected via

$$x = x' + vt' \qquad 35$$

$$t = t' \qquad 36$$

FORM INVARIANCE.

Usually construct the derivatives of the transformation equations using the chain rule to link the frames!

$$x = x' + vt'$$

$$\frac{dx}{dt} = \frac{dx'}{dt'}\underbrace{\frac{dt'}{dt}}_{1} + v\underbrace{\frac{dt'}{dt}}_{1}$$

$$u = u' + v \qquad 37$$

One more

$$\frac{du}{dt} = \frac{du'}{dt'}\frac{dt'}{dt} + 0$$

$$\frac{du}{dt} = \frac{du'}{dt'}$$

$$a = a'$$

so

$$f \equiv ma = ma' \equiv f' \qquad 38$$

Newton's 2nd Law is invariant with respect to Galilean Trans.