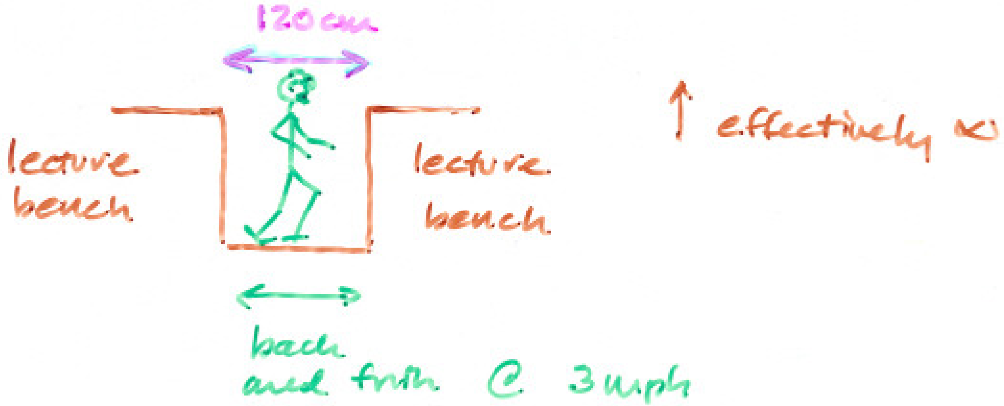
How about me in the well...



193 lbs dripping wet $\Rightarrow$ $m = 87.5$ kg.

3 mph $\Rightarrow$ $v = 1.34$ m/s

$$E = \tfrac{1}{2} m v^2 = \tfrac{1}{2}(87.5)(1.34)^2 = 78.6 \text{ J}$$

what quantum energy level am I in?

$$E_n = \frac{n^2 \hbar^2 \pi^2}{2 m L^2} \Rightarrow n = \sqrt{2mE} \cdot \frac{L}{\hbar \pi}$$

$$n \simeq 4.3 \times 10^{35}$$

remember Bohr's Correspondence Principle?

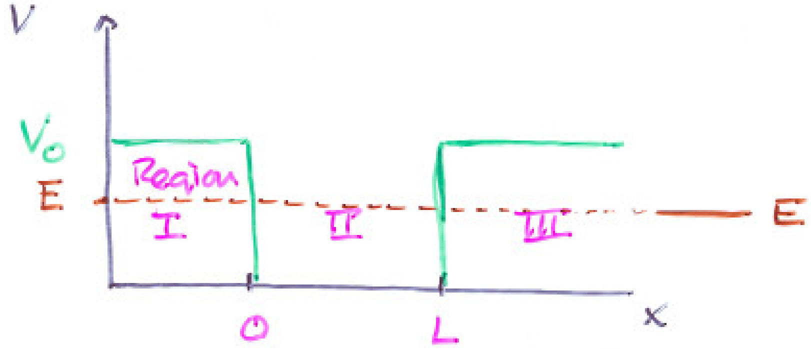
what's my probability of being at some point in well?

$$\psi_n = \sqrt{2/L} \sin\left(\frac{n\pi x}{L}\right) \rightarrow P \propto \sin^2\left(\frac{n\pi x}{L}\right)$$

$$\sin^2(1.1 \times 10^{36} x)$$

zeroes at $x = 3 \times 10^{-36}$ m

FINITE SQUARE WELL — famous problem!



classically... with $E < V_0$ if the particle is in region II: IT WOULD BE TRAPPED

Sch Eq:
$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

in region II ... $V(x) = 0$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$$

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi = -k^2\psi$$

general solutions are:

$$\psi_{II}(x) = Ce^{ikx} + De^{-ikx}$$

— usual standing waves

quantum mechanics has a funny situation...

regions I and II $\qquad x < 0 \quad V(x) = V_0$

$\qquad\qquad\qquad\qquad x > L \quad = V_0$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V_0\psi = E\psi$$

$$-\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}(E - V_0)\psi$$

$$\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}(V_0 - E)\psi$$

$$\frac{d^2\psi}{dx^2} = +\alpha^2\psi \qquad \alpha^2 = \underbrace{\frac{2m}{\hbar^2}(V_0 - E)}_{\text{which is } +}$$

first note:

THIS HAS A SOLUTION!

DECIDEDLY <u>NON</u> - classical

THESE GENERAL SOLUTIONS ARE

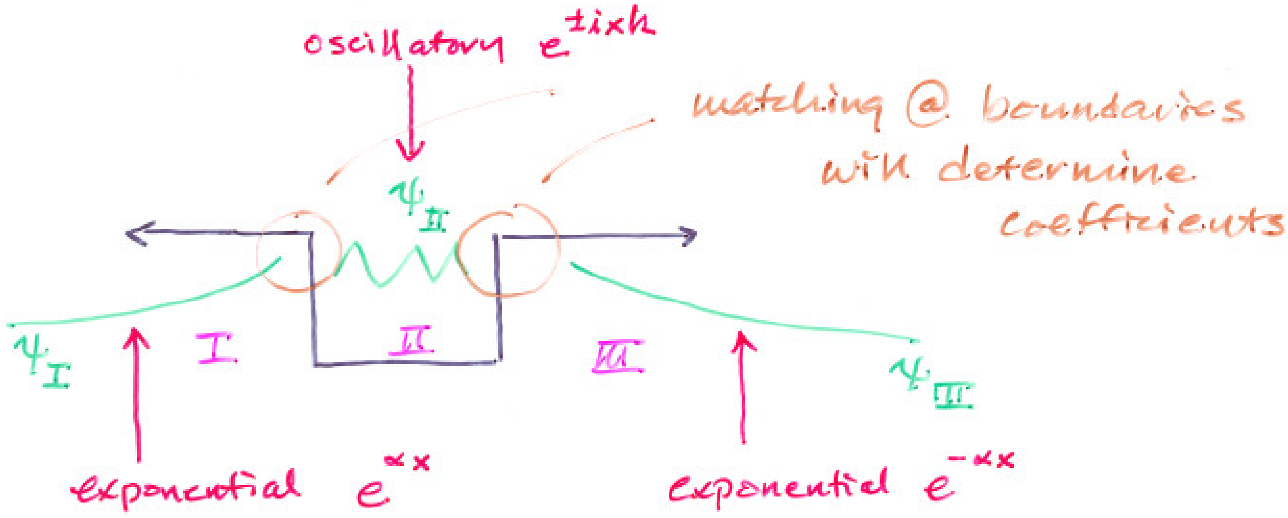
$$\psi_I = Ae^{\alpha x} + \cancelto{0}{Be^{-\alpha x}} \qquad \text{to keep } \psi_I \text{ finite, } x \to -\infty$$

$$\psi_{III} = \cancelto{0}{Ae^{\alpha x}} + Be^{-\alpha x} \qquad \psi_{III} \text{ finite, } x \to \infty$$

note! each DECAYS EXPONENTIALLY

So, there are solutions in all 3 regions

and they must match at the boundaries.



THIS IS COMPLICATED TO SOLVE... THE "ANSWER" IS ON NEXT PAGE.

The energy:

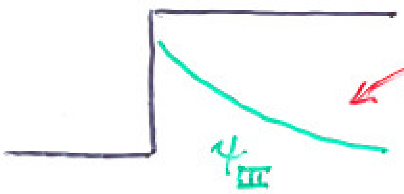
$$E_n = \frac{n^2 \pi^2 \hbar^2}{2m(L + 2/\alpha)^2} \qquad n = 1, 2 \ldots$$

complicated... $\alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$

is energy-<u>dependent</u>

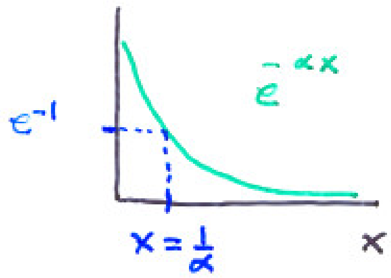
must be solved <u>numerically</u>

cannot be solved analytically.



in $e^{-\alpha x}$

α characterizes how far into the wall the wavefunction penetrates



$e^{-1} = 0.368$

$1/\alpha$ rule of thumb characterization of an exponential

$$\alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

So, obviously...

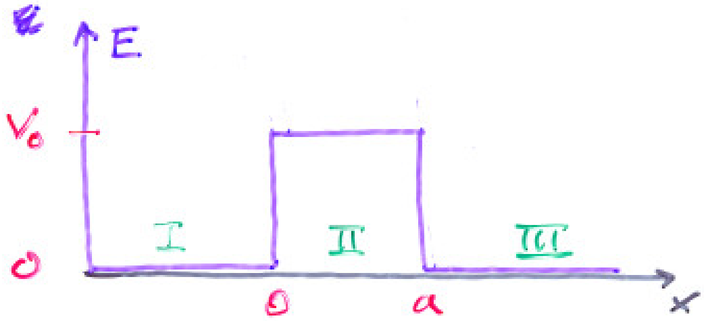
$$1/\alpha = \frac{\hbar}{\sqrt{2m(V_0 - E)}}$$

proportional to $\hbar$ $\Rightarrow$ tiny, but finite non-classical effect.

SO: the wavefunction literally leaks into classically forbidden regions

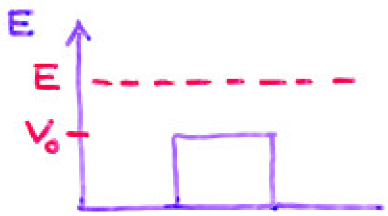
$\rightarrow$ many real-world consequences.

BARRIERS & TUNNELING

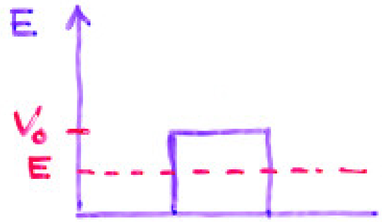


Particle incident from the LEFT with energy E.

2 circumstances:

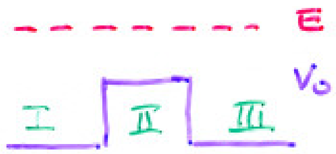


$E > V_0$



$0 < E < V_0$

A.



I. $$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \qquad V=0$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0 \qquad k^2 \equiv \frac{2mE}{\hbar^2}$$

general solution:

$$\psi_I(x) = Ae^{ikx} + Be^{-ikx}$$

II. $$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V_0)\psi = 0 \qquad q^2 \equiv \frac{2m(E-V_0)}{\hbar^2} > 0$$

general solution:

$$\psi_{II}(x) = Ge^{iqx} + Fe^{-iqx}$$

III. same Sch. Eq. as I.

general solution:

$$\psi_{III}(x) = Ce^{ikx} + De^{-ikx}$$

$D = 0$

Boundary conditions say:

$\psi(x)$'s and $\frac{d\psi(x)}{dx}$'s are continuous

at $x=0$: $\psi_I(0) = \psi_{II}(0)$

$$A + B = G + F$$

$x=a$: $\psi_{II}(a) = \psi_{III}(a)$

$$Ge^{iqa} + Fe^{-iqa} = Ce^{ika}$$

$x=0$: $\frac{d\psi(0)}{dx}_I = \frac{d\psi(0)}{dx}_{II}$

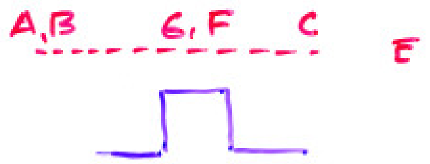
$$Aik\,e^{ik0} + (B)(-ik)e^{-ik0} = iqGe^{iq0} - iqFe^{-iq0}$$

$$k(A-B) = q(G-F)$$

$x=a$: $\frac{d\psi_{II}(a)}{dx} = \frac{d}{dx}\psi_{III}(a)$

$$iqGe^{iqa} - iqFe^{-iqa} = ikCe^{ika}$$

4 equations, 5 unknowns... A, B, G, F, C



Form ratios:

$$\left|\frac{C}{A}\right|^2 \rightarrow \text{ like a transmission past the barrier}$$

$$T \equiv \left|\frac{C}{A}\right|^2 = \frac{4E(E-V_0)}{V_0^2 \sin^2 qa + 4E(E-V_0)}$$

$$\left|\frac{B}{A}\right|^2 \rightarrow \text{ like a reflection(!)}$$

$$R \equiv \left|\frac{B}{A}\right|^2 = \frac{V_0^2 \sin^2 qa}{V_0^2 \sin^2 qa + 4E(E-V_0)}$$

which is not zero for $V_0 \neq 0$

notice that if $V_0 = 0$ $\quad T \rightarrow 1$ & $R \rightarrow 0$

Think Classically.

$$E = \frac{p^2}{2m} + V_0$$

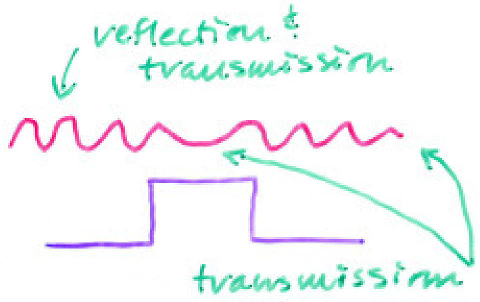
$$p = \sqrt{2m(E-V_0)}$$

momentum reduced as particle passes by V_0

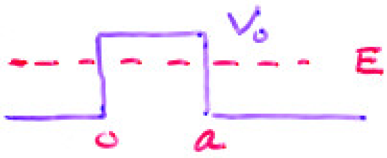
Quantum mechanically

$$\lambda_I = \frac{2\pi\hbar}{p} = \frac{2\pi\hbar}{\sqrt{2mE}}$$

$$\lambda_{II} = \frac{2\pi\hbar}{\sqrt{2m(E-V_0)}}$$



B.



I. $\psi_I(x) = A e^{ikx} + B e^{-ikx}$ as before

II. $q^2 = \frac{2m(E - V_0)}{\hbar^2}$

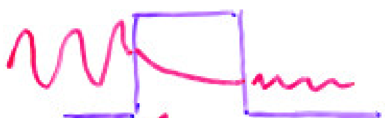
BUT... $q = \frac{\sqrt{2m(E-V_0)}}{\hbar}$ is imaginary

$$q = \frac{i}{\hbar}\sqrt{2m(V_0 - E)}$$

$$\equiv \frac{i}{2\delta} \; ; \quad \delta \equiv \frac{1}{\sqrt{8m(V_0-E)}}$$

$$\psi_{II}(x) = G e^{-\frac{x}{2\delta}} + F e^{\frac{x}{2\delta}}$$

III. $\psi_{III}(x) = C e^{ikx}$ as before



exponentiates ... but still penetrates

$\boxed{\text{TUNNELING}}$

$$T = \frac{4E(V_0 - E)}{V_0^2 \sinh^2 \frac{a}{2\delta} + 4E(V_0 - E)}$$

$\neq 0$

ie tunneling happens

$$R = \frac{V_0^2 \sinh^2 \frac{a}{2\delta}}{V_0^2 \sinh^2 \frac{a}{2\delta} + 4E(V_0 - E)}$$

classically? particle would bounce of and go back.

THIS IS VERY REAL... RESPONSIBLE FOR MANY PHENOMENA.

NUCLEAR PHYSICS

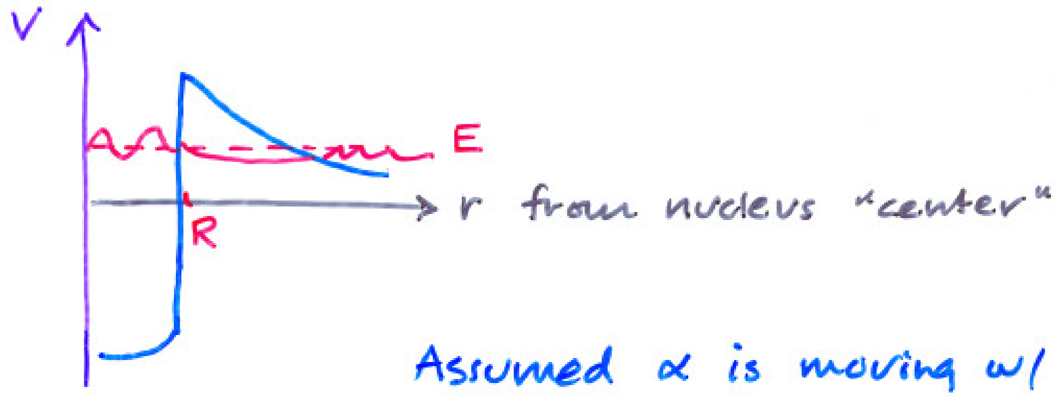
the protons in the nucleus are all +

→ they are still bound together with a force which OVERCOMES the Coulomb repulsion.

but decays from the nucleus do happen

→ explained in 1928 by George Gamow & independently by Ronald Gurney & Edward Condon

potential acting on an α particle (He nucleus):



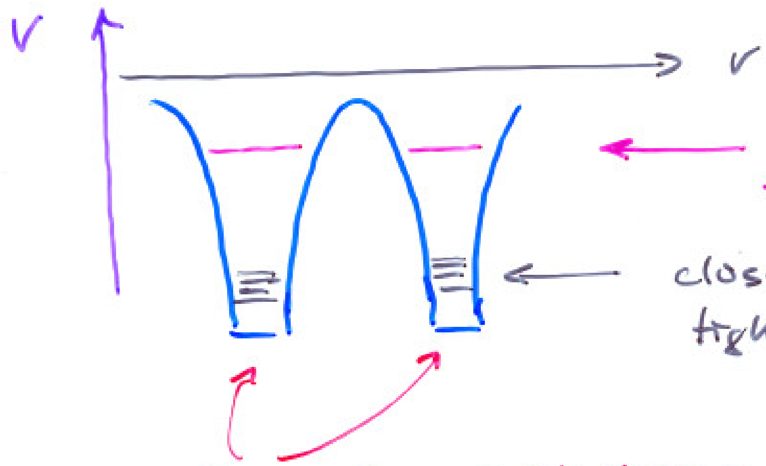
Assumed α is moving w/ some energy E inside R ... and Tunnels through the potential barrier

FUSION

$$\underbrace{{}^2H + {}^2H}\ \rightarrow\ {}^3H + n + 6.4 \times 10^{-13}\,J$$

a Coulomb barrier keeping deuterons apart.. unless they TUNNEL through it.

MOLECULES



outer electrons can tunnel from one atom to other

close-in electrons are tightly bound

deeply bound electron potentials

such electrons are shared within the molecule

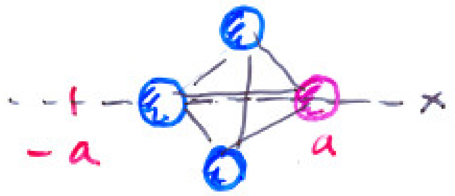
.. lose their identity

Scanning Tunnel Microscopy (STM)

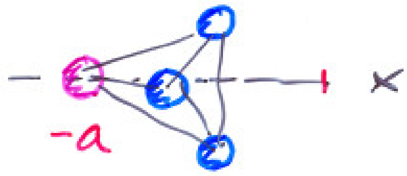
creates tiny current → highly sensitive to distance

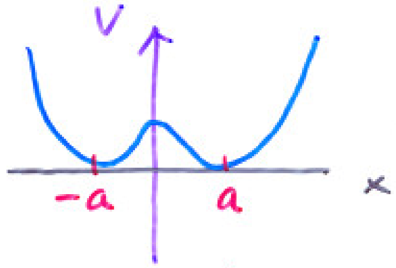
- can detect distances & measure peaks and valleys
- can actually deposit single atoms on a substrate

eg. ammonia NH_3



or



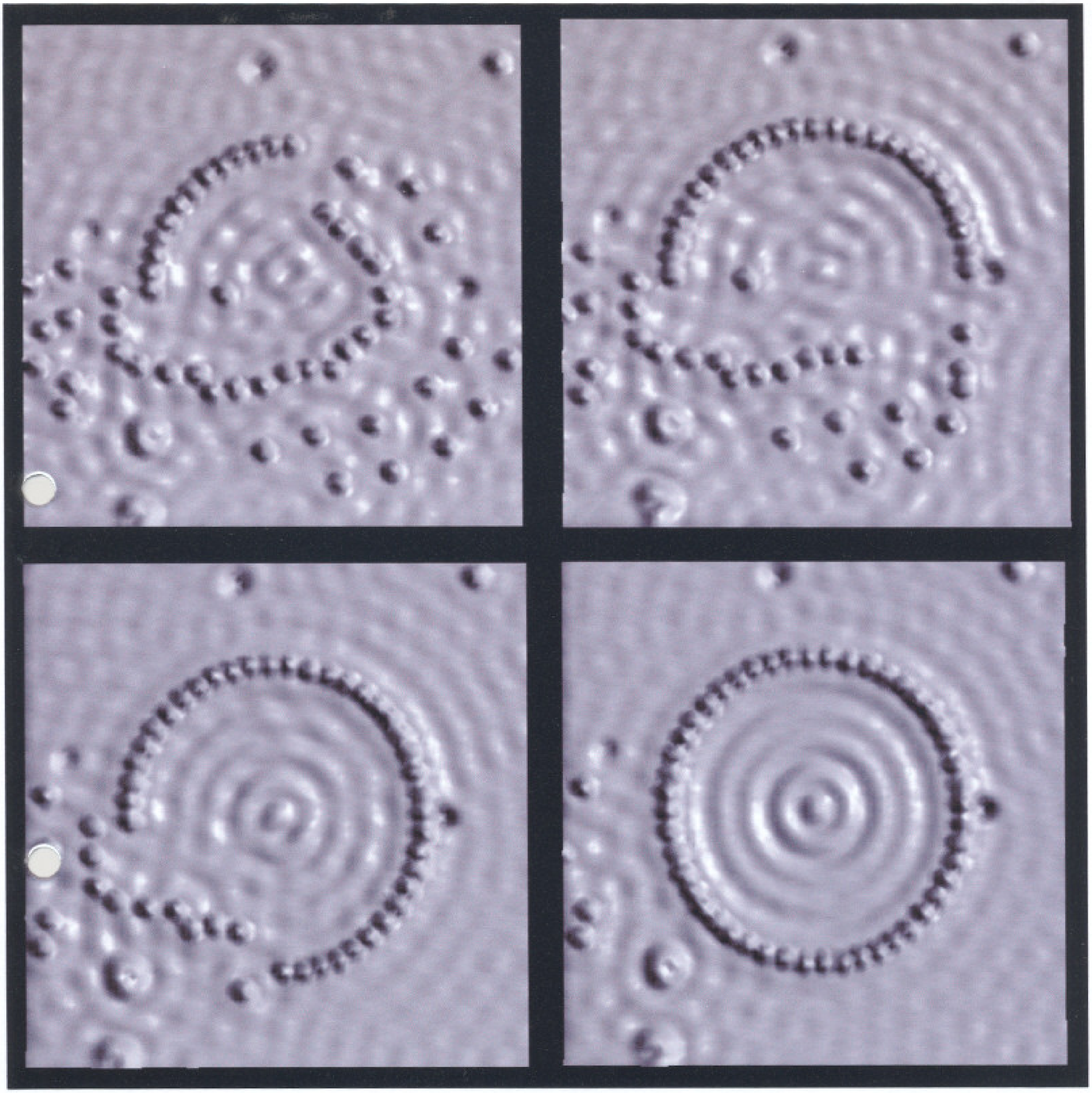


Sometimes the N is here.

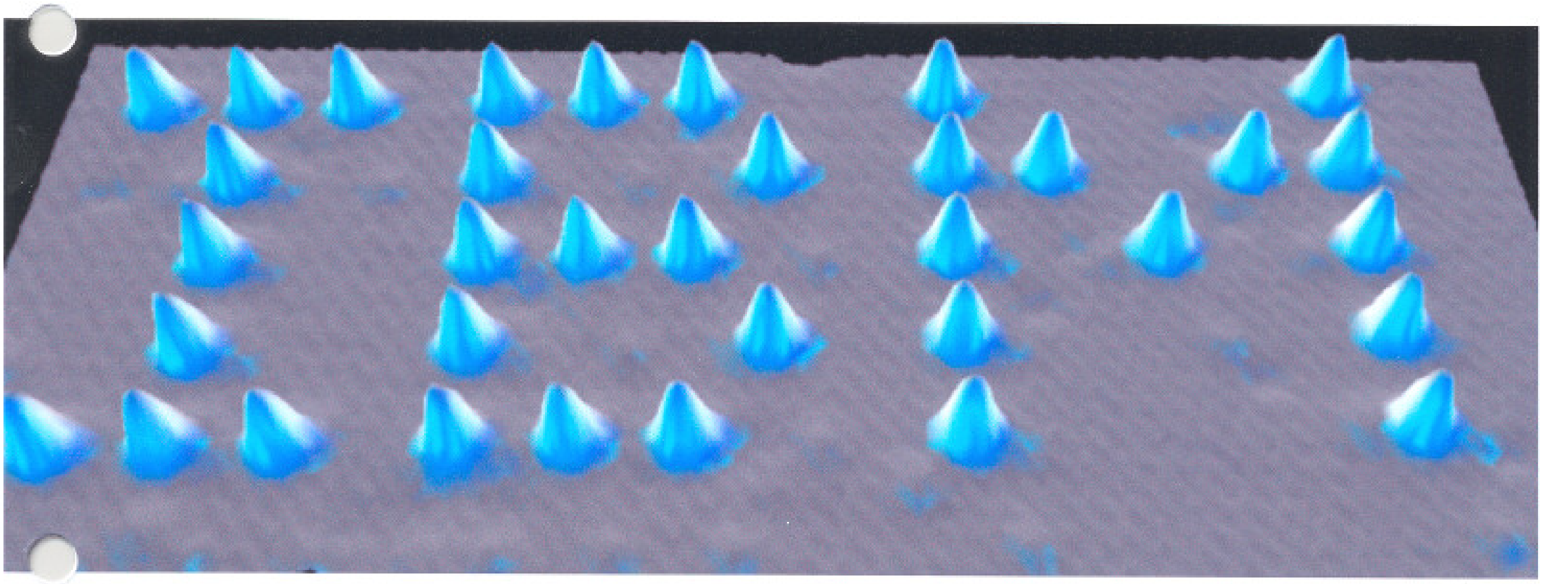
Sometimes, it's here.

tunnels to get from one to the other.

& emits radiation



iron on copper



Xenon on Nickel