Find C: by normalizing the probability

$$1 = \int d^3\vec{v}\; C\, e^{-Bv^2}$$

$$= C \int_{-\infty}^{\infty} dv_x\, e^{-Bv_x^2} \int_{-\infty}^{\infty} dv_y\, e^{-Bv_y^2} \int_{-\infty}^{\infty} dv_z\, e^{-Bv_z^2}$$

$$\int_{-\infty}^{\infty} dx\, e^{-Bx^2} = \sqrt{\pi/B}$$

gives $$C = \left(\frac{B}{\pi}\right)^{3/2}$$

Find B: use physics of the ideal gas.

$$\langle v^2\rangle = \left(\frac{B}{\pi}\right)^{3/2} \int d^3\vec{v}\; v^2 e^{-Bv^2}$$

$$= -\left(\frac{B}{\pi}\right)^{3/2} \frac{d}{dB} \int d^3\vec{v}\; e^{-Bv^2}$$

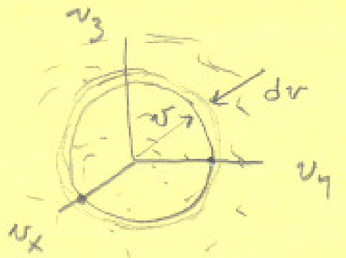
$$\langle v^2\rangle = -\left(\frac{B}{\pi}\right)^{3/2} \frac{d}{dB} \left(\frac{\pi}{B}\right)^{3/2} = \frac{3}{2B}$$

so,

$$B = \frac{2}{3}\langle v^2\rangle = \frac{m}{2kT}$$

$$f(v) = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT}$$ Maxwell distribution of molecular velocities

Speed distribution ... $f(\vec{v})$ is velocity because $f(\vec{v})\, d^3\vec{v}$ is the prob.



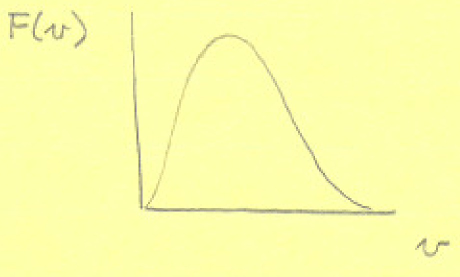
count number of molecules inside a shell of dv thickness to get speed count.

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$F(v)\, dv = f(\vec{v})\, 4\pi v^2\, dv$$

$$F(v)\, dv = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}\, dv$$

the MB speed distribution.



what's the peak?

$$v^*: \quad \frac{d F(v)}{dv} = 0$$

$$4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \left[2v\, e^{-mv^2/2kT} - v^2 \frac{1}{2}\frac{m}{kT}\, 2v\, e^{-mv^2/2kT} \right] = 0$$

$$2v - v^3 \frac{m^2}{kT} = 0 \quad \Rightarrow \quad v = 0 \text{ and } v^2 = \frac{2kT}{m}$$

$$v^* = \sqrt{\frac{2kT}{m}}$$

or $\frac{1}{2} m v^{*2} = kT = \frac{2}{3} \langle K \rangle$

average?

$$\langle v \rangle = \int v F(v)\, dv$$

$$= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty v^3 e^{-mv^2/2kT} dv$$

class of integrals.

$$I_n = \int_0^\infty x^n e^{-ax^2} dx = \frac{[(n-1)/2]!}{2a^{(n+1)/2}} \quad n \text{ odd}$$

$$= \frac{1.3.5 \cdots (n-1)}{2^{(n/2)+1} a^{n/2}} \sqrt{\frac{\pi}{a}} \quad n \text{ even.}$$

$$I_3 = \frac{\left(\frac{3-1}{2}\right)!}{2a^2} = \frac{1}{2a^2}$$

$$\langle v \rangle = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \frac{1}{2} \left(\frac{2kT}{m}\right)^2$$

$$= 4\pi \sqrt{\frac{1}{4}\left(\frac{m}{2\pi kT}\right)^3 \left(\frac{2kT}{m}\right)^4}$$

$$= 4\pi \sqrt{\frac{2kT}{m\pi^3}} = \frac{4}{\sqrt{2\pi}} \sqrt{\frac{kT}{m}}$$

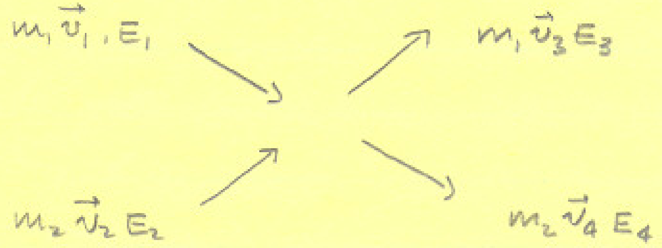
$$\frac{\langle v \rangle}{v^*} = \sqrt{\frac{4}{\pi}} \simeq 1.13$$

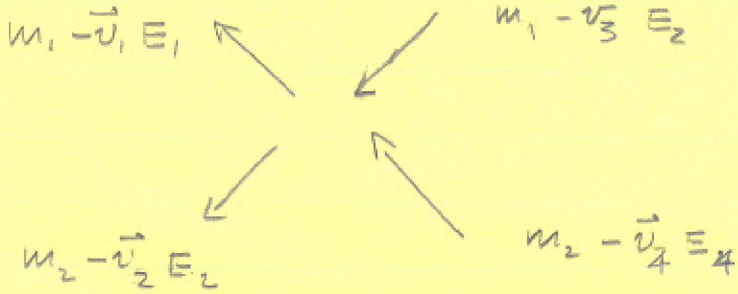
mean above peak for any temperature

Standard to assume that the molecules are point-like and non-interacting, so $E = \frac{1}{2} m v^2$

$$F(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 \underbrace{e^{-E/kT}}_{\text{an important quantity}}$$

Imagine the collision of two molecules.





Rate is proportional to the densities of projectiles and targets

$$R(12 \to 34) = c\, n(E_1)\, n(E_2)$$

$$R(34 \to 12) = c'\, n(E_3)\, n(E_4)$$

"microscopic reversibility" $\Rightarrow$ $c = c'$ (no loss)

In equilibrium: $R(12 \to 34) = R(34 \to 12)$

so $$n(E_1)\, n(E_2) = n(E_3)\, n(E_4) \quad ①$$

also: $$E_1 + E_2 = E_3 + E_4 \quad ②$$

Take log ①

$$\ln n(E_1) + \ln n(E_2) = \ln (n E_3 + \ln n(E_4))$$

how can this be compatible with ② ?

only if: $$\ln n(E) = A - \beta E$$

$$n(E) = e^{A-\beta E}$$

but $$e^{A} = e^{A-\beta(0)}$$

so $$\ln n(0) = A \Rightarrow e^{A} = n(0)$$

$$n(E) = n(0)\, e^{-\beta E}$$

Since this must become the MB distribution $\beta = 1/kT$

Go back and think about the gas from this perspective

$$n(E)\,dE = g(E)\,F_{MB}(E)\,dE$$
$$= g(E)\,A\,e^{-E/kT}$$

need to find density of states $g(E)$

Presume uniform distribution $S(v)\,dv = C\;4\pi v^2 dv$ ← proportional to the shell volume

~~where $C = \left(\frac{m}{2\pi kT}\right)^{3/2}$~~

each v corresponds to an E, so

$$g(E)\,dE = S(v)\,dv = C\;4\pi v^2 dv$$

so,

$$n(E)dE = g(E)F_{MB}(E)dE = C\;4\pi v^2 dv\;F_{MB}(E)$$
$$= C'\;4\pi v^2 dv\;e^{-E/kT}$$

$$\#\text{ particles per unit volume} = n(v)\,dv$$

$$\frac{N}{V} = \int_0^\infty n(v)\,dv = C'\int_0^\infty 4\pi v^2 e^{-mv^2/2kT}\,dv$$

$$= \frac{4\pi C'}{2^2\left(\frac{m}{2kT}\right)}\sqrt{\frac{\pi 2kT}{m}} = C'\left(\frac{2\pi kT}{m}\right)^{3/2}$$

$$C' = \frac{N}{V}\left(\frac{m}{2\pi kT}\right)^{3/2}$$

so,

$$n(v)\,dv = 4\pi \frac{N}{V}\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}\,dv$$

what we had before

In terms of energy

$$F(v)\,dv = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT}\, v^2\,dv$$

$$E = \frac{1}{2}mv^2$$

$$dE = \frac{2mv}{2}\,dv = mv\,dv$$

$$2Em = m^2v^2 \Rightarrow mv = \sqrt{2Em}$$

$$\frac{dE}{\sqrt{2Em}} = dv \qquad \text{and} \qquad v^2\frac{dE}{\sqrt{2Em}} = v^2\,dv$$

$$\frac{2E}{m}\frac{dE}{\sqrt{2Em}} = \frac{\sqrt{2}\sqrt{E}\,dE}{(m)^{3/2}}$$

$$F(v)\,dv \longrightarrow 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-E/kT}\,\frac{\sqrt{2}\sqrt{E}\,dE}{m^{3/2}}$$

$$= \sqrt{\frac{{}^{2}16\pi^2\; 2}{(2\pi kT)^3}}$$

$$F(E)\,dE = 2\pi\frac{1}{(\pi kT)^{3/2}}E^{1/2}e^{-E/kT}\,dE$$

This is a famous probability distribution in physics.. the Maxwell-Boltzmann Distribution

$$F_{MB} = A e^{-E/kT}$$

* probability that a single member of a large ensemble at temperature T has an energy E. *

$e^{-E/kT}$ is called the Boltzmann Factor.

$n(E)\,dE$ the distribution in number of states with energies between E and E+dE

$$n(E) = g(E)\, F_{MB}(E) \qquad (A)$$

where $g(E)$ is the number of states per unit energy range — called the Density of states.

This can be for continuous situations (Ⓐ) like energies and velocities of molecules in a gas, or DISCRETE situations.. like quantum ones!

$$n_i = g_i\, F_{MB}(E)$$

then g_i counts the number of states at a given E
→ The degeneracy factor of QM situations.

This is classical: distinguishable molecules.

What constitutes a point at which classical physics is not right to use?

The termal energy of the particles $\frac{3}{2} kT = \frac{1}{2} \frac{p^2}{2m}$

So, this p can also be thought of quantum mechanically.

$$p = \frac{h}{\lambda} \qquad , \qquad \lambda = \frac{h}{p}$$

If this λ is $\ll$ average distance between molecules, then classical.

$$\frac{1}{2} \frac{p^2}{m} = \frac{3}{2} kT$$

$$p = \sqrt{3mkT}$$

so $$\lambda = \frac{h}{\sqrt{3mkT}} \ll \sqrt[3]{\frac{V}{N}}$$

so, when

$$\left(\frac{N}{V}\right) \frac{h^3}{(3mkT)^{3/2}} \ll 1$$ MB statistics okay.

very high T

very small m