PHYSICS 215

– fall 2006

www.pa.msu.edu/courses/PHY215/

CLASSICAL PHYSICS?

TRADITIONALLY... NEWTON'S "LAWS"
MAXWELL'S EQUATIONS
ENERGY CONSERVATION

kinetic energy
potential energy

KE + PE = constant

for conservative forces

PHY215 begins the abstraction & dismantling of all of these ideas...

- Einstein's Special Theory of Relativity

  completely abstracting the notions of SPACE & TIME

- Quantum Mechanics

  completely abstracting the notions of what it means to KNOW.

TAKE NEWTON'S "LAWS"

$$\sum_{\substack{\text{all forces}\\ \text{on a body}}} \vec{F} = m\vec{a}$$

no. Newton didn't say that... Euler did later.

$$\sum_{i}^{\text{all forces}} \vec{F}_i = \frac{d\vec{p}}{dt}$$

where $\vec{p} = m\vec{v}$

"quantity of motion"

Simple, right?

$\vec{a}$ — change of velocity with respect to time

$\vec{v}$ — change of distance with respect to time

SO... $\vec{a}$ involves meter sticks

& clocks

EINSTEIN WILL MESS WITH THAT!

... acceleration with respect to what?

ABSOLUTE SPACE

&

ABSOLUTE TIME

EINSTEIN WILL MESS WITH THAT!

WHAT ABOUT m?

dunno... inertia? "stuff"?

how to define?

EINSTEIN WILL MESS WITH THAT!

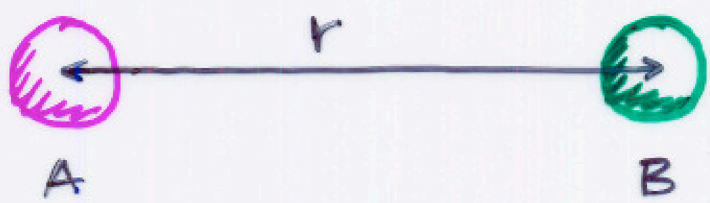
WHAT ABOUT $\vec{F}$?

dunno... how to define?

2nd LAW... kinda circular

OKAY.

WE GOT GRAVITY FROM NEWTON...



$$F_{AB} = G \frac{m_A m_B}{r^2}$$

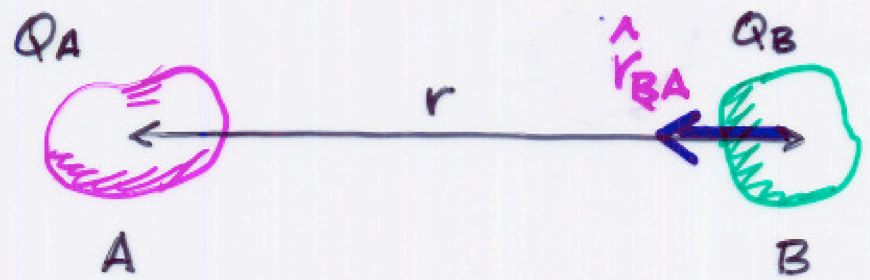
!

a force transmitted instantaneously across space

ACTION AT A DISTANCE

EINSTEIN WILL MESS WITH THAT!

ELECTRICITY & MAGNETISM



$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N m^2}$

$$\vec{F}_B = \frac{1}{4\pi\epsilon_0} \frac{Q_A Q_B}{r_{AB}^2} \hat{r}_{BA}$$

Coulomb's Electrostatic "Law"

great example of Action at a Distance to Coulomb, Ampere et al.

even a "Coulomb's" law for magnetism...

FARADAY happened

then

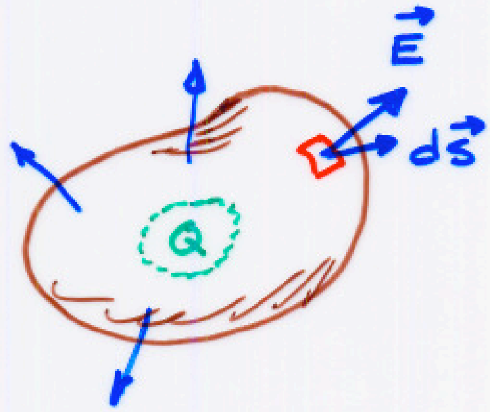
MAXWELL happened

} electromagnetic fields & Maxwell's Equations

MAXWELL'S EQUATIONS 1863

in "integral form"

1.

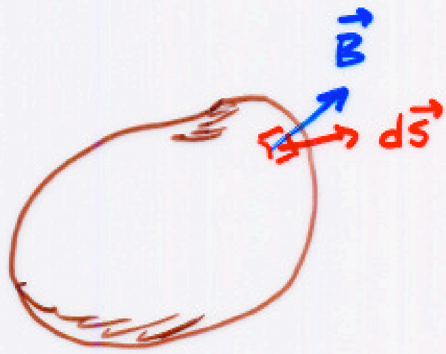


$$\epsilon_0 \oint_{\text{surface}} \vec{E} \cdot d\vec{s} = Q$$

Gauss' "Law"

for electrostatics

2.

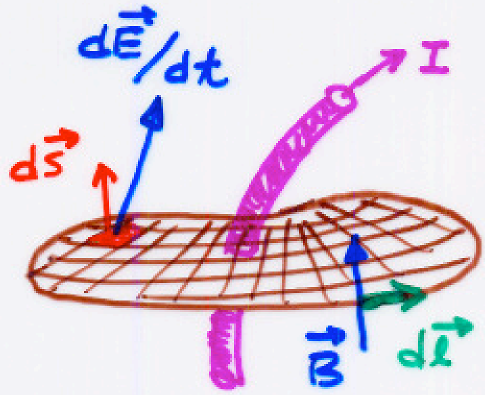


$$\oint_{\text{surface}} \vec{B} \cdot d\vec{s} = 0$$

Gauss' "Law" for magnetism

$\Rightarrow$ no "magnetic monopoles"

3.



arbitrary, closed mathematical loop

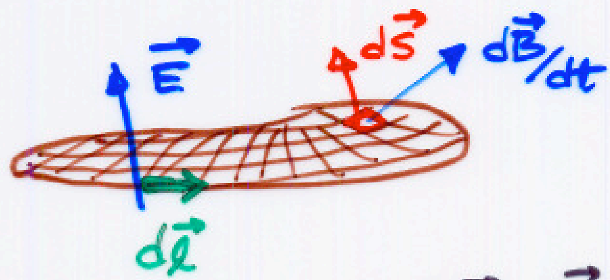
enclosing a net current, I

& a time varying $\vec{E}$

$$\oint_{\text{loop}} \vec{B}\cdot d\vec{\ell} = \mu_0 \epsilon_0 \frac{d}{dt} \iint_{\text{surface}} \vec{E}\cdot d\vec{s} + \mu_0 I \quad *$$

Generalized Ampere's "Law"

4.



$$\oint \vec{E}\cdot d\vec{\ell} = -\frac{d}{dt} \iint_{\text{surface}} \vec{B}\cdot d\vec{s}$$

Faraday's Law

In each of 3 & 4, can write in terms of FLUXES:

$$\iint \vec{E}\cdot d\vec{s} \equiv \Phi_E \quad \text{electric flux}$$

$$\iint \vec{B}\cdot d\vec{s} \equiv \Phi_B \quad \text{magnetic flux}$$

*($\mu_0 = 4\pi \times 10^{-7}\ N\cdot C^{-2}\cdot s^2$ permeability constant)

FIELD RESULT = SOURCES

E's & B's over extended regions

extended charge, current, changing fields

(5.) $$\vec{F} = \frac{d\vec{p}}{dt} = q\vec{E} + q\vec{v}\times\vec{B}$$

Lorentz Force... for particulate charges
(more in a bit)

MORE USEFUL → M.E.'s for particular points

so called "differential form" of Maxwell's Equations not calculate them all... hint at it:

$$\oint \vec{E}\cdot d\vec{s} = q/\epsilon_0$$

$$\oint \vec{B}\cdot d\vec{s} = 0$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \left(I + \epsilon_0 \frac{d\Phi_E}{dt} \right)$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

REMEMBER... $\vec{E}$ from electric potential?

$$E_l = -\frac{dV}{dl}$$ for l^{th} coordinate

$$\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$$

so: $$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k} \right)$$

where

$$V = V(x, y, z)$$ a scalar function of coordinates

GRADIENT $\vec{\nabla} \equiv \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

$\vec{E} = -\vec{\nabla} V$ (or $\vec{E} = \text{grad}\, V$)

$\vec{\nabla}$ is a versitile operator: $\vec{A}$ arbitrary vector

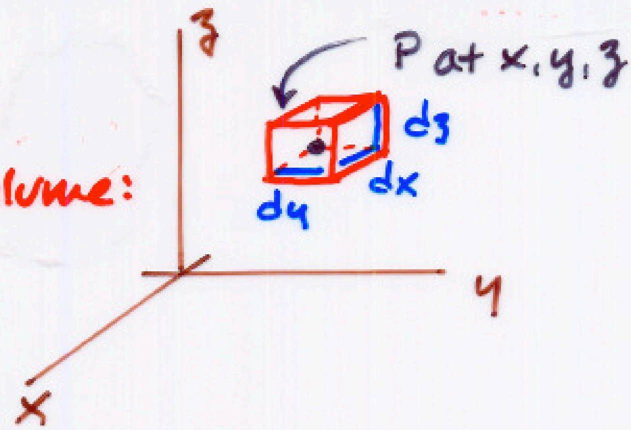
THE DIVERGENCE:

$$\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

THE CURL:

$$\vec{\nabla}\times\vec{A} = \hat{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{j}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

a little, teeny volume:



remember surface vectors point out

BACK FACE AREA: $d\vec{S} = -\hat{\imath}\, dy\, dz$

FRONT FACE AREA: $d\vec{S} = +\hat{\imath}\, dy\, dz$

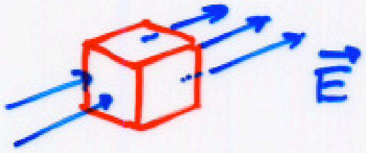
Assume $\exists\ \vec{E}$ .. at back face, $\vec{E}(x,y,z)$ which <u>changes</u> by the front face...

$$x \longrightarrow x + dx$$

$$\vec{E}(x,y,z) + \underbrace{\left(\frac{\partial \vec{E}}{\partial x}\right) dx}$$

value at back

change, going from back to front.. along x



FLUX THROUGH ENTIRE VOLUME:

$$\oint \vec{E} \cdot d\vec{S}$$

due to just these 2 faces:

$$\underset{\text{back, front}}{\int \vec{E} \cdot d\vec{S}} = \vec{E} \cdot (-\hat{\imath}\, dy\, dz) + \left(\vec{E} + \frac{\partial \vec{E}}{\partial x} dx\right) \cdot (+\hat{\imath}\, dy\, dz)$$

cancel

$$= dx\, dy\, dz \left(\frac{\partial \vec{E}}{\partial x} \cdot \hat{\imath} \right)$$

$$= dx\, dy\, dz\, \frac{\partial}{\partial x} \left(\vec{E} \cdot \hat{\imath} \right) = dx\, dy\, dz\, \frac{\partial E_x}{\partial x}$$

the other 4 faces' contributions look similar...

$$\oint \vec{E} \cdot d\vec{S} = dx\, dy\, dz \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right)$$

$$= dx\, dy\, dz \left(\vec{\nabla} \cdot \vec{E} \right)$$

charge inside?

presume charge density ρ

total charge in  $q = \rho \, dx\, dy\, dz$

so, 1st M.E. $\oint \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$

$$dx\,dy\,dz\,(\vec{\nabla} \cdot \vec{E}) = \frac{1}{\epsilon_0}(\rho \, dx\, dy\, dz)$$

and get: $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

Maxwell's Equation for Gauss "Law" ... in differential form ... tiny regions

THE OTHERS FOLLOW...

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$$ ← current density

THE PAY-OFF... FREE SPACE

no, ρ or $\vec{J}$

$$\vec{\nabla}\cdot\vec{E} = 0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ ← time changing $\vec{B} \Rightarrow \perp \vec{E}$

$$\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0 \frac{\partial \vec{E}}{\partial t}$$ ← ditto $E \leftrightarrow B$

look at particular component... x

$$\mu_0\epsilon_0 \frac{\partial E_x}{\partial t} = \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}$$

remember this