go to the $\vec{\nabla}\times\vec{E}$ equation... curl it again:

$$\underbrace{\vec{\nabla}\times\vec{\nabla}\times\vec{E}}_{} = \vec{\nabla}\times\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

vector calculus identity:

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = \vec{\nabla}(\underbrace{\vec{\nabla}\cdot\vec{E}}_{=0}) - \nabla^2\vec{E}$$

$$\nabla^2\vec{E} = \frac{\partial^2\vec{E}}{\partial x^2} + \frac{\partial^2\vec{E}}{\partial y^2} + \frac{\partial^2\vec{E}}{\partial z^2}$$

$=0$ Gauss' "Law" in free space

so:

$$\nabla^2\vec{E} = \frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$ from 4^{th} M.E.

$$\nabla^2\vec{E} - \mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2} = 0$$

(ditto for a $\vec{B}$ one also)

* A WAVE EQUATION *

general wave equation: $\frac{\partial^2 \varphi}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 \varphi}{\partial t^2} = 0$

velocity of the wave

so, $\frac{1}{v^2} = \epsilon_0 \mu_0$

$$v = \sqrt{\frac{1}{\epsilon_0 \mu_0}}$$

$\mu_0 = 4\pi \times 10^{-7}\ N \cdot C^{-2} \cdot s^2$

$\epsilon_0 = 8.8542 \times 10^{-12}\ C^2 \cdot N^{-1} \cdot m^{-2}$

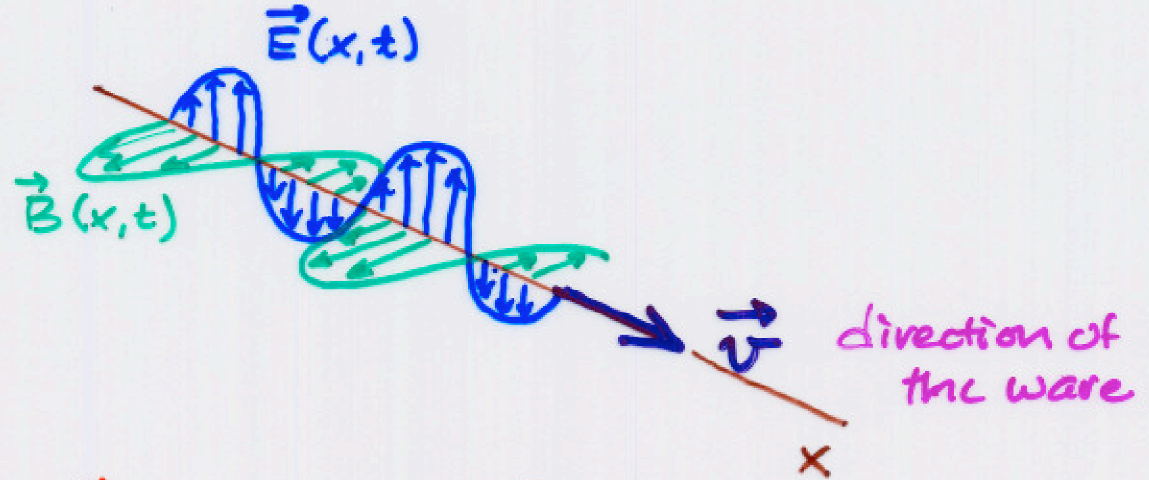
$$v = \frac{1}{\sqrt{(4\pi \times 10^{-7}) \cancel{N} \cdot \frac{s^2}{\cancel{C^2}} (8.8542 \times 10^{-12}) \cancel{C^2} \cdot \cancel{N^{-1}} \cdot m^{-2}}}$$

$v = 2.998 \times 10^8\ m/s = c$!

bit of a surprise for Maxwell...
he knew the recent measured
value for c

ALL WAVE EQUATIONS HAVE

$\cos k(x - vt)$ - like solutions



the $\frac{d\vec{E}}{dt}$ creates the $\vec{B}$

&

the $\frac{d\vec{B}}{dt}$ creates the $\vec{E}$

and so on, and so on...

-- as you know--

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$ Poynting's Vector

$$|\vec{S}| = \frac{\text{energy}}{(\text{area})(\text{time})}$$

& momentum $\quad p = \frac{\text{energy density}}{c}$

$$\text{Intensity} = |\langle S \rangle|$$

$$I = \frac{1}{\mu_0} B_{rms} E_{rms}$$

from ME ... $|B| = \frac{|E|}{c}$

$$I = \frac{E_{rms}^2}{\mu_0 c} = \epsilon_0 E_{rms}^2 c$$

OKAY. GREAT. WAVES... what actually waves?

THE ETHER

a very odd substance

more in a bit.

It's ALL ABOUT INVARIANCE

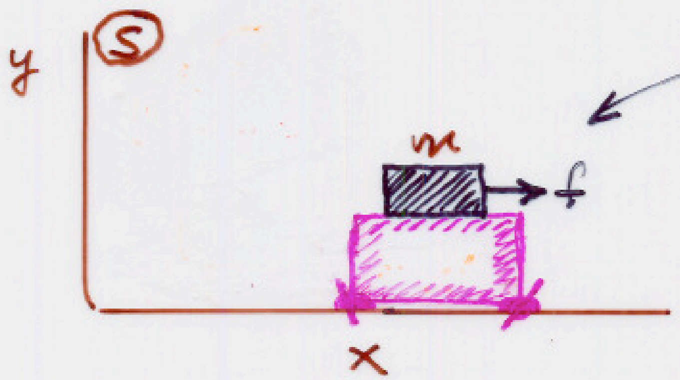
→ "physics," as reflected in equations, shouldn't be different for different observers... the "form" of the equations should not be different

(Galileo... Einstein)

CONSIDER 2 SPECIAL FRAMES OF REFERENCE

inertial rest frames

⇒ constant, relative velocities



f applied in S

EVERYONE KNEW THAT THE CONNECTION BETWEEN S & S' WAS

$$\left.\begin{aligned} x &= x' + vt \\ y &= y' \\ t &= t' \end{aligned}\right\} \text{"Galilean Transformations"}$$

IN (S) Newton's 2^{nd} "LAW" describes the motion of the block...

$$f = ma$$

FOR (S'), IT SHOULD BE THE SAME

$$f' = m'a' \quad \ldots \text{ check...}$$

(presume $m = m'$)

$$x = x' + vt'$$

$$\underbrace{\frac{dx}{dt}}_{u} = \underbrace{\frac{dx'}{dt'}}_{u'} \cdot \underbrace{\frac{dt'}{dt}}_{1} + v\underbrace{\frac{dt'}{dt}}_{1}$$

$$u = u' + v$$

$$\frac{du}{dt} = \frac{du'}{dt'} \underbrace{\frac{dt'}{dt}}_{1} + \underbrace{\frac{dv}{dt}}_{0}$$

$$a = a'$$

so, $\quad f = ma = ma' = f'$

Newton's 2^{nd} "LAW" is INVARIANT wrt G.T. ✓

EVERYONE

KNEW

THIS

BY 1890 .. 2 BROAD, SUCCESSFUL THEORIES:

1. MECHANICS .. NEWTON'S "LAWS"

GRAVITATIONAL "LAW"

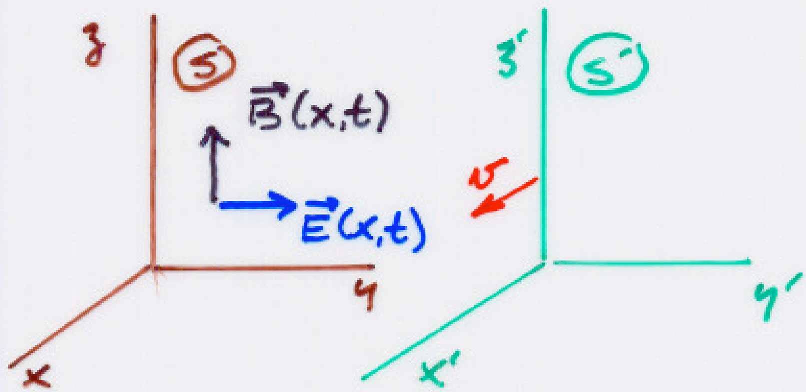
MECHANICAL & HEAT ENERGIES

2. ELECTRICITY & MAGNETISM

... MAXWELL'S EQUATIONS

there was an embarrassment of sorts...

an EM wave in (S)



what's it's form in (S')?

remember... Newton works

for G.T.

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

$\Rightarrow \quad F' = F$

THE DOMAIN OF H.A. Lorentz

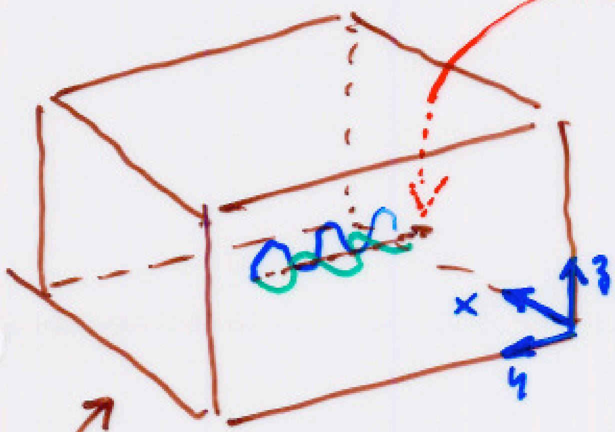
"the King of Electromagnetics"

HE HAD A THEORY...

REMEMBER THE "WHAT'S WAVING" QUESTION?

WHY... THE ETHER, THAT'S WHAT

EM waves in the fabric of the ether



m

THE universe — full of ether — sits still...

a fixed coordinate system

EVERYTHING in the universe with mass

$F = ma$ ✓

Lorentz was an atomist... MAXWELL WAS NOT

he worked out that charged "ions" or "electrons" existed

acceleration ⟶ EM waves in ether

↓

applied forces to other ions/electrons

THAT'S THE LORENTZ FORCE -- the connection between his DUAL WORLDS --

ions/electrons feel both $\vec{F} = m\frac{d^2\vec{x}}{dt^2}$

and $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$

OKAY.

G.T. $\Rightarrow$ $F = F'$

HOW ABOUT EM?

$$F = qE - quB \quad = \quad F' = qE' - qu'B'$$

KNOW: $u' = u - v$

$$qE' - qu'B' = qE' - quB' + qvB'$$

$$E' - u'B' = \underline{E' + vB'} - \underline{uB'}$$

$$E' - u'B' = \underline{E} - \underline{uB}$$

Assign: $E = E' + vB'$ & $B = B'$

$\Rightarrow$ BY G.T.

WHAT ABOUT M.E. ?

look at $\vec{\nabla} \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$

where:

$$\vec{E} = \vec{E}(x,t)\,\hat{j}$$
$$\vec{B} = \bar{B}(x,t)\,\hat{k}$$

so:

$$\vec{\nabla} \times \vec{B} = -\frac{\partial B}{\partial x}\,\hat{j}$$

and:

$$\frac{\partial \vec{E}}{\partial t} = \frac{\partial E}{\partial t}\,\hat{j}$$

so, this M.E. is

$$\frac{\partial B}{\partial x} = -\frac{1}{c^2}\frac{\partial E}{\partial t}$$

TRANSFORM TO OTHER FRAME AS BEFORE...

Show this, D1

$$\frac{\partial B'}{\partial x'} = -\frac{1}{c^2}\frac{\partial E'}{\partial t'} + \frac{1}{c^2}\left[v\frac{\partial E'}{\partial x'} - v\frac{\partial B'}{\partial t'} + v^2\frac{\partial B'}{\partial x'} \right]$$

WHICH DOES NOT HAVE THE FORM OF

$$\frac{\partial B'}{\partial x'} = -\frac{1}{c^2}\frac{\partial E'}{\partial t'}$$

MAXWELL'S EQUATIONS ARE NOT

INVARIANT

WITH RESPECT TO GALILEAN TRANSFORMATIONS

!

NEWTON'S ... <u>are</u>.

THIS IS WHERE SPECIAL RELATIVITY REALLY

BEGINS...

problems with electromagnetism

&

INVARIANCE

Something very strange here about Maxwell's Eqs.

Leading Lorentz to ask -- what does it take to make Maxwell's equations INVARIANT?

HE FOUND (for relative, inertial frames w/ velocity along x):

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$$\gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}}$$

CALLED -- THE LORENTZ TRANSFORMATIONS 1899

FOR LORENTZ -- THE ETHER — still exists

EM WAVES — in ether w/ ME holding in original form

SPEED OF LIGHT — c, in ether

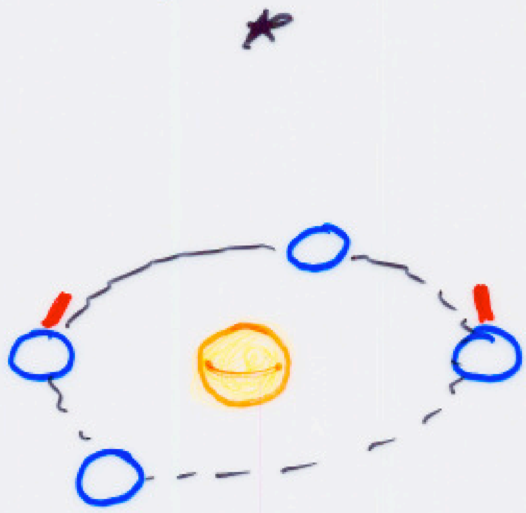
(also written by Voigt (1887) and Larmor (1898))

Experimental Situation, cira 1900...

1. Aberration of starlight

Bradley ~ 1728

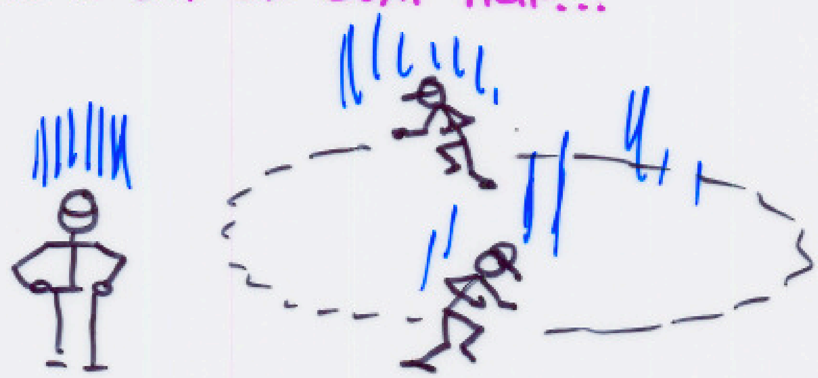
telescope in chimney — looks only @ zenith



trying to detect stellar parallax...

GOT A SURPRISE

FIGURE IT OUT ON BOAT TRIP...

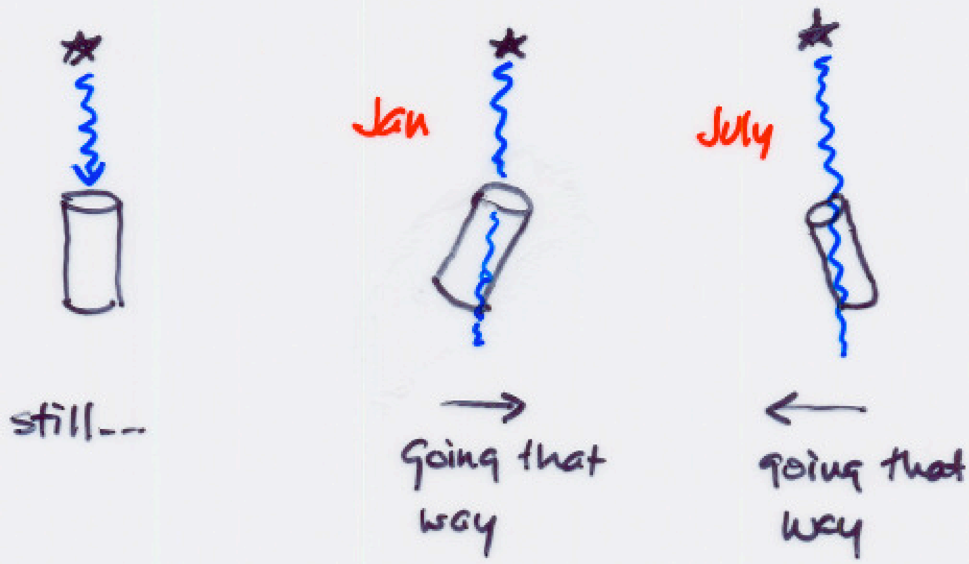


tilt your head forward to keep rain off your face...

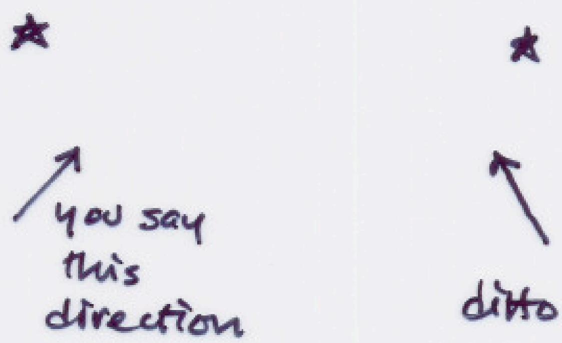
SUPPOSE: LIGHT ~ RAIN

TRACK ~ EARTH'S ORBIT

HAT ~ TELESCOPE



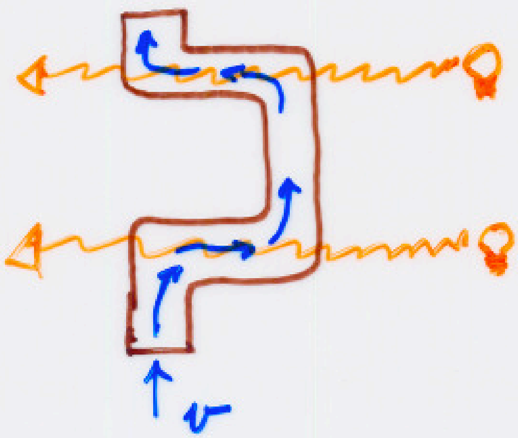
THE APPARENT DIRECTION OF THE STAR IS WHERE THE TELESCOPE POINTS



SUPPOSE YOU DRAGGED RAIN WITH YOU → no tilt.

⇒ STELLAR ABERATION SUGGESTS THAT EARTH MOVES THROUGH THE ETHER...

2. Fizeau's measurement of ether drag



Light through water:

← with current

← against current

"classically" -- $u = \frac{c}{n} \pm v$ if ether "dragged" by water

Fizeau found -- $u = \frac{c}{n} \pm f v$

not ϕ

not 1

JEESE. <u>some</u> dragging ?

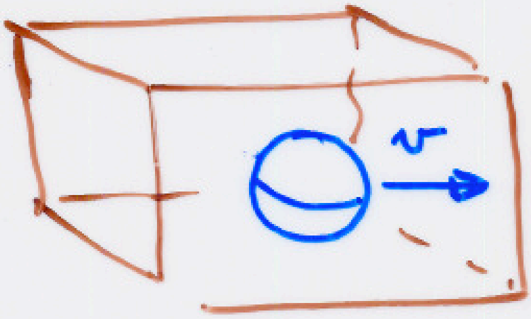
THE ETHER? --- complicated puppy to say the least

(remember — waving TRANSVERSELY to support light... more RIGID than steel, but everywhere inside all objects, outer space, etc.)

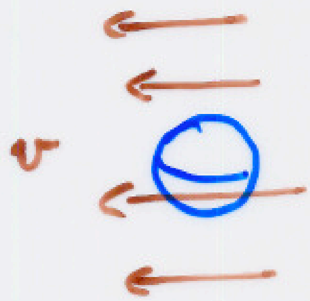
3. Michelson - Morley Experiment
(from 1887 – today...)

INCREDIBLY PRECISE INTERFEROMETRIC MEASUREMENTS

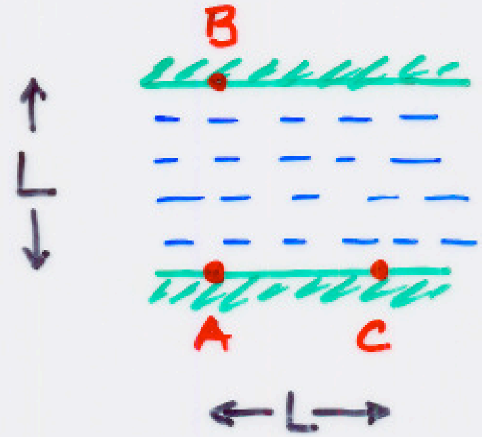
TO DETERMINE THE SPEED OF THE EARTH RELATIVE
TO THE ETHER...



ON EARTH... AN "ETHER WIND"



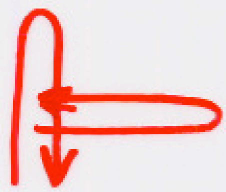
THE (CLEVER) IDEA ...



SWIMMERS EACH CAPABLE OF

u

RELATIVE TO WATER



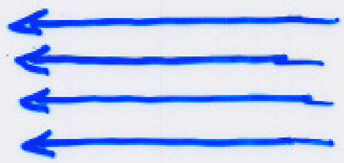
ABA : "$\perp$"

ACA : "$\parallel$"

· STILL WATER ·

Time ABA: $T_{\perp}(\text{still}) = \dfrac{2L}{u}$

Time ACA: $T_{\parallel}(\text{still}) = \dfrac{2L}{u}$



RIVER FLOWS WITH v

$T_{\parallel}$:

AC...

$$V^R_{Ground} = u - v$$

CA...

$$V^L_{Ground} = u + v$$

ROUND TRIP

$$T_{\parallel} = \frac{L}{V_G^R} + \frac{L}{V_G^L} = \frac{L}{u-v} + \frac{L}{u+v}$$

$$T_{\parallel} = \frac{2L}{u} \frac{1}{1 - v^2/u^2}$$

and

$$T_{\perp} = \frac{2L}{u} \frac{1}{\sqrt{1 - v^2/u^2}}$$

show DZ