Go to the $\nabla \times E$ equation... curl it again:

$$\nabla \times \nabla \times E = \nabla \times \left(-\frac{\partial E}{\partial x}\right)$$

Vector calculus identity:

$$\nabla \times \nabla \times E = \nabla (\nabla \cdot E) - \nabla^2 E$$

$$= 0$$ Gauss' "Law" in free space

So:

$$\nabla^2 E = \frac{2}{\epsilon_0} (\nabla \times B)$$

$$\mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$ from 4th M.E.

$$\nabla^2 E - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial x^2} = 0$$

(ditto for a $B$ curl also)

* A WAVE EQUATION *
general wave equation: \[ \frac{\partial^2 \phi}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2 \phi}{\partial t^2} = 0 \]

velocity of the wave

So, \[ \frac{1}{v^2} = \varepsilon_0 \mu_0 \]

\[ \mu_0 = 4\pi \times 10^{-7} \text{ N} \cdot \text{C}^{-2} \cdot \text{s}^2 \]
\[ \varepsilon_0 = 8.8542 \times 10^{-12} \text{ C}^2 \cdot \text{N}^{-1} \cdot \text{m}^{-2} \]

\[ v = \frac{1}{\sqrt{(4\pi \times 10^{-7}) \frac{\text{N} \cdot \text{s}^2}{\text{C}^2} \left( 8.8542 \times 10^{-12} \right) \text{C}^2 \cdot \text{N}^{-1} \cdot \text{m}^{-2}}} \]

\[ v = 2.998 \times 10^8 \text{ m/s} = c ! \]

bit of a surprise for Maxwell... he knew the recent measured value for c
ALL WAVE EQUATIONS HAVE

\[ \cos k(x-ut) - \text{like solutions} \]

\[ \vec{E}(x,t) \]

\[ \vec{B}(x,t) \]

\[ \vec{v} \quad \text{direction of the wave} \]

the \( \frac{d\vec{E}}{dt} \) creates the \( \vec{B} \)

\( \vec{\xi} \)

the \( \frac{d\vec{B}}{dt} \) creates the \( \vec{E} \)

and so on, and so on...

-- as you know --

\[ \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \]

Painting's Vector

\[ |\vec{S}| = \frac{\text{energy}}{(\text{area})(\text{time})} \]

\( \vec{S} \) = momentum

\[ \rho = \frac{\text{energy density}}{c} \]
Intensity = \| \langle S \rangle \|

I = \frac{1}{\mu_0} B_{\text{rms}} E_{\text{rms}}

from ME... \|B\| = \frac{1|E|}{c}

\[ I = \frac{E_{\text{rms}}^2}{\mu_0 c} = \epsilon_0 \frac{E_{\text{rms}}^2}{\mu_0 c} \]

\[ \sqrt{2} \]

\text{OKAY. GREAT. WAVES... what about waves?}

\text{THE ETHER}

\text{a very odd substance}

\text{move in a bit.}
It's ALL ABOUT INVARIANCE

→ "physics," as reflected in equations, shouldn't be different for different observers... the "form" of the equations should not be different

(Galileo... Einstein)

CONSIDER 2 SPECIAL FRAMES OF REFERENCE

inertial rest frames

⇒ constant, relative velocities

f applied in S
EVERYONE KNEW THAT THE CONNECTION BETWEEN $\mathbb{S}$ AND $\mathbb{S}'$ WAS

$$x = x' + vt$$
$$y = y'$$
$$t = t'$$

"Galilean Transformations"
IN (5) Newton's 2nd "LAW" describes the motion of the block...

\[ f = ma \]

FOR (5'), IT SHOULD BE THE SAME

\[ f' = m'a' \quad \text{... check...} \]

\[ (\text{presume } m = m') \]

\[ x = x' + vt' \]

\[ \frac{dx}{dt'} = \frac{dx'}{dt''} \frac{dt''}{dt'} + v \frac{dx'}{dt'} \]

\[ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \]

\[ u \quad \downarrow \quad 1 \quad 1 \]

\[ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \]

\[ u' \quad v \quad u \]

\[ u = u' + v \]

\[ \frac{du}{dt} = \frac{du'}{dt'} \frac{dt'}{dt} + \frac{dv}{dt} \]

\[ \downarrow \quad \downarrow \quad \downarrow \]

\[ 1 \quad 1 \quad 0 \]

\[ a = a' \]

\[ \text{so,} \quad f = ma = ma' = f' \]

Newton's 2nd "LAW" is INVariant wrt G.T. \(\checkmark\)
Everyone
Knew
This
By 1890... 2 broad, successful theories:

1. Mechanics: Newton's "Laws"
   - Gravitational "Law"
   - Mechanical & Heat Energies

2. Electricity & Magnetism
   - Maxwell's Equations

there was an embarrassment of sorts...
The domain of H.A. Lorentz

"the king of electromagnetics"

He had a theory...
REMEMBER THE "WHAT'S WAVING" QUESTION?

WHY... THE ETHER, THAT'S WHAT

EM waves in the fabric of the ether

THE universe — full of ether — sits still...

a fixed coordinate system

EVERYTHING in the universe with mass
\[ F = ma \]

Lorentz was an atomist... MAXWELL WAS NOT

he worked out that "ions" or "electrons" existed

acceleraton \rightarrow EM waves in ether

applied forces to other ions/electrons
THAT'S THE LORENTZ FORCE --- the connection between his DUAL WORLDS.
ions/electrons feel both  
\[ \mathbf{F} = m \mathbf{a} \]
\[ \text{and} \quad \mathbf{F} = q \mathbf{E} + q \mathbf{u} \times \mathbf{B} \]

OKAY.

G.T. \(\Rightarrow\) \( \mathbf{F} = \mathbf{F}' \)

How about \( \text{EM?} \)

\[ \mathbf{F} = q \mathbf{E} - q \mathbf{u} \mathbf{B} = \mathbf{F}' = q \mathbf{E}' - q \mathbf{u}' \mathbf{B}' \]

\( \text{KNOW:} \quad \mathbf{u}' = \mathbf{u} - \mathbf{v} \)

\[ q \mathbf{E}' - q \mathbf{u}' \mathbf{B}' = q \mathbf{E}' - q \mathbf{u} \mathbf{B}' + q \mathbf{u} \mathbf{B}' \]

\[ \mathbf{E}' - \mathbf{u}' \mathbf{B}' = \mathbf{E}' + \mathbf{v} \mathbf{B}' - \mathbf{u} \mathbf{B}' \]

\[ \mathbf{E}' - \mathbf{u}' \mathbf{B}' = \mathbf{E} - \mathbf{u} \mathbf{B} \]

Assign: \( \mathbf{E} = \mathbf{E}' + \mathbf{v} \mathbf{B}' \quad \frac{1}{c} \quad \mathbf{B} = \mathbf{B}' \)

\( \Rightarrow \) BY G.T.
WHAT ABOUT M.E.?

look at \[ \bar{D} \times \bar{B} = \frac{1}{c^2} \frac{\partial \bar{E}}{\partial x} \]

where:
\[ \bar{E} = \bar{E}(x,t) \hat{j} \]
\[ \bar{B} = \bar{B}(x,t) \hat{k} \]

so:
\[ \bar{D} \times \bar{B} = -\frac{\partial \bar{B}}{\partial x} \hat{j} \]

and:
\[ \frac{\partial \bar{E}}{\partial x} = \frac{\bar{E}}{\partial x} \hat{j} \]

so, this M.E. is
\[ \frac{\partial \bar{B}}{\partial x} = -\frac{1}{c^2} \frac{\partial \bar{E}}{\partial x} \]

TRANSFORM TO OTHER FRAME AS BEFORE...

Show this, D1

\[ \frac{\partial \bar{B}'}{\partial x'} = -\frac{1}{c^2} \frac{\partial \bar{E}'}{\partial x'} + \frac{1}{c^2} \left[ v \frac{\partial \bar{E}'}{\partial x'} - v \frac{\partial \bar{B}'}{\partial x'} + v^2 \frac{\partial \bar{B}'}{\partial x'} \right] \]

which does not have the form of
\[ \frac{\partial \bar{B}'}{\partial x'} = -\frac{1}{c^2} \frac{\partial \bar{E}'}{\partial x'} \]
MAXWELL'S EQUATIONS ARE NOT INVAARIANT WITH RESPECT TO GALILEAN TRANSFORMATIONS!

NEWTON'S... ARE.

THIS IS WHERE SPECIAL RELATIVITY REALLY BEGINS...

PROBLEMS WITH ELECTROMAGNETISM & INVARIANCE
Something very strange here about Maxwell's Eqs.

Leading Lorentz to ask... what does it take to make Maxwell's equations INVARIANT?

He found (for relative, inertial frames w/ velocity along x):

\[ x' = \gamma (x - vt) \]
\[ y' = y \]
\[ z' = z \]
\[ t' = \gamma (t - \frac{v}{c^2}x) \]

**Called... THE LORENTZ TRANSFORMATIONS** 1899

For Lorentz... the ether - still exists

EM waves - in ether w/ ME holding in original form

Speed of light - c, in ether

(also written by Voigt and Larmor) 1887 1898
Experimental Situation, cira 1900...

1. Aberration of starlight
   Bradley ~ 1728
   
   ![Diagram of telescope in chimney looking at zenith](image)
   
   telescope in chimney - looks only @ zenith
   
   ![Diagram of stars and people](image)
   
   trying to detect stellar parallax...
   
   got a surprise
   
   figure it out on boat trip...
   
   ![Diagram of people standing on boat](image)
   
   tilt your head forward to keep rain off your face...
Suppose: Light ~ Rain
Track ~ Earth's Orbit
Hat ~ Telescope

\[ \text{Jan} \rightarrow \text{going that way} \]
\[ \text{July} \leftarrow \text{going that way} \]

**The apparent direction of the star is where the telescope points**

Suppose you dragged rain with you → no tilt.

⇒ Stellar aberration suggests that Earth moves through the ether...
2. Fizeau's measurement of ether drag

\[
\text{Light through water:} \quad \leftarrow \text{with current} \quad \leftarrow \text{against current}
\]

\[
\text{"classically"} \quad u = \frac{c}{n} \pm v
\]

\[
\text{if ether "dragged" by water}
\]

\[
\text{Fizeau found} \quad u = \frac{c}{n} \pm f v
\]

\[
\text{not } \phi \quad \text{not 1}
\]

JEESE. some dragging?

THE ETHER? ... complicated puppy to say the least

(remember - waving TRANVERSELY to support light... more RIGID than steel, but everywhere inside all objects, outer space, etc.)
3. Michelson - Morley Experiment
(from 1887 - today...)

INCREDIBLY PRECISE INTERFEROMETRIC MEASUREMENTS TO DETERMINE THE SPEED OF THE EARTH RELATIVE TO THE ETHER...

ON EARTH... AN "ETHER WIND"
THE (CLEVER) IDEA...

Swimmers each capable of $u$ relative to water
Aborted

ABA: "⊥"
ACA: "∥"

Still Water

Time ABA: $T_L(\text{still}) = \frac{2L}{u}$

Time ACA: $T_M(\text{still}) = \frac{2L}{u}$
RIVER FLOWS WITH $v$

$T_{ii} = \frac{L}{v_x^R} + \frac{L}{v_x^L} = \frac{L}{u-v} + \frac{L}{u+v}$

$T_{ii} = \frac{2L}{u} \left( \frac{1}{1-v^2/u^2} \right)$

and

$T_{ii} = \frac{2L}{u} \sqrt{1-u^2/v^2}$

Show D2