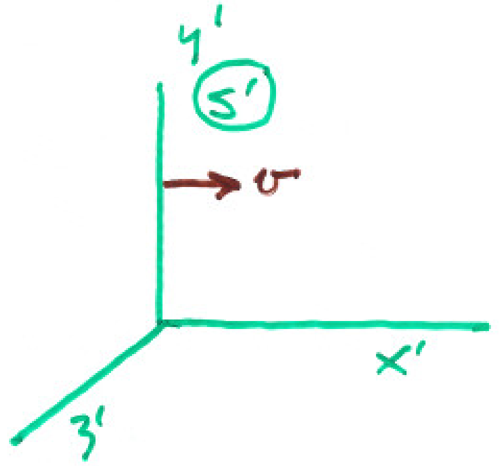
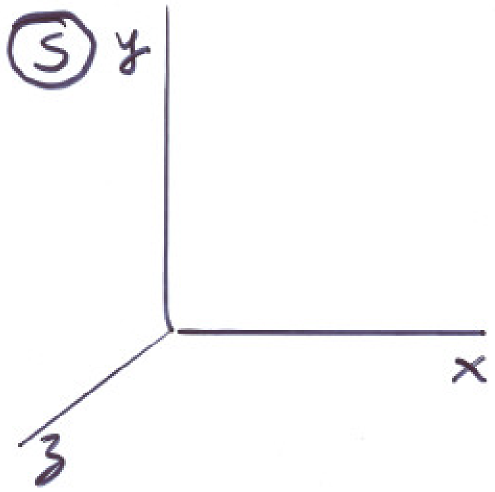
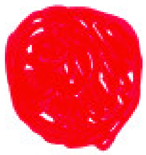

y'
S'
v
x'
z'



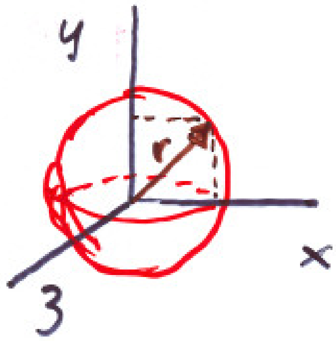
BECAUSE... THE SECOND POSTULATE SAYS
SPEED OF LIGHT IS c IN S
and c IN S'



FOR (S):

PICK POINT ON WAVEFRONT

WITH $z=0$



$r = ct$

AND

$r = \sqrt{x^2+y^2}$

$ct = \sqrt{x^2+y^2}$

THE LOCUS OF POINTS ON WAVEFRONT IN $z=0$ PLANE:

$$c^2t^2 = x^2 + y^2$$

$$x^2 + y^2 - (ct)^2 = 0$$

FOR (S')? SAME FORM...

$$x'^2 + y'^2 - (ct')^2 = 0$$

HOW TO GO FROM ONE TO THE OTHER?

TRY G.T. ... $x' = x - vt$

$$y' = y$$

$$z' = z$$

$$t' = t$$

$$x'^2 + y'^2 - (ct')^2 = 0$$

PLUG IN:

$$(x - vt)^2 + y^2 - (ct)^2 = 0$$

$$x^2 + \underbrace{v^2t^2 - 2xvt}_{} + y^2 - (ct)^2 = 0$$

spoils it from looking like

$$x^2 + y^2 - (ct)^2 = 0$$

CONCLUDE: P2 not compatible with G.T.

DEMAND THAT P1 & P2 WORK:

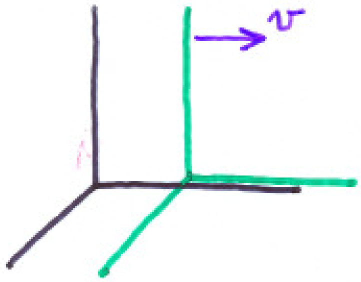
$$x'^2 + y'^2 - (ct')^2 = 0$$

$$\Big\downarrow \text{ some transformation}$$

$$x^2 + y^2 - (ct)^2 = 0$$

ASSUME A LINEAR TRANSFORMATION:

$$\left.\begin{aligned} x' &= \alpha x + \eta t \\ t' &= \epsilon x + \gamma t \end{aligned}\right\} \text{ determine } \alpha, \eta, \epsilon, \gamma$$



BOUNDARY CONDITIONS –

$x' = 0$ when $x = vt$

☆ Results: $\alpha = \gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$

$$\epsilon = -\frac{\gamma v}{c^2} = \frac{-v/c^2}{\sqrt{1 - v^2/c^2}}$$

D3 Show This

So, the transformations which enforce identical observations of the light are:

$$x' = \gamma(x - vt)$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right), \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

THE SAME TRANSFORMATIONS THAT LORENTZ FOUND WHICH LEAVE MAXWELL'S EQUATIONS INVARIANT!

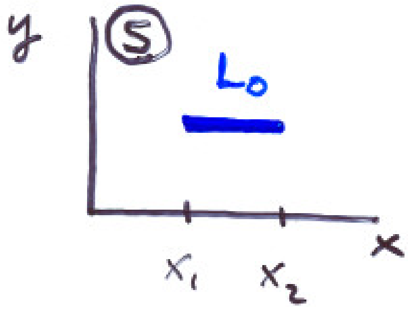
BUT, STARTING ONLY FROM HIS TWO POSTULATES

everyone else: motivated by DATA

THIS...IS THEORETICAL PHYSICS AT ITS (SPOOKIEST) BEST!

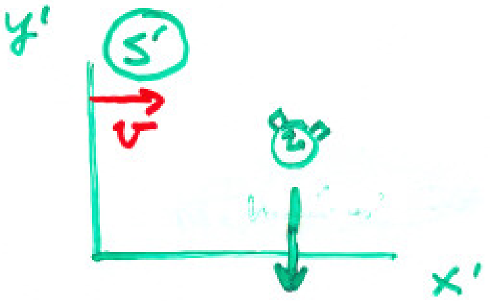
CONSEQUENCES ARE RADICAL...

Imagine a measuring stick at rest in (S)

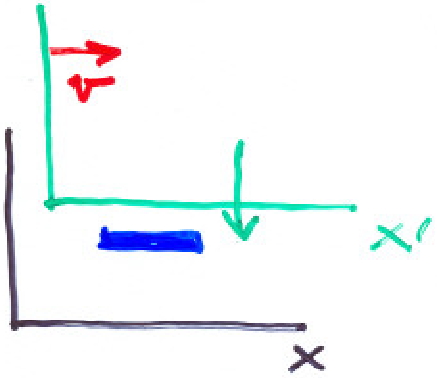


you may make a length measurement
at your leisure --- independent of time.

$L_0 = x_2 - x_1$ the "PROPER LENGTH"
(at rest in the measured Rest Frame)



FOR S' TO MEASURE THE LENGTH OF THE STICK – START & STOP A CLOCK...



MARKER AT REST IN (S'), get: $L' = v\Delta t'$

a proper time

THE INTERVAL ALSO MEASURED IN (S): $L_0 = v\Delta t$

$$\frac{L'}{L_0} = \frac{v\Delta t'}{v\Delta t} = \frac{\Delta t'}{\Delta t}$$

$$L' = L_0 \left(\frac{\Delta t'_0}{\Delta t} \right)$$

PRE-EINSTEIN? $\Delta t' = \Delta t \Rightarrow L' = L_0$

POST-EINSTEIN? $\Delta t = \gamma \Delta t' \Rightarrow L' = \frac{L_0}{\gamma}$

OBJECT AT REST OF LENGTH L_0 IS MEASURED TO BE SHORTER IN A CO-MOVING FRAME

• **LENGTH CONTRACTION** •

50. for co-moving, inertial rest frames, Einstein's postulates require:

1. clocks appear to run slower in frame moving relative to observer
2. lengths appear to be shorter in frames moving relative to observer
3. SPACE & TIME transformations between co-moving frames must be via Lorentz transformations
4. the notion of simultaneity is no longer an absolute concept.

Hermann Minkowski speech in 1908

"The views of space and time which I wish to lay before you have sprung from the soil of experimental physics and herein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.

WHAT IS INVARIANT??

- SPEED OF LIGHT
- THE "INTERVAL" *

* $$s^2 = \underbrace{x^2 + y^2 + z^2}_{\text{Euclidean}} \underbrace{- (ct)^2}_{\text{new!}}$$

an invariant "length" ... in SPACETIME

in a co-moving frame...

$$s^{2\prime} = x'^2 + y'^2 + z'^2 - (ct')^2$$

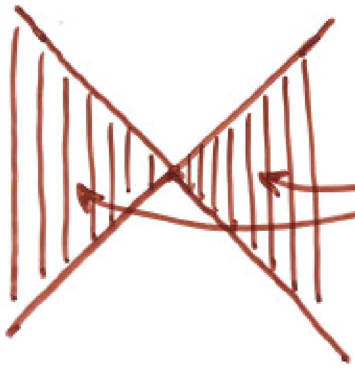
and they are INVARIANT with respect to Lorentz Transformations

$$s'^2 \xrightarrow{L.T.} s^2 = s'^2$$

(for light, $s^2 = 0$)

A NEW GEOMETRY... THE WORLD IS NOT EUCLIDEAN

... IT'S MINKOWSKIAN



ABSOLUTE "ELSEWHERE"

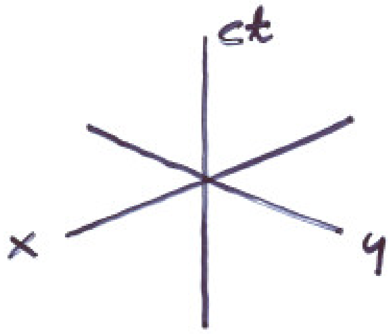
no causal influence possible

from or to... NOW

4 DIMENSIONS

- space & time are on equal footing

SPACETIME DIAGRAMS...

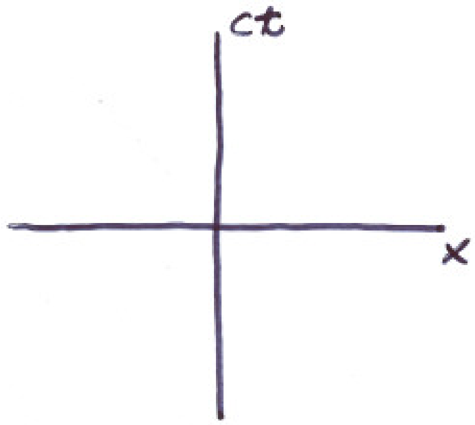


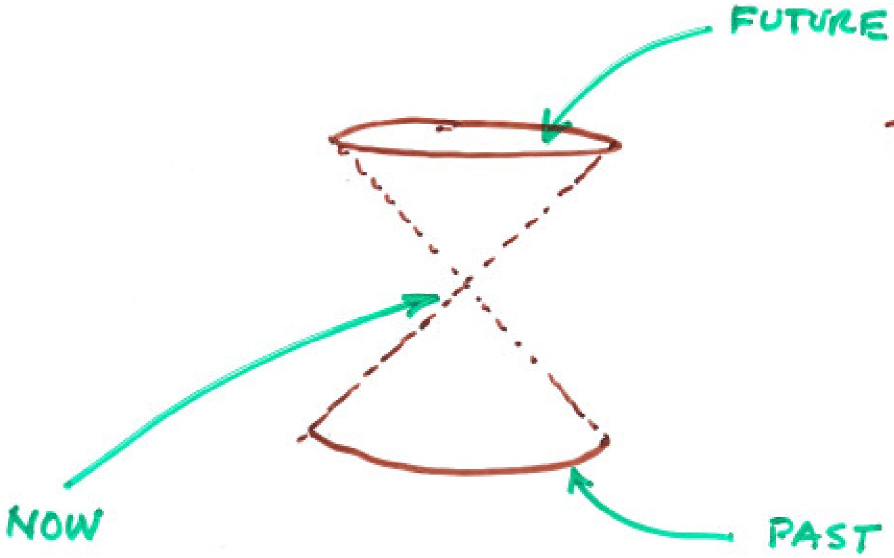
where's z ?

dunno... cannot picture it

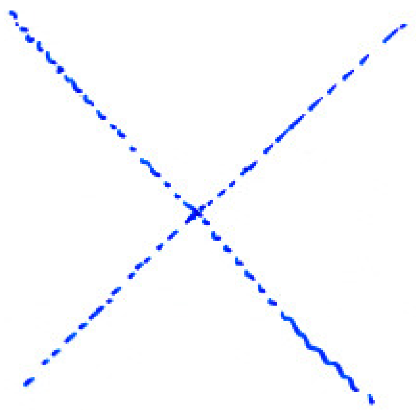
THE BEGINNING OF NON-VISUALIZABLE ABSTRACTION IN PHYSICS

made easier...





THE "LIGHT CONE"

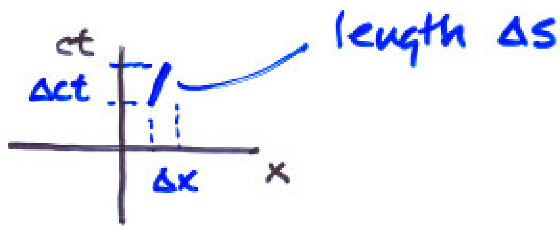


lines where $ct = x$

the trajectory for light

FINITE INTERVALS...

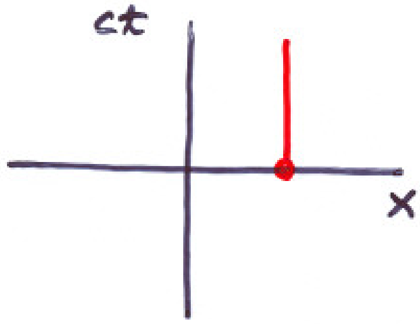
$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - (c\Delta t)^2$$



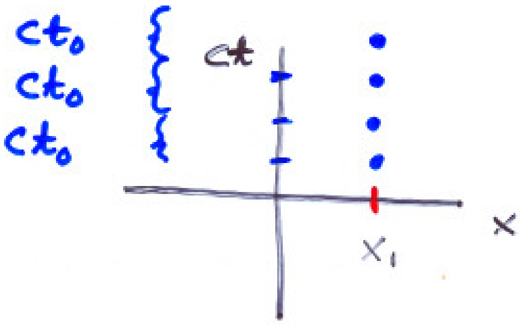
3 kinds: $\Delta s^2 = 0 \Rightarrow$ "light-like"

$\Delta s^2 > 0 \Rightarrow$ "space-like"

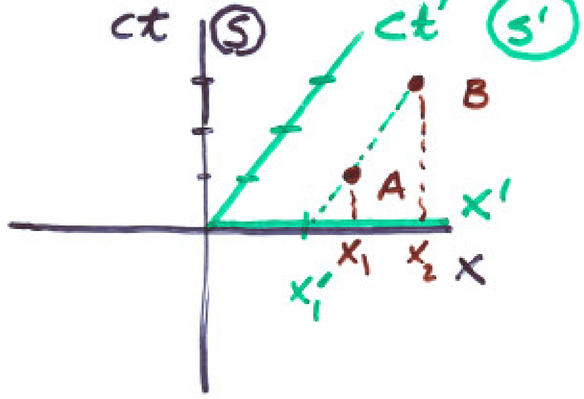
$\Delta s^2 < 0 \Rightarrow$ "time-like"



OBJECT SITTING STILL AT $x = x_1$



A CLOCK SITTING STILL A $x = x_1$ AND TICKING WITH Δt

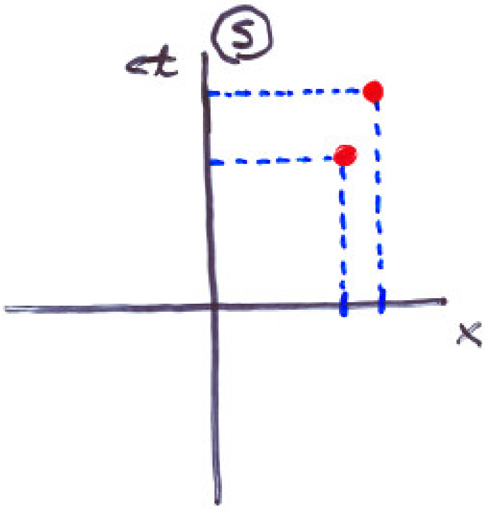


GALILEAN TRANSFORM.

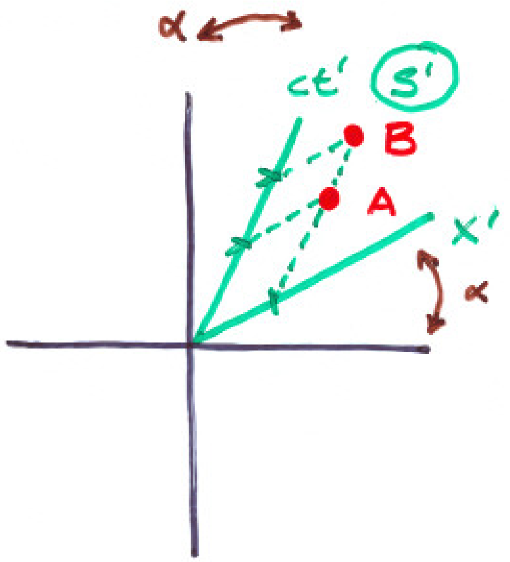
- A BEFORE B IN S & S'
- A & B AT SAME PLACE IN S'
- A & B AT DIFFERENT PLACES IN S
- SAME TIMES IN BOTH

LORENTZ TRANSFORMATIONS

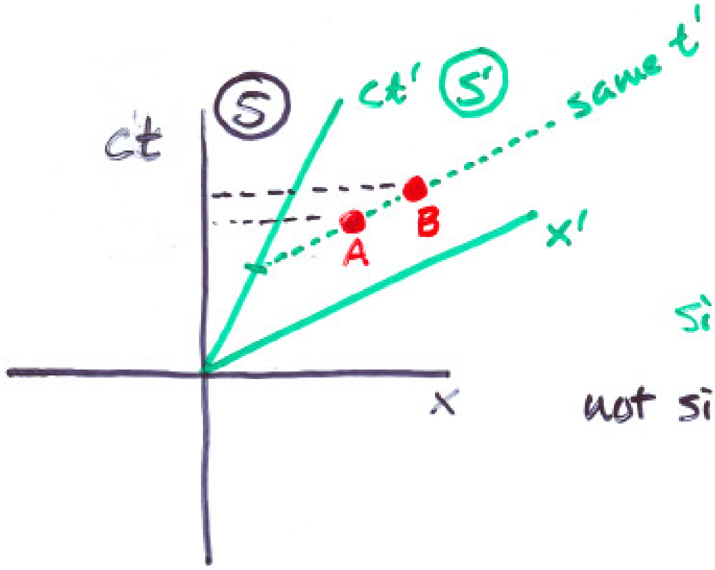
"tilt" both space axis and time axis



S
ct
x

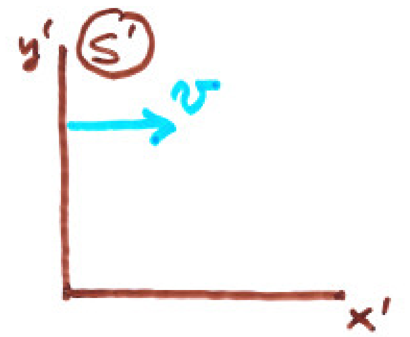


$\alpha = \tan^{-1}\left(\frac{v}{c}\right)$



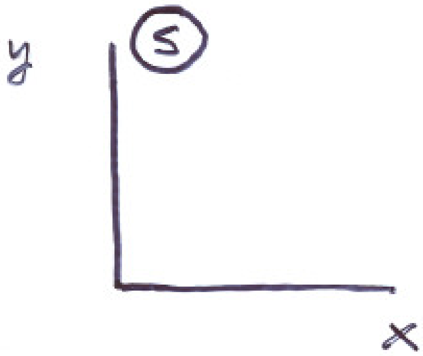
simultaneous in S'

not simultaneous in S



y'
S'
v
x'

Velocities ... have to get your head right!



v: velocity of S' wrt S

u: velocity of arrow in S

u': velocity of arrow in S'

Three velocities to keep track of...

VELOCITIES

Remember: GT $u' = u - v$

velocity of (S) relative to (S')

speed of something in (S')

speed of something in (S)

LORENTZ TRANSFORMATIONS

$$x' = \gamma(x - vt) \qquad \gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\frac{dx'}{dt} = \gamma\left(\frac{dx}{dt} - v\right) \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$= \gamma(u - v) \qquad \beta \equiv v/c$$

$$t' = \gamma\left(t - \frac{\beta}{c}x\right)$$

$$\frac{dt'}{dt} = \gamma\left(1 - \frac{\beta}{c}u\right)$$

$$\frac{dx'}{dt'} = u' = \frac{dx'}{dt}\frac{dt}{dt'} = \frac{\gamma(u-v)}{\gamma\left(1 - \frac{\beta}{c}u\right)}$$

$$u' = \frac{u - v}{1 - \frac{\beta}{c}u}$$

RELATIVISTIC TRANSFORMATION OF VELOCITIES

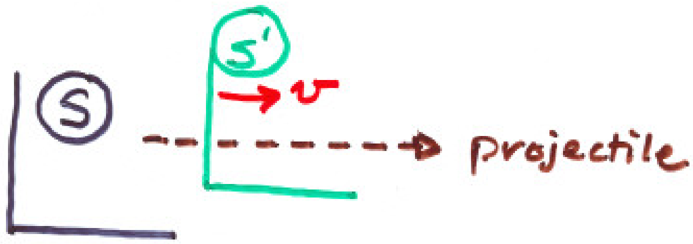
BY THE WAY... u's transverse to v will have to change as well BECAUSE they are

$\frac{d(\text{something})}{dt}$

$$u_x = \frac{u_x' + v}{1 + (\beta/c)u_x'}$$

$$u_y = \frac{u_y'}{\gamma[1 + \beta/c\, u_x']}$$

$$u_z = \frac{u_z'}{\gamma[1 + \beta/c\, u_x']}$$



u as determined by S

u' as determined by S'

found $u' = \dfrac{u - v}{1 - \frac{\beta}{c} u}$

or

$$u = \frac{u' + v}{1 + \frac{\beta}{c} u'}$$

EXTREMES:

1. β and u' very small $\longrightarrow$ wrt c

$$u \sim \frac{u' + v}{1 + 0} = u' + v$$ Galilean Trans.

2. $u' \sim c$ -- ie, a flashlight inside S'

$$u = \frac{c + v}{1 + \frac{\beta}{c} c} = \frac{c + v}{1 + \beta} = \frac{c + v}{1 + v/c} = c$$