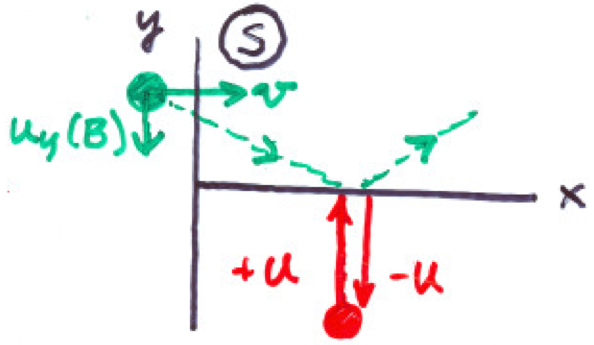
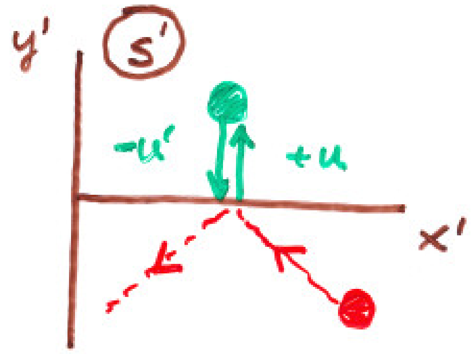
MASS

very subtle points... standard derivation:

identical balls

each throws their ball straight forward — collide & recoil elastically





They practice... so they can each push their balls at same speeds

$$u = u'$$

KEEP TRACK OF VELOCITIES!

v is frame speed of S' wrt S
(and x-component of B in S)
etc

$$\beta = v/c \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

in S calculate from S' quantities:

$$u_y(B) = \frac{u'_y(B)}{\gamma\left(1 + v\,u'_x(B)/c^2\right)}$$

$$u'_x(B) = 0$$
$$u'_y(B) = u'$$

$$u_y(B) = \frac{u'}{\gamma}$$

INSIST ON MOMENTUM CONSERVATION IN EACH FRAME.

$$\underbrace{y \text{ component before}}_{\text{all objects in } S} = \underbrace{y \text{ component after}}_{\text{all objects in } S}$$

$$m_A u - m_B u_y(B) = -m_A u + m_B u_y(B)$$
$$m_A u - m_B \frac{u'}{\gamma} = -m_A u + m_B \frac{u'}{\gamma}$$

$u = u' \Rightarrow \quad m_B = \gamma m_A$

NOT HOW THORNTON & REX DO THIS! subtle difference...

OKAY.

if u is very -- vanishingly small... take

$$m_A = m_0$$

mass at rest in frame

"REST MASS" → INVARIANT...

THEN FOR ANY TWO FRAMES YOU CAN INTERPRET

$$m(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

m_0 is an intrinsic property

which is called the Relativistic Mass. Not universally acknowledged as a useful concept.

ahem... continuing...

$$\vec{p} \equiv m\vec{v}$$

$$\vec{p} = m_0 \gamma \vec{v}$$

AN EVEN MORE SUBTLE POINT... FORCE

continuing to imagine $m(v)$...

$$\vec{F} = \frac{d}{dt}\vec{p} \qquad \text{still}$$

$$= m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dv}\frac{dv}{dt}$$

$$= m\vec{a} + \vec{v}\frac{dm}{dt}$$

$$\vec{F} = m_0\gamma\left(\frac{d\vec{v}}{dt} + \frac{v\,{}^{dv}/_{dt}}{c^2 - v^2}\vec{v}\right)$$

when $\vec{v}$ is not in the same direction as $\vec{F}$, this gets messy and unpleasant.

THE ONLY SATISFACTORY TREATMENT IS TO USE A TENSORIAL TREATMENT IN 4-DIMENSIONS

... "beyond the scope of this course..."

→ grad school

OKAY. THE TEE-SHIRT EQUATION...

CONTINUE TO RELY ON THE WORK-ENERGY THEOREM...

Work done = ΔKE

$$K = \int_{v=0}^{v=v} F\,ds$$

$$= \int_{v=0}^{v=v} \frac{d}{dt}(mv)\frac{ds}{dt}\,dt$$

$$= \int v \frac{d}{dt}(mv)\,dt$$

$$= \int v\,d(mv)$$

$$= \int v\,d\left[\frac{m_0 v}{(1-v^2/c^2)^{1/2}}\right]$$

$$= m_0 \int v \left[\frac{1}{(1-v^2/c^2)^{1/2}} + \frac{v^2/c^2}{(1-v^2/c^2)^{3/2}}\right] dv$$

$$= m_0 \int_0^v \frac{v\,dv}{(1-v^2/c^2)^{3/2}}$$

$$= m_0 c^2 \left.\frac{1}{(1-v^2/c^2)^{1/2}}\right|_0^v$$

$$K = m_0 c^2 \left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)$$

repeat: $$K = m_0 c^2 \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right)$$

expand.. $$\frac{1}{\sqrt{1 - v^2/c^2}} = 1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots$$

$$K = m_0 c^2 \left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots - 1 \right)$$

if $v \ll c$ -- higher powers ignored..

$$K(v \ll c) = m_0 c^2 \left(\frac{1}{2}\frac{v^2}{c^2} \right)$$
$$= \frac{1}{2} m_0 v^2$$

<u>nice</u>.

go back...

$$K = m_0 c^2 (\gamma - 1)$$
$$= m_0 \gamma c^2 - m_0 c^2$$
$$K = mc^2 - m_0 c^2$$

New idea about the energy of an object:

$mc^2 = K + m_0 c^2$ = energy of motion + "rest energy"

$\equiv$

TOTAL ENERGY $\quad E = mc^2$

SO: THESE TWO FUNDAMENTAL RELATIONSHIPS EMERGE:

$$\left.\begin{aligned} E &= \gamma m_o c^2 \\ p &= \gamma m_o v \end{aligned}\right\}$$ they connect... like the interval

$$E^2 = \gamma^2 m_o^2 c^4 = \frac{m_o^2 c^4}{1 - v^2/c^2}$$

$$= \frac{m_o^2 v^2 c^2}{1 - v^2/c^2} + m_o^2 c^4$$ using p...

$$E^2 = p^2c^2 + m_o^2 c^4$$

$$(m_o c^2)^2 = (pc)^2 - E^2$$ ← is an "interval" in energy/momentum space

$$(m_o c^2)^2 = \underbrace{(p_x c)^2 + (p_y c)^2 + (p_z c)^2}_{\text{like space coordinates}} - \underbrace{E^2}_{\text{like time coordinate}}$$

↑ "rest energy^2"

NOW. I'LL DROP THE m_o DESIGNATION (unless necessary to make a point) → the mass of an elementary particle is an invariant – a defining characteristic.

$$E^2 = p^2c^2 + m^2c^4$$

$$E = \sqrt{p^2c^2 + m^2c^4}$$ ← problematic later $\pm E$??!!

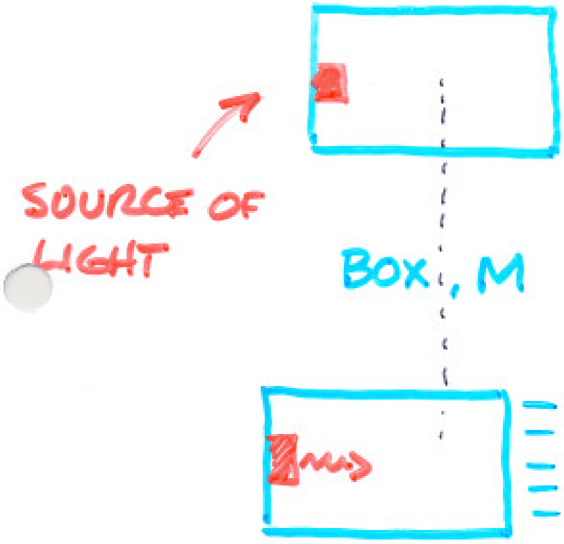
NON-CLASSICAL CONCEPT:

AN OBJECT OF NATURE WITHOUT MASS...

$$m = 0 \Rightarrow E = pc$$

which, indeed, became a useful concept a few years after 1905.

AN EXAMPLE OF EINSTEIN'S:



SOURCE EMITS LIGHT

OF ENERGY $E = pc$

(massless "photons")

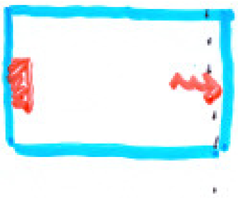
BOX RECOILS

MOMENTUM CONSERVED: (BOX, nonrelativistic)

$$|-Mv_B| = p = E/c \Rightarrow v_B = \frac{E}{Mc}$$

BOX $\vec{p}$ = photon $\vec{p}$

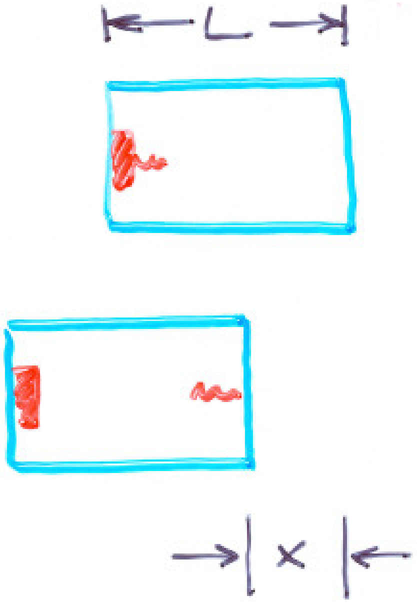
WHEN RADIATION HITS OTHER SIDE, BOX <u>STOPS</u>



com doesn't change $\rightarrow$ BOX, M moved LEFT, something had to compensate!

AS IF MASS IS TRANSPORTED TO THE RIGHT

BY THE LIGHT!



- Time for light to go

LEFT to RIGHT: $t = L/c$

(ignoring the fact that back moves towards light!)

So, distance block moves:

$$x = v_B t = v_B (L/c)$$

$$= \left(\frac{E}{Mc^2}\right) L$$

That the center of mass is unchanged:

M(displacement left) = a (displacement R)

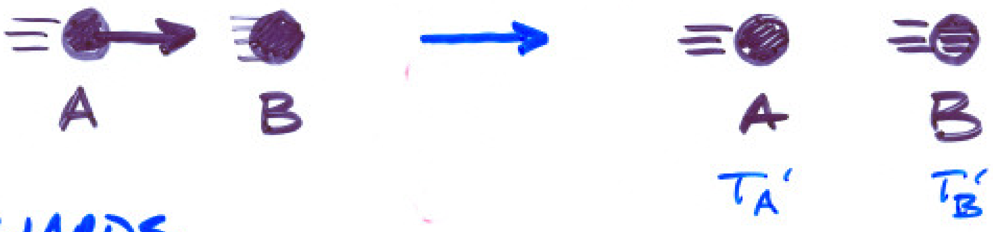
$$Mx = M\left(\frac{E}{Mc^2}\right)L = aL$$

so: $a = E/c^2$ AS IF LIGHT TRANSPORTS MASS: $m_L = E/c^2$

E BEHAVES LIKE AN INERTIA

ENERGY CONSERVATION... RELATIVITY STYLE!

kinetic E: T_A T_B



BILLIARDS...

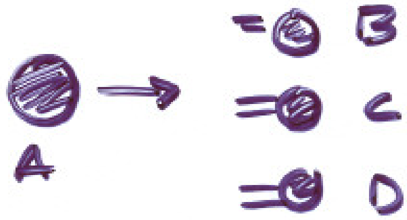
ENERGY BEFORE = ENERGY AFTER

$$T_A + m_A c^2 + T_B + m_B c^2 = T_A' + m_A c^2 + T_B' + m_B c^2$$

CANCEL

$\Rightarrow$ WOULD NOT DETECT IN SIMPLE COLLISIONS...

"DECAYS"



$$m_A c^2 = m_B c^2 + m_C c^2 + m_D c^2 + \underbrace{T_B + T_C + T_D}_{\text{HOW MUCH?}}$$

depends on

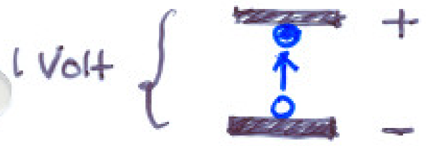
MASS DIFFERENCES

$(m_A - m_B - m_C - m_D)c^2$ energy

UNITS ARE A PAIN:

$m(\text{proton}) \cong 10^{-27}\,\text{kg}$ way too many zeros!

USE "electron volts" FOR ATOMIC, NUCLEAR, HIGH ENERGY, ASTROPHYSICS CALCULATIONS!



PROTON OF CHARGE

$+e = +1.602 \times 10^{-19}\,C$

ACCELERATED OVER 1 VOLT:

WORK DONE $= qV$

$= (1.602 \times 10^{-19}\,C)(1\,V) = 1.602 \times 10^{-19}\,J$

$\equiv 1\,eV$ one electron-volt

MASSES.. "rest masses":

eV/c^2

ENERGIES.. "rest energy":

eV

electron: $m_e = 9.1 \times 10^{-31}\,kg$

$E_0 = m_e c^2 = (9.1 \times 10^{-31}\,kg)(3.0 \times 10^8\,m/s)^2 \left(\frac{1\,eV}{1.6 \times 10^{-19}\,J}\right)$

$E_0 = 511{,}900\,eV = 0.51\,MeV$

"$m_e = 0.51\,MeV/c^2$"