CALCULATIONS:

Conserve momentum.

Conserve relativistic energy.

**Problem**

Charged "pion" decays into 2 particles most of the time...

\[ \pi^\pm \rightarrow \mu^\pm + \nu_\mu \]

Pion \rightarrow Muon + neutrino

The facts:

\[ m_\pi = 139.57 \text{ MeV/c}^2 \Rightarrow E_0(\pi) = 139.57 \text{MeV} \]

\[ m_\mu = 105.45 \text{ MeV/c}^2 \Rightarrow E_0(\mu) = 105.45 \text{MeV} \]

\[ T_\mu = 2.2 \times 10^{-6} \text{ s} \Rightarrow m_\nu = 0 \]

\[ P_\pi^0 = 0 \]

⇒ π at rest.

(a) What is the momentum of the muon?

(b) On average, how far does the muon travel before it decays?
a) conserve momentum

\[ \vec{P}_\pi = \vec{P}_\mu + \vec{P}_\nu \]
\[ 0 = \underbrace{\vec{P}_\mu}_{\Theta \rightarrow \mu} \rightarrow \nu \]

not enough: conserve relativistic energy as well...

\[ E_\pi = E_\mu + E_\nu \]
\[ m_\pi c^2 + T_\pi = m_\mu c^2 + T_\mu + m_\nu c^2 + T_\nu \]
\[ \uparrow \quad \sqrt{E_\mu^2 \text{ total}} \quad \uparrow \quad \sqrt{E_\nu^2 \text{ total}} \]
\[ = 0 \quad \Rightarrow \quad \text{total} \]

remember:

\[ E^2 = p^2 c^2 + m^2 c^4 \]
\[ E_\nu^2 = P_\nu c^2 \quad E_\mu^2 = P_\mu c^2 + m_\mu^2 c^4 \]

but: \[ |P_\nu| = |P_\mu| = p \quad \Rightarrow \]

\[ E_\nu = pc \quad E_\mu = \sqrt{p^2 c^2 + m_\mu^2 c^4} \]

\[ m_\pi c^2 = \sqrt{p^2 c^2 + m_\mu^2 c^4} + pc \]

\[ pc = 2.995 \text{ MeV} \quad \text{or} \quad p = 2.995 \text{ MeV/c} \]
b) \[ e \xrightarrow{\text{muon}} \mu \]

d on average

OKAY, THIS IS A STANDARD PROBLEM WITHIN A PROBLEM!

Protons, mostly:

Upper atmosphere

Collisions with \( N_2 \)

Protons, neutrons

Lots of particles — including \( \mu \)

\( \mu \)s thick \( \approx 20,000 \text{ m} \)

Mt Everest \( \approx 8800 \text{ m} \)

Muons will decay at rest according to

\[ N(t) = N(0) e^{-t/\tau} \]

\( \tau \) is the "lifetime"

Fraction remaining after time \( t \)

\( \rightarrow \) in rest frame of \( \mu \)

Non-relativistically — a \( \mu \) would travel, on average,

\[ d = v \tau \]
\[ v = 0.99c \quad \Rightarrow \quad \tau \approx 7 \]

\[ d = (0.99)(3 \times 10^8)(2.2 \times 10^{-6}) \]

\[ d \approx 650 \text{ m} \]

But we "see" them on the Earth \( \sim 1 \mu \text{cm/min} \)
ON EARTH, WE SEE $\mu$'S "CLOCK" DILATED

$\tau = \delta \tau_\mu \sim 7 \times$ muons "proper" lifetime

SO, IT TRAVELS, ON AVERAGE...

\[ d = v \delta \tau \]

\[ \approx (7)(650 \text{ m}) = 4550 \text{ m} \] (for 1 lifetime)

WHAT'S THE DEAL-- $\mu$ SAYS 1 LIFETIME--
deccay in upper atmosphere, never reaching earth!

MUON'S REST FRAME:

SEES: earth rushing toward it at $v = 0.99c$

with atmosphere Lorentz contracted

\[ d_\mu = d_e \frac{1}{\gamma} \text{ same factor} \]
The $\mu$ travels $d = \nu \tau$

\[
p = \gamma m_{\mu} \nu
\]

\[
d = \frac{p}{\gamma m_{\mu}}
\]

\[
= \frac{pc}{m_{\mu} c^2}
\]

\[
d = \frac{(pc)(ct)}{m_{\mu} c^2}
\]

\[
= \frac{29.95 \text{ MeV}}{105.45 \text{ MeV}} \left( \frac{3 \times 10^8 \text{ m/s}}{(2.2 \times 10^{-6} \text{ s})} \right)
\]

\[
d = (0.28)(660 \text{ m}) = 185 \text{ m}
\]

**Handy Rules of Thumb:**

\[
E = mc^2
\]

\[
\gamma = \frac{E}{mc^2}
\]

\[
E^2 = \frac{1}{1 - \beta^2} m^2 c^4
\]

\[
\Rightarrow \beta^2 = \frac{p^2 c^2}{E^2} \quad \text{or}
\]

\[
\beta = \frac{pc}{E}
\]
**BINDING ENERGY:**

\[ \vec{p}_1 = -\vec{p}_2 \]

\[ M c^2 = E_1 + E_2 = 2E \]

\[ M c^2 = 2 \gamma m c^2 \]

rest energy of the \( 2m \) system

so, \( M > 2m \).

\[ T(2m) = (M - 2m) c^2 \]

available kinetic energy

"old" notion of "conservation of mass" wrong.

TRUE FOR ALL SYSTEMS --

firecracker

excited atomic system

dissociated molecular system

\{ masses of products less than mass of original \}
back to "binding energy"…

For a composite system to stay together, \( M \neq m \)

\[ E(\text{system}) < Mc^2 + mc^2 \]

Previously… you said that \( E \) of bound system was negative.

\[ E(\text{system}) = Mc^2 + mc^2 - B \]

\[ \uparrow \text{Binding energy} \]
\[ = \text{energy you must supply to un-bind the system.} \]

Stay relativistic here… \( E(\text{system}) \Rightarrow M(\text{system})c^2 \)

\[ M_s = M + m - \frac{B}{c^2} \leq M + m \]

So:

The mass of a hydrogen atom is less than the mass of a proton & an electron, so it stays together.

"mass deficit"
Okay, it gets strange... and solves a problem:

Before

\[
\begin{array}{c}
\text{M} \\
\uparrow \\
\text{M} \\
\text{M}
\end{array} \leftrightarrow \text{M} \quad \text{M} \\
\text{M} \quad \text{M}
\]

After

\[
\begin{array}{c}
\text{M} \\
\text{M}
\end{array}
\]

Classically... you would have said:

\[2 \left( \frac{1}{2} M v^2 \right) = \text{PE(spring)}\]

Relativistically...

\[2M\beta^2 c^2 + mc^2 = M_5 c^2 = 2Mc^2 + mc^2\]

\[2Mc^2 (\gamma - 1) \neq 0 \quad \text{something missing}\]

\[2M\beta^2 c^2 + mc^2 = 2Mc^2 + mc^2 + B = M_5 c^2\]

\[2Mc^2 (\gamma - 1) = B\]

\[B/c^2 = 2M (\gamma - 1) \rightarrow M_5\]

Can interpret the "potential energy" of spring differently
\[ 2M \delta c^2 + \mu c^2 = 2Mc^2 + (\mu + \delta \mu)c^2 \]
\[ 8mc^2 = 2M(Y-1) \]

the spring gains mass = "potential energy"

Can take a relativistic point of view:

- there is energy in form of mass-energy
- energy associated with motion
- that's it.

Proton: \[ m = 10^{-27} \text{kg} \text{ or } 900 \text{ MeV}/c^2 \]

Made up of quarks... and

\[ 2m_u \leq m_d \leq 0.008 \text{ MeV}/c^2 \]

and a massless set of "gluons"

+ \( E(\text{gluons}) \) of motion

makes up the inertia of proton

E's last 1905 paper title:

"Does the inertia of a Body Depend on its Energy Content?"

Yup.
A LITTLE DOPPLER DO 'YA...

\[ \Delta t_0 = \frac{N}{f} \]

\[ \chi' = \frac{\text{total distance}}{N} = \frac{v \Delta t' + c \Delta t'}{N} \]

\[ = \frac{v \Delta t' + c \Delta t'}{\frac{c}{\Delta t_0}} \]

in \( s' \): \[ f' = \frac{c}{\chi'} \]
so,
\[ f' = \frac{\Delta k_0}{\Delta k} \cdot \frac{f}{1 + \nu/c} \]
\[ = \frac{1}{\gamma} \cdot \frac{f}{1 + \nu/c} \]
\[ f' = f \cdot \frac{\sqrt{1 - \nu^2/c^2}}{\sqrt{1 + \nu/c}} \]

RELATIVISTIC
DOPPLER EFFECT
→ separating \((v \rightarrow -v)\)

VERY DIFFERENT IN PRINCIPLE FROM ACOUSTIC
DOPPLER EFFECT

\[ f' = f \cdot \frac{v \pm v_s}{v \pm v_s} \]
\[ f' = f \cdot \frac{\text{"c"} \pm v_s}{\text{"c"} \pm v_s} \]

where \text{"c"} is the speed of sound in the medium

• in classical: source can go faster than \text{"c"}

• in classical: a distinction between source & receiver
  ... because they move relative to an "ether"
\[
f' = f \frac{\sqrt{1 - v/c}}{\sqrt{1 + v/c}} \Rightarrow f' > f \quad \text{or} \quad \lambda' < \lambda
\]

\textcircled{RED SHIFT}

stellar spectra (galactic, actually) \rightarrow

show shift toward RED

ALL GALAXIES ARE OBSERVED TO DO THIS RELATIVE TO EARTH

SUGGESTING THAT:

EARTH IS AT THE CENTER OF THE UNIVERSE.

right?

wrong.

The truth is even stranger... and more wonderful than that! More Einstein!
EXAMPLES...

PROBLEM 19. A rocket ship carrying passengers blasts off to go from NYC to LA, a distance of 5000 km. How fast must it go to have its length contracted by 1%?

Remember

\[ L = \frac{L_0}{\gamma} \]

\( L_0 \) proper length, at rest in \( S' \)

how do I remember? well “contraction” \( \rightarrow \) “smaller”

and \( \gamma > 1 \)

So, \( \frac{L}{L_0} = 0.99 = \frac{1}{\gamma} = \sqrt{1 - \frac{v^2}{c^2}} \)

so \( \gamma = 0.14 = \frac{v}{c} \), no

\( v = 0.14c \)
Problem 20. Astronomers discover a planet orbiting a star 20 light-years away. How fast must a rocket ship go if the trip is to take no more than 40 years for the astronauts? How long will that take on Earth?

Pressure \( u = \beta c \) for the entire trip.

In \( S' \): the rocket - the distance is Lorentz-contracted as the planet rushes toward it...

\[
d' = \frac{d}{\gamma} = 40 \text{ ly} \cdot \sqrt{1 - \beta^2}
\]

So in \( S' \):

\[
u = \frac{2d'}{40 \text{ ly}} = \frac{40 \text{ ly} \cdot \sqrt{1 - \beta^2}}{40 \text{ ly}} = \frac{40 \sqrt{1 - \beta^2}}{40}
\]

\[
u = c \sqrt{1 - \beta^2}
\]

\[
\beta = \sqrt{1 - \beta^2} \quad \Rightarrow \beta = \sqrt{1 - \beta^2} \approx 0.71c
\]

For time elapsed on Earth...

\[
t' = 40 \text{ y}
\]

- and time is dilated on earth

\[
t = \gamma t' = \frac{1}{\sqrt{1 - \beta^2}} \times 40 \text{ y} = 56.6 \text{ y}
\]
Problem 32. A proton and an antiproton are moving toward one another, each at 0.8c with respect to the collision point. How fast are they moving with respect to one another?

**Velocity-addition formula:**

\[ u' = \frac{u_x + v}{1 + \frac{u_x v}{c^2}} \]  

or:

\[ u' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} \]

Imagine sitting on proton, and watching antiproton come at you. The relative speed of the frames: 

\[ v = -0.8c \text{ and } u_x = 0.8c \]

\[ u_x' = \frac{0.8c - 0.8c}{1 - (0.8c)(-0.8c/c^2)} = 0.976c \]

**Vimpy p\bar{p} collider:**

\[ \gamma \text{ only } \sim 5 \]

At Fermilab, \( \gamma \sim 10^6 \)

\( v \) of proton and antiproton is \( \sim 260 \text{ mph slower than the speed of light.} \)

see homework #66