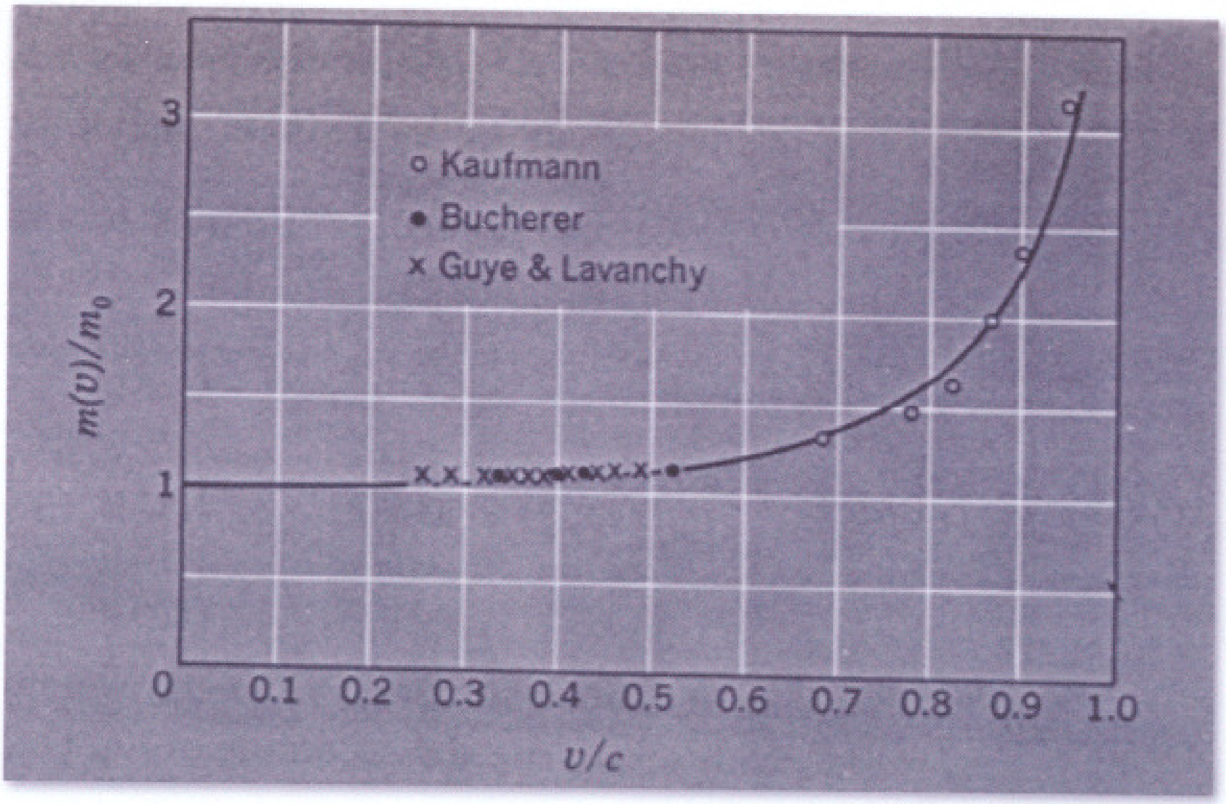

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CALCULATIONS:

conserve momentum.

conserve relativistic energy.

$\boxed{\text{PROBLEM}}$ charged "pion" decays into 2 particles most of the time...

$$\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}$$

pion → muon + neutrino

the facts: $m_{\pi} = 139.57 \text{ MeV}/c^2 \Rightarrow E_0(\pi) = 139.57 \text{ MeV}$

$m_{\mu} = 105.45 \text{ MeV}/c^2 \Rightarrow E_0(\mu) = 105.45 \text{ MeV}$

$\tau_{\mu} = 2.2 \times 10^{-6} \text{ s}$; $m_{\nu} = 0$ (! not any more !)

$\vec{p}_{\pi} = 0 \Rightarrow \pi$ at rest.

a) what is the momentum of the muon?

b) on average, how far does the muon travel before it decays?

a) conserve momentum

$$\vec{P}_\pi = \vec{P}_\mu + \vec{P}_\nu$$

$$0 =$$

$$\vec{P}_\mu = -\vec{P}_\nu \qquad\qquad \mu \longleftarrow \overset{\pi}{\bullet} \longrightarrow \nu$$

not enough: conserve relativistic energy as well...

$$E_\pi = E_\mu + E_\nu$$

$$m_\pi c^2 + \underset{=0}{\underset{\uparrow}{T_\pi}} = \underbrace{m_\mu c^2 + T_\mu}_{\sqrt{E_{\mu\,\text{total}}^2}} + \underbrace{\underset{=0}{\underset{\uparrow}{m_\nu c^2}} + T_\nu}_{\sqrt{E_{\nu\,\text{total}}^2}}$$

remember:

$$E^2 = p^2c^2 + m^2c^4$$

$$E_\nu^2 = P_\nu^2 c^2 \qquad\qquad E_\mu^2 = P_\mu^2 c^2 + m_\mu^2 c^4$$

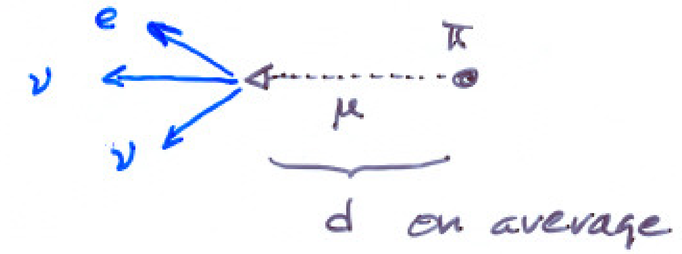
but: $|P_\nu| = |P_\mu| \equiv p$. so:

$$E_\nu = pc \qquad\qquad E_\mu = \sqrt{p^2c^2 + m_\mu^2 c^4}$$

$$m_\pi c^2 = \sqrt{p^2c^2 + m_\mu^2 c^4} + pc$$

$$pc = 29.95 \text{ MeV} \quad \text{or} \quad p = 29.95 \text{ MeV}/c$$

b)



OKAY. THIS IS A STANDARD PROBLEM WITHIN A PROBLEM!

muons will decay at rest according to

$$N(t) = N(0)\,\underbrace{e^{-t/\tau}}_{\text{fraction remaining after time } t}$$

← the "lifetime"

→ in rest frame of μ

NONRELATIVISTICALLY ... a μ would travel, on average,

$d = v\tau$ $\qquad\qquad v \simeq 0.99c \Rightarrow \gamma \doteq 7$

$d = (0.99)(3\times10^{8})(2.2\times10^{-6})$

$d \sim 650\,\text{m}$

BUT WE "SEE" THEM ON THE EARTH ~ $1\mu/\text{cm}/\text{min}$

RELATIVISTICALLY...

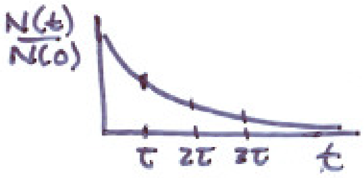
ON EARTH, WE SEE μ's "CLOCK" DILATED

$$\tau = \gamma\tau_\mu \sim 7\times \text{ muon's "proper" lifetime}$$

SO, IT TRAVELS, ON AVERAGE...

$$d = v\gamma\tau$$
$$\hat{=} (7)(650\,m) \simeq 4600\,m$$

(for 1 lifetime)



$1/e$ for each $t = \tau \sim 1/3$

WHAT'S THE DEAL... μ SAYS 1 LIFETIME...

decays in upper atmosphere, never reaching earth!

MUON'S REST FRAME:

SEES: earth rushing toward it at $v \simeq 0.99c$

with atmosphere Lorentz contracted

$$d_\mu = d_e \frac{1}{\gamma}$$ ← same factor

back to b)

The μ travels $d = v\gamma\tau$

relativistic momentum $p = \gamma m_\mu v$

$$d = \frac{p}{\gamma m_\mu}\gamma\tau$$

$$= \frac{p\tau}{m_\mu c^2}c^2$$

$$d = \frac{(pc)(c\tau)}{m_\mu c^2}$$

$$= \frac{29.95\ \text{MeV}}{105.45\ \text{MeV}}(3\times10^8\ \text{m/s})(2.2\times10^{-6}\ \text{s})$$

$$d = (0.28)(660\ \text{m}) = 185\ \text{m}$$

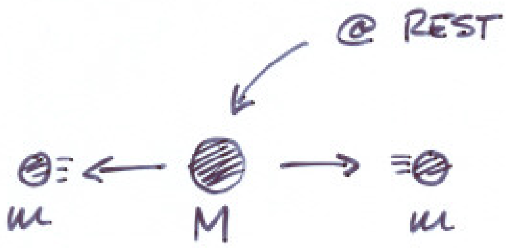
HANDY RULES OF THUMB:

$$E = m\gamma c^2 \qquad E^2 = \frac{1}{(1-\beta^2)}m^2c^4$$

$$\gamma = \frac{E}{mc^2} \qquad \Rightarrow \beta^2 = \frac{p^2c^2}{E^2} \text{ or}$$

$$\beta = \frac{pc}{E}$$

BINDING ENERGY:



ALREADY POINTED OUT: $\vec{p}_1 = -\vec{p}_2$

$$Mc^2 = E_1 + E_2 = 2E$$

$$Mc^2 = 2\gamma mc^2$$

rest energy of the 2m system

so, $M > 2m$ ...

$$\text{\&} \quad T(2m) = (M - 2m)c^2$$

available kinetic energy

"old" notion of "conservation of mass" wrong.

TRUE FOR ALL SYSTEMS --

firecracker

excited atomic system

dissociated molecular system

masses of products less than mass of original back to "binding energy"...

FOR A COMPOSITE SYSTEM TO STAY TOGETHER, M & m

$$E(\text{system}) < Mc^2 + mc^2$$

PREVIOUSLY... YOU SAID THAT E OF BOUND SYSTEM WAS NEGATIVE.

$$E(\text{system}) = Mc^2 + mc^2 - B$$

Binding energy = energy you must supply to un-bind the system.

STAY RELATIVISTIC HERE... $E(\text{system}) \Rightarrow M(\text{system})c^2$

$$M_s = M + m - B/c^2 \quad < M + m$$

SO:

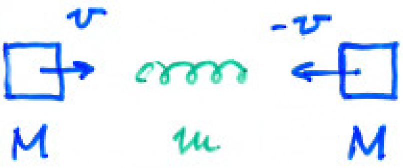
THE MASS OF A HYDROGEN ATOM IS LESS THAN THE MASS OF A PROTON & AN ELECTRON

SO IT STAYS TOGETHER

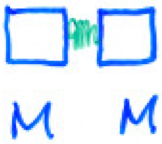
"mass deficit"

OKAY. IT GETS STRANGE... AND SOLVES A PROBLEM:

BEFORE



AFTER



CLASSICALLY... YOU WOULD HAVE SAID:

$$2\left(\tfrac{1}{2}MV^2\right) = PE(\text{spring})$$

RELATIVISTICALLY...

$$2M\gamma c^2 + mc^2 = M_s c^2 = 2Mc^2 + mc^2$$

$$2Mc^2(\gamma - 1) \neq 0 \qquad \text{something missing}$$

$$2M\gamma c^2 + mc^2 = 2Mc^2 + mc^2 + B = M_s c^2$$

$$2Mc^2(\gamma-1) = B$$

$$B/c^2 = 2M(\gamma - 1) \longrightarrow m_B$$

can interpret the "potential energy" of spring differently

$$2M\gamma c^2 + mc^2 = 2Mc^2 + (m + \delta m)c^2$$

$$\delta m c^2 = 2M(\gamma - 1)$$

the spring gains mass = "potential energy"

Can take a relativistic point of view:

- there is energy in form of mass-energy
- energy associated with motion
- that's it.

PROTON: $m \sim 10^{-27}$ kg or ~ 900 MeV/c^2

made up of quarks... uud

$2m_u \simeq m_d \simeq 0.008$ MeV/c^2

and a massless set of "gluons"

+ E(gluons) of motion

makes up the inertia of proton

E's last 1905 paper title:

"Does the Inertia of a Body Depend on It's Energy Content"

yup.

A LITTLE DOPPLER DO 'YA...

source of light → emits N waves @ frequency f → TAKES $\boxed{\Delta t_0 = N/f}$

FROM S':

Δt_0 is $\Delta t'$ in S'

$$\underbrace{\underbrace{v\Delta t'}_{} \quad \underbrace{c\Delta t'}_{}}_{\text{for N waves}}$$

$$\lambda' = \frac{\text{total distance}}{N} = \frac{v\Delta t' + c\Delta t'}{N}$$

$$= \frac{v\Delta t' + c\Delta t'}{f\,\Delta t_0}$$

from S'

in S': $f' = c/\lambda'$

so,
$$f' = \frac{\Delta t_0}{\Delta t'} \frac{f}{1 + v/c}$$
$$= \frac{1}{\gamma} \frac{f}{1 + v/c}$$
$$f' = f \frac{\sqrt{1 - v^2/c^2}}{1 + v/c}$$
$$f' = f \frac{\sqrt{1 - v/c}}{\sqrt{1 + v/c}}$$

RELATIVISTIC DOPPLER EFFECT

→ separating $(v \to -v)$

VERY DIFFERENT IN PRINCIPLE FROM ACCOUSTIC DOPPLER EFFECT

$$f' = f \frac{v \pm v_0}{v \pm v_s}$$
$$f' = f \frac{\text{"}c\text{"} \pm v_{s'}}{\text{"}c\text{"} \pm v_s}$$

where "c" is the speed of sound in the medium

- in classical: source can go faster than "c"
- in classical: a distinction between source & receiver ... because they move relative to an "ether"

$$f' = f \frac{\sqrt{1 - v/c}}{\sqrt{1 + v/c}} \Rightarrow f' > f$$

or $\underbrace{\lambda' < \lambda}_{\text{a "RED SHIFT"}}$

- stellar spectra (galactic, actually) → show shifting toward RED

ALL GALAXIES ARE OBSERVED TO DO THIS RELATIVE TO EARTH

SUGGESTING THAT:

EARTH IS AT THE CENTER OF THE UNIVERSE.

right?

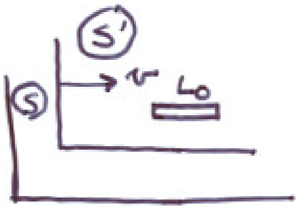
wrong.

The truth is even stranger.. and more wonderful than that! More Einstein!

EXAMPLES...

PROBLEM 19. A rocket ship carrying passengers blasts off to go from NYC to LA, a distance of 5000 km. How fast must it go to have it's length contracted by 1%?

Remember



L_0 proper length, at rest in S'

$$L = \frac{L_0}{\gamma}$$

how do I remember? well "contraction" ⇒ "smaller"

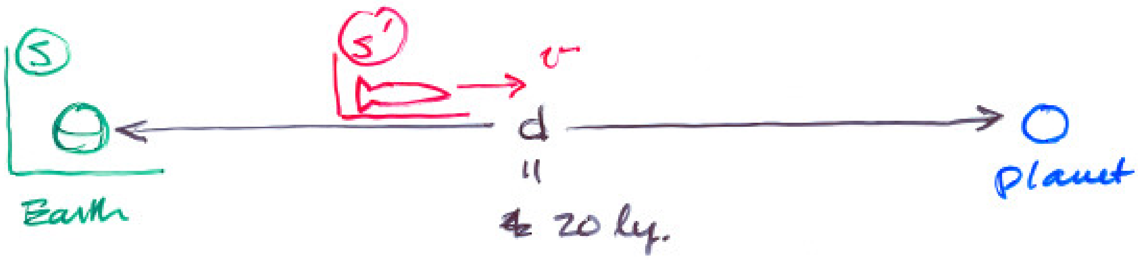
and $\gamma > 1$

So,
$$\frac{L}{L_0} = 0.99 = \frac{1}{\gamma} = \sqrt{1 - v^2/c^2}$$

so $\beta = 0.14 = v/c$, so

$$v = 0.14c$$

Problem 20. Astronomers discover a planet orbiting a star 20 light-years away. How fast must a rocket ship go if the Round trip is to take no more than 40 years for the astronauts? How long will ~~that~~ take on Earth?



presume $v = \beta c$ for the entire trip

In S' - the rocket - the distance is Lorentz-contracted as the planet rushes toward it...

$$d' = \frac{d}{\gamma} = \cancel{40}\ 20\,ly\sqrt{1-\beta^2}$$

so in S':

$$v = \frac{2 \cdot d'}{40\,y.} = \frac{40\,l\text{-}y}{40\,y}\sqrt{1-\beta^2}$$

$$v = c\sqrt{1-\beta^2}$$

$$\beta = \sqrt{1-\beta^2} \Rightarrow \beta = \sqrt{1/2} \sim 0.71c$$

For time elapsed on earth..

$t' = 40y$ — and time is dilated on earth

$$t = \gamma t' = \frac{1}{\sqrt{1-\beta^2}}\,40y = 56.6y$$

Problem 32. A proton and an antiproton are moving toward one another, each at 0.8c with respect to the collision point. How fast are they moving with respect to one another?

A velocity-addition problem.

$$u_x = \frac{u_x' + v}{1 + u_x' v/c^2} \qquad \text{away}$$

or:

$$u_x' = \frac{u_x - v}{1 - u_x' v/c^2}$$

Imagine sitting on proton (S′) and watching antiproton (S) come at you. The relative speed of the frames is $v = -0.8c$ and the $u_x = 0.8c$

$$u_x' = \frac{0.8c - 0.8c}{1 - (0.8c)(-0.8c/c^2)} = 0.976c$$

Wimpy $p\bar{p}$ collider..

γ only ~ 5

At Fermilab $\gamma \sim 1067$

v of protons and antiprotons is $\sim$ 260 mph slower than the speed of light.

see homework #66