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Niels Bohr... one of the quantum heroes.

A young post doc for JJ... did not get along.

Migrated to Manchester to work with the more pleasant Rutherford ~ 1912.

a little of this and a little of that...

‡ learned about the Balmer / Rydberg formula → like lightning - he immediately understood Rutherford's model.



total flaw in any "planetary" model: the electron has to radiate since it's accelerating.

Right?

Bohr was 27 years old... he knew his Planck
he knew his Einstein
He was up on developments.

Rutherford had no time for theoreticians... but Bohr:
"He's different! He's a football player!"

Bohr spent 3 months in Manchester

- doing experiments
- worrying about how α particles lose energy as they pass through matter.



a model with
harmonically bound
electrons which
vibrate... in units of h .

not quite

In July 1912 -- he finished and rushed home
to be married in early August.

- kept writing

- changed honeymoon plans to go back
to England to work.

gotta like that!

Born in Denmark to teach -- and in February 1913
learned of Bolzano formula

By March 6 had a paper -- 1st of a trilogy --
off to Rutherford for submission to

Philosophical Magazine

The rest... as they say... is history.

Here's what Bohr did:

Postulates:

- an electron in an atom moves in circular orbits around the nucleus. — obeying classical dynamics.
- only specific orbits are allowed. *
- the electron does not radiate EM energy.
- EM energy is only radiated when an electron drops from an outer orbit to an inner one. **

* Bohr quantized the angular momentum of the orbits!

$$L = n\hbar$$

$$\hbar = \frac{h}{2\pi}$$

$$n = 1, 2, 3, \dots$$

not quite like Planck's oscillators.

**

$$hf = E_i - E_f$$

E 's are energies of orbits



assume $M_N \gg m_e$ -- effectively infinitely massive.

$$\Sigma F_c = m_e a_e$$

$$\frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r^2} = m \frac{v^2}{r}$$

from $L = n\hbar = mvr$

$$v = \frac{n\hbar}{mr}$$

$$\frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r^2} = \frac{m}{r} \left(\frac{n\hbar}{mr} \right)^2$$

$$r_n = 4\pi\epsilon_0 \frac{n^2 \hbar^2}{mZe^2}$$

$$n = 1, 2, 3, \dots$$

↑
"quantum number"

$$v_n = \frac{n\hbar}{mr} = \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{n\hbar}$$

Energy.

$$U = - \int_{r}^{\infty} \frac{Ze^2}{4\pi\epsilon_0 r^2} dr = - \frac{Ze^2}{4\pi\epsilon_0 r}$$

negative, since the Coulomb force is +... to remove the electron would require work.

$$K = \frac{1}{2}mv^2 = \frac{Ze^2}{4\pi\epsilon_0 2r} \quad (\text{from } \Sigma F = m\frac{v^2}{r})$$

$$E = K + U$$

$$E = - \frac{Ze^2}{4\pi\epsilon_0 2r} = -K$$

$$\text{using } r_n = \frac{4\pi\epsilon_0 n^2 \hbar^2}{mZe^2}$$

$$E_n = - \frac{mZ^2e^4}{(4\pi\epsilon_0)^2 \hbar^2} \frac{1}{n^2} \quad n = 1, 2, 3, \dots$$

For hydrogen? $Z = 1$ lowest, $n = 1$

$$E_1 = - \frac{(9 \times 10^9)^2 (9.11 \times 10^{-31}) (1.6 \times 10^{-19})^4}{(2)(1.05 \times 10^{-34})^2}$$

$$E_1 = - 2.17 \times 10^{-18} \text{ J} = - 13.6 \text{ eV}$$

Radiation...

$$f = \frac{E_i - E_f}{h}$$

$$f = \underbrace{\left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{m z^2 e^4}{4\pi h^3}} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$f = R \cdot c \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

↗ calculated... $R = 1.1 \times 10^7 \text{ m}^{-1} = R_{\infty}$

→ THE RYDBERG CONSTANT IN BALMER FORMULA.

So, let's collect results so far:

important quantity... The so-called "fine structure constant"
is

$$\alpha \equiv \frac{e^2}{4\pi\epsilon_0\hbar c}$$

dimensionless: e, c, \hbar

characterizes the "strength"
of the electromagnetic
interaction

$$\alpha = \frac{(9 \times 10^9)(1.6 \times 10^{-19})^2}{(1.05 \times 10^{-34})(3 \times 10^8)} \approx \frac{1}{137} \sim 0.007$$

Radius...

$$r_n = \frac{n^2}{Z} \left(\frac{\hbar}{m c \alpha} \right)$$

a basic radius is defined called the Bohr Radius

$$a_0 \equiv \frac{\hbar}{m c \alpha} \quad (r_1 \text{ for } Z=1)$$

$$a_0 = \frac{(1.05 \times 10^{-34})}{(9.1 \times 10^{-31})(3 \times 10^8)(1/137)} = 0.53 \times 10^{-10} \text{ m}$$

basically the radius of the ground state hydrogen
atom... known for a century.

$$a_n = \frac{n^2}{Z} a_0$$

energy

$$E_n = - \frac{e^2 z^2}{8\pi\epsilon_0 a_0 n^2} = - \frac{\hbar^2 z^2}{2m a_0^2 n^2}$$

or

$$= - \frac{1}{2} m \left(\frac{z\alpha c}{n} \right)^2$$

or

$$= -13.6 \frac{z^2}{n^2} \text{ eV}$$

One of the first apparent failures of Bohr's theory had to do with ionized He

In effect Bohr's theory would suggest

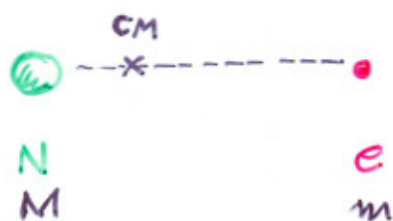
$$\frac{R_{\infty\text{He}}}{R_{\infty\text{H}}} = \frac{\left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{m z_{\text{He}}^2 e^4}{4\pi\hbar^3 c}}{\left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{m z_{\text{H}}^2 e^4}{4\pi\hbar^3 c}} = \frac{z_{\text{He}}^2}{z_{\text{H}}^2} = 4$$

But, experiment suggested 4.0016

Bohr replied:

A 2-body problem... original theory presumed

$$M_p = \infty.$$



In mechanics you can deal with the CM displacement by using the "reduced mass" as the single mass replacement for m_e .

$$\frac{1}{\mu} = \frac{1}{m} + \frac{1}{M}$$

$$\mu = \frac{mM}{M+m} = \frac{m}{1+m/M} \approx m \left(1 - \frac{m}{M}\right)$$

$$m_e = 9.1094 \times 10^{-31} \text{ kg}$$

$$m_p = 1.6726 \times 10^{-27} \text{ kg}$$

} modern numbers

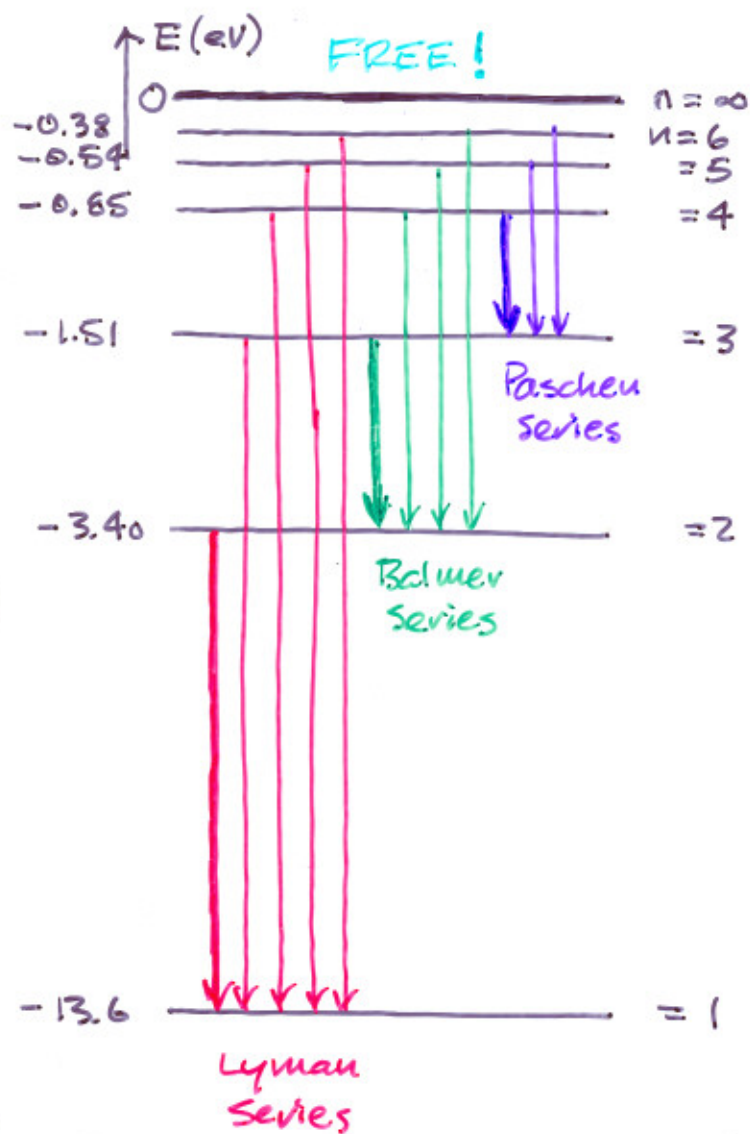
In

$$\frac{R_{\infty \text{He}}}{R_{\infty \text{H}}} = \frac{\left(1 - \frac{m_e}{4M_p}\right)}{\left(1 - \frac{m_e}{M_p}\right)} \frac{Z_{\text{He}}^2}{Z_{\text{H}}^2} = 4.00162 \quad (= 4.0016 \text{ expt})$$

(numbers available to Bohr gave 4.00163...)

THIS CLINCHED IT FOR MANY PEOPLE - instantaneously, Bohr is a major figure

Bohr's model predicted all possible H spectra



$\Rightarrow \text{He}^+$ is just the same!

EXCEPT

$$\frac{1}{\lambda_{\text{He}^+}} \approx 4 \frac{1}{\lambda_{\text{H}}}$$



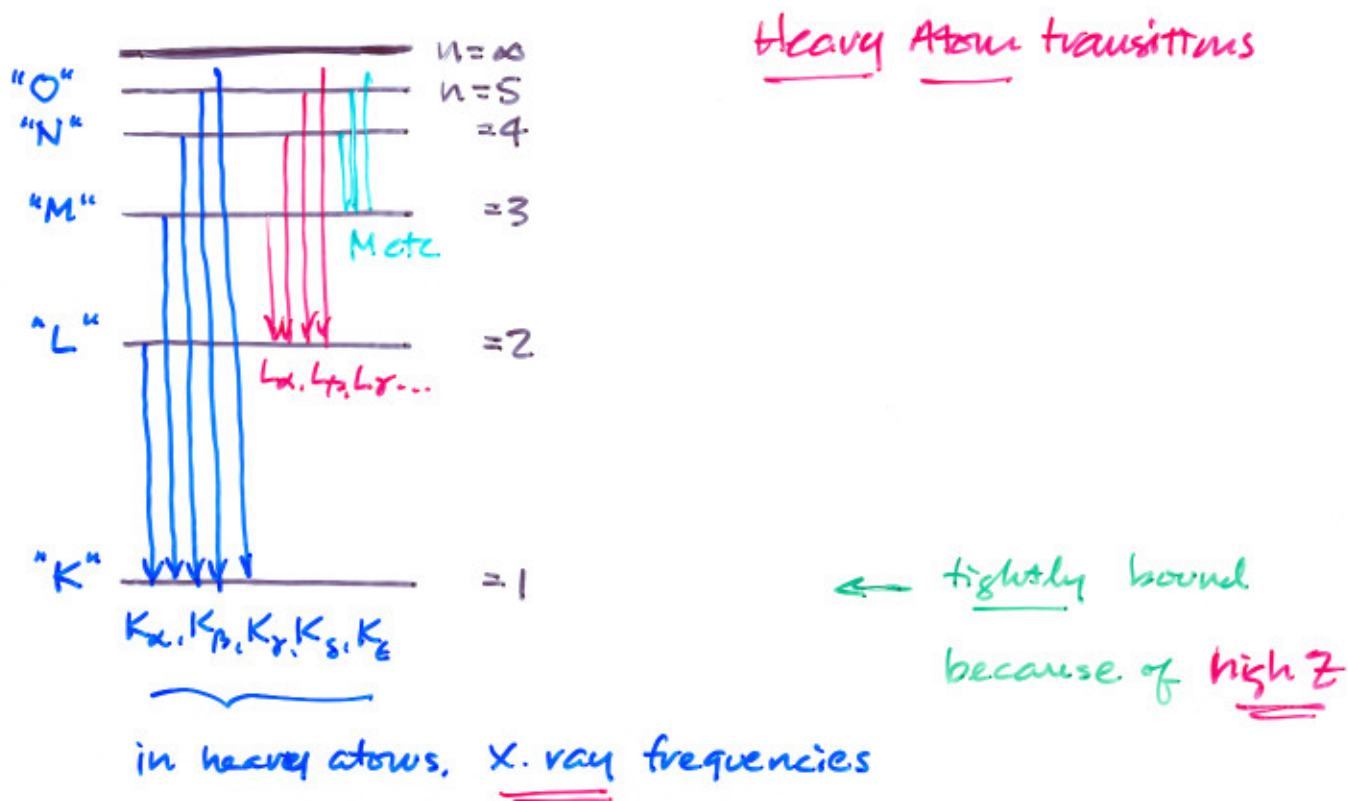
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WARTIME MADNESS

During this time (1913) Bohr/Rutherford model stimulated much thinking

H.G.J. Moseley -- a young lecturer at Manchester studied X-ray emission and absorption

nomenclature:



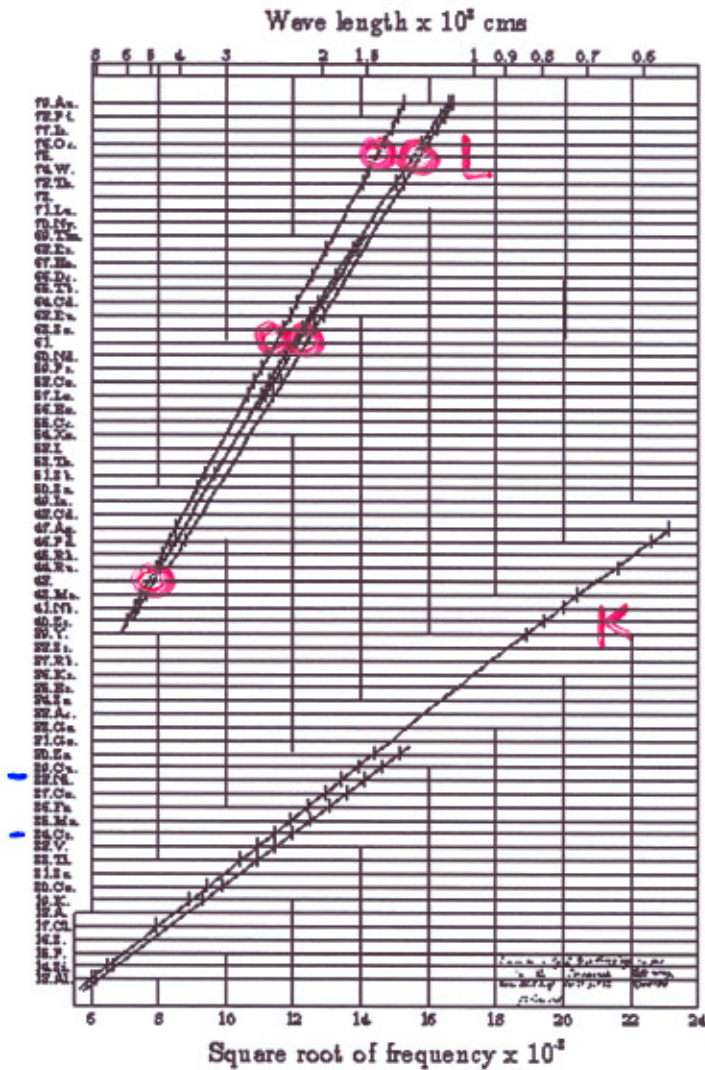
Moseley set up a little train of elements --



rhodium
1925

promethium
1945

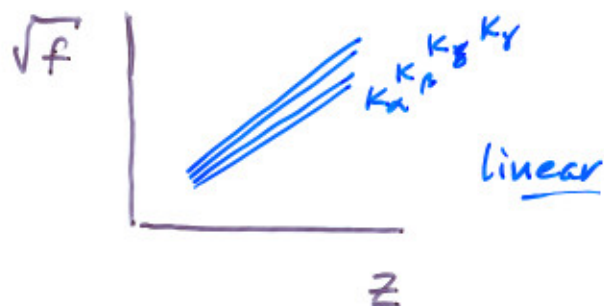
technetium
1937



He found empirically

$$f_k \propto (Z-1)^2$$

Moseley's Law.



* Moseley Plot *

∴ he found that atomic number assumptions were wrong --

Nickel was previously listed before cobalt... Moseley found that to be backwards

Could find missing slots → new elements required.

He could explain using Bohr's ideas --

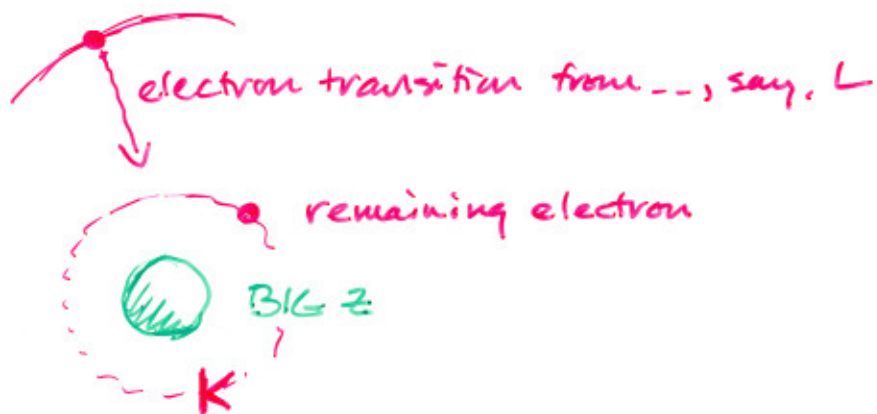
but with a bit of -- a reach

also Bohr:

$$E_n = - \frac{Z^2 e^4}{(4\pi\epsilon_0)^2} \frac{m_e}{2\hbar^2} \frac{1}{n^2} = hf$$

So, for transitions to empty K slot --

$$f = \frac{Z^2 e^4}{4\pi(4\pi\epsilon_0)^2} \frac{m_e}{\hbar^3} \left(1 - \frac{1}{n_i^2}\right) \quad i = 2, 3, \dots$$



The K shell is so tightly bound .. that the L-shell electron sees not Z , -- but effectively $(Z-1)$... the remaining electron shields the + charge by $1e$.

sure.

So, he said

$$f = \frac{(Z-1)^2 e^4}{4\pi(4\pi\epsilon_0)} \frac{m_e}{h^3} \left(1 - \frac{1}{Z^2}\right)$$

Hence, Moseley's Law.

-- least support to Bohr's theory.

which really is only appropriate for 1 electron atoms...

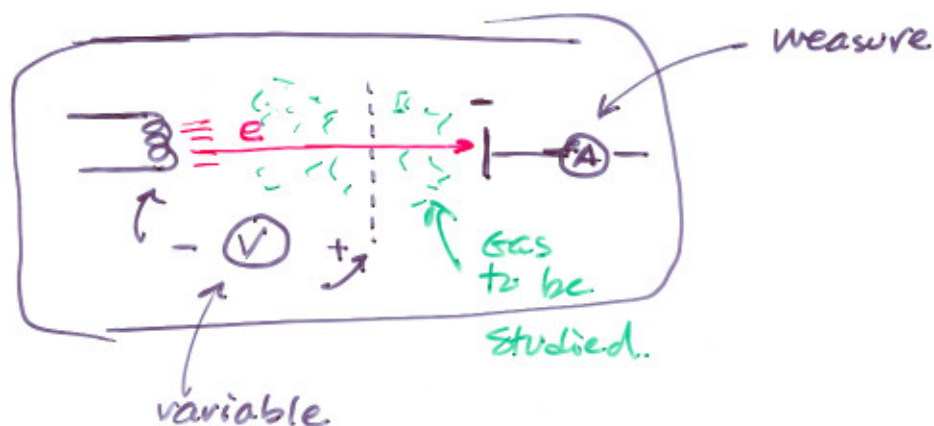
Disastrous Gallipoli Campaign



Gallipoli casualties			
	Died	Wounded	Total
Australia	8,709	19,441	28,150
New Zealand	2,721	4,852	7,553
The United Kingdom	21,255	52,230	73,485
France (estimated)	10,000	17,000	27,000
India	1,358	3,421	4,779
Newfoundland	49	93	142
Total Allies	44,072	97,037	141,109
Ottoman Empire	86,692	164,617	251,309



Frank-Hertz Experiment 1914.
(James) (Gustav)

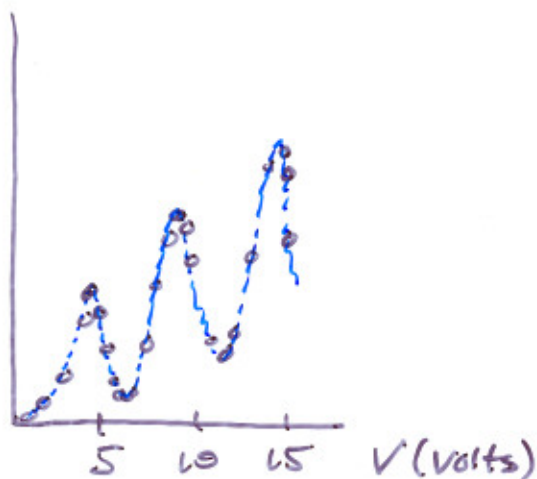


Hg

They ran up the voltage and measured current.

↑
increases ——— increases

until a critical voltage.



When voltage was
4.88 volts

Hg spectra visible

Each bump \Rightarrow ionization
of a particular
atomic level —

an inelastic collision of $e \Rightarrow$ no current

Showed that energy levels are quantized.