Bohr thought deeply...

enunciated "Correspondence Principle" in 1913

... and tweaked it for a decade.

In the limiting case of large quantum numbers, frequencies and intensities of radiation calculated from classical physics must agree with those calculated from quantum physics.

Classically, the frequency of an electron in orbit is

$$f_{\text{classical}} = \frac{v}{2\pi r}$$

$$= \frac{n\hbar / m_e r}{2\pi r}$$

$$= \frac{n\hbar}{2\pi m_e} \frac{1}{r^2} = \frac{n\hbar}{2\pi m_e}\left(\frac{e^2 m_e}{4\pi\epsilon_0 n^2 \hbar^2}\right)^2 \quad (*)$$

How about quantum theory-wise?

$$f_q = \frac{e^4}{4\pi(4\pi\epsilon_0)^2} \frac{m_e}{\hbar^3}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] \quad \text{one level transition}$$

$$= \frac{e^4}{4\pi(4\pi\epsilon_0)^2} \frac{m_e}{\hbar^3} \frac{2n-1}{n^2(n-1)^2}$$

If n is very large $\quad \frac{2n-1}{n^2(n-1)^2} \rightarrow \frac{2n}{n^4} = \frac{2}{n^3}$

$$f_q \rightarrow \frac{e^4}{2\pi(4\pi\varepsilon_0)^2} \frac{m_e}{\hbar^3} \frac{1}{n^3} \qquad *$$

so $f_q \rightarrow f_{classical}$

This "correspondence" of quantum $\rightarrow$ classical is sort of like the correspondence of relativistic $\rightarrow$ classical as $\beta \rightarrow 0$ -- comforting

$h \rightarrow 0$ is more complicated, actually, but also a classical limit

But. The conceptual upheaval coming would not leave this comfort alone!

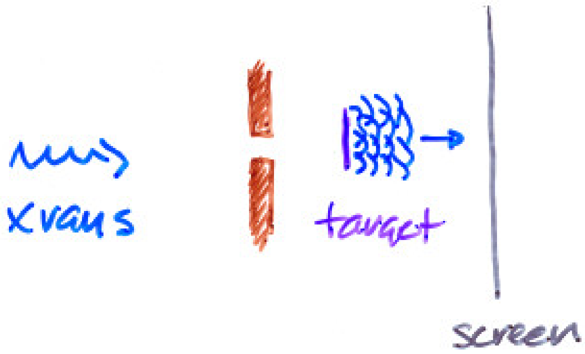
X-ray DIFFRACTION

- from discovery... to actual confirmation of x-rays as EM waves... long time!

  expected short λ, so diffraction required tiny "slits"

  Crystals suggested by Max von Laue

  sure enough... 1912 showed diffraction patterns



- explained by father-son team of Brits

  W.L. Bragg - son ... "theory"

  W.H. Bragg - father — "experiment"

  a model of crystals →

  study materials

  analyze x rays

From van Laue's Nobel Lecture

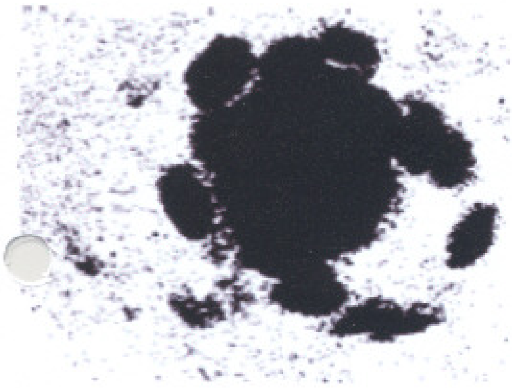


Fig. 1.

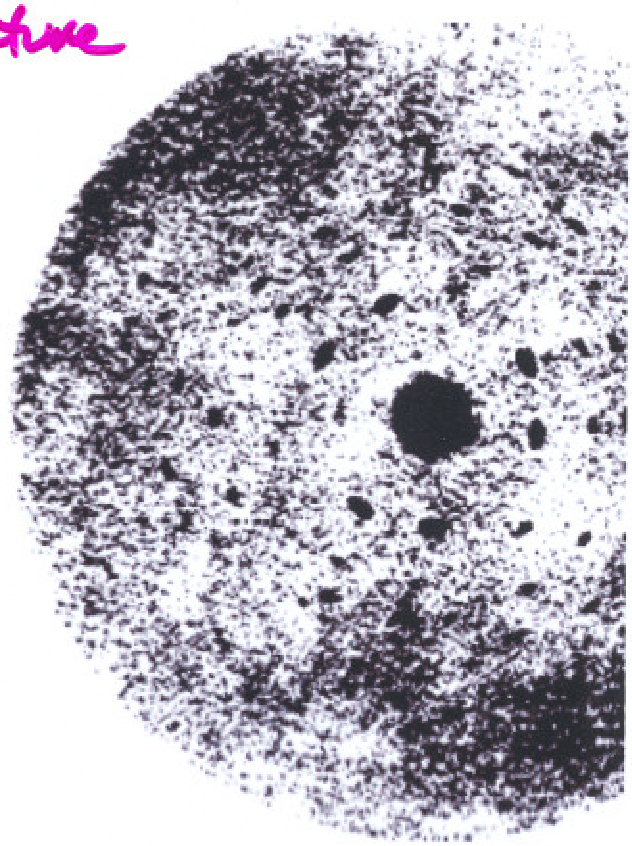
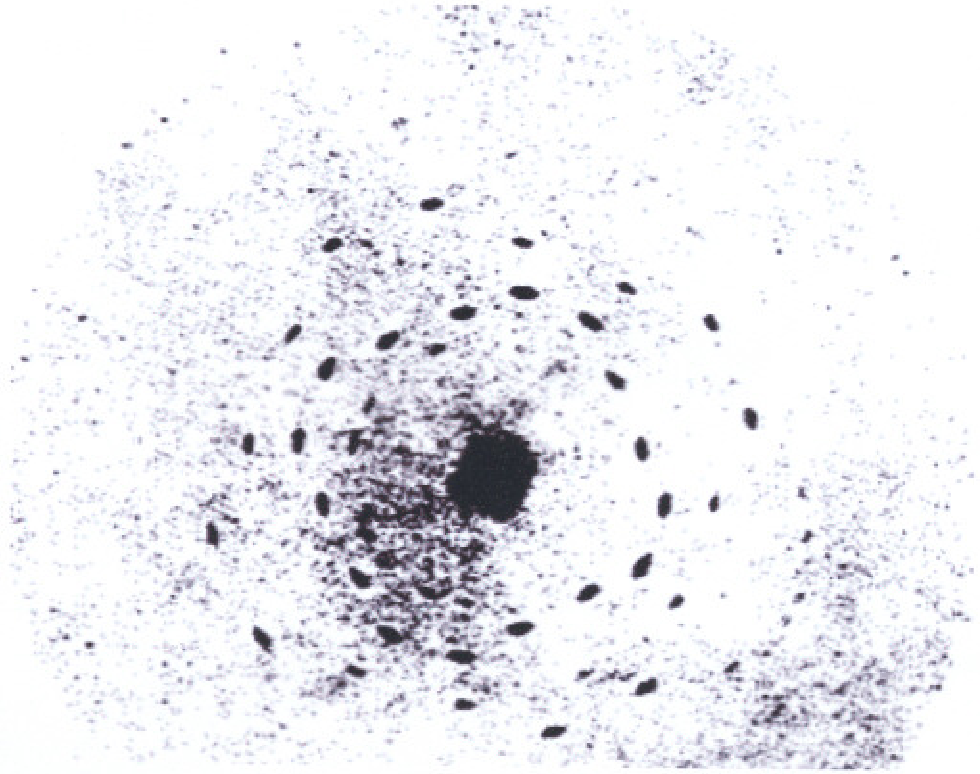
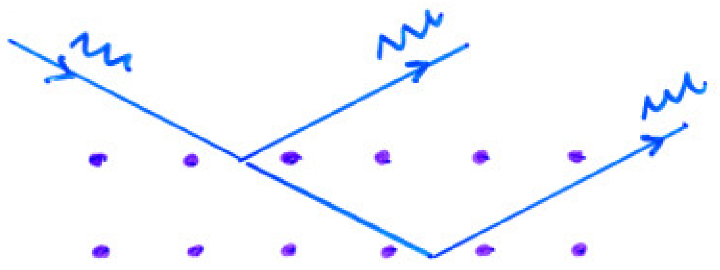


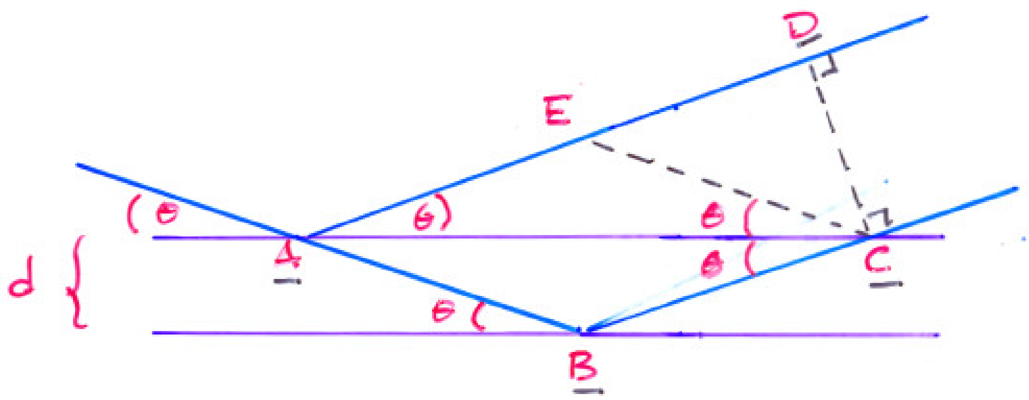
Fig. 2.



Imagine the following arrangement of scattering centers



when these constructively interfere — bright spots



D-E in phase

phase difference $= \left(\frac{2\pi}{\lambda}\right) \cdot$ (difference in path length)

$$= \left(\frac{2\pi}{\lambda}\right)(ABC - AD)$$

$$ABC - AD = (AB + BC) - (\underbrace{AE}_{= BC} + ED)$$

$$= AB - ED$$

$$= \frac{d}{\sin\theta} - \frac{d}{\sin\theta}\cos 2\theta = \frac{d}{\sin\theta}(1 - \cos 2\theta)$$

$$= 2d\sin\theta$$

$$\text{phase difference} = \left(\frac{2\pi}{\lambda}\right) 2d \sin\theta$$

$\|$

$2\pi n$

for constructive interference

$$\therefore \quad n = \frac{1}{\lambda}\, 2d \sin\theta$$

$$\lambda = \frac{2d}{n} \sin\theta$$

$$n\lambda = 2d \sin\theta$$

Bragg Condition
Bragg Law

Notice that this is like reflection from a plane ... Bragg Plane

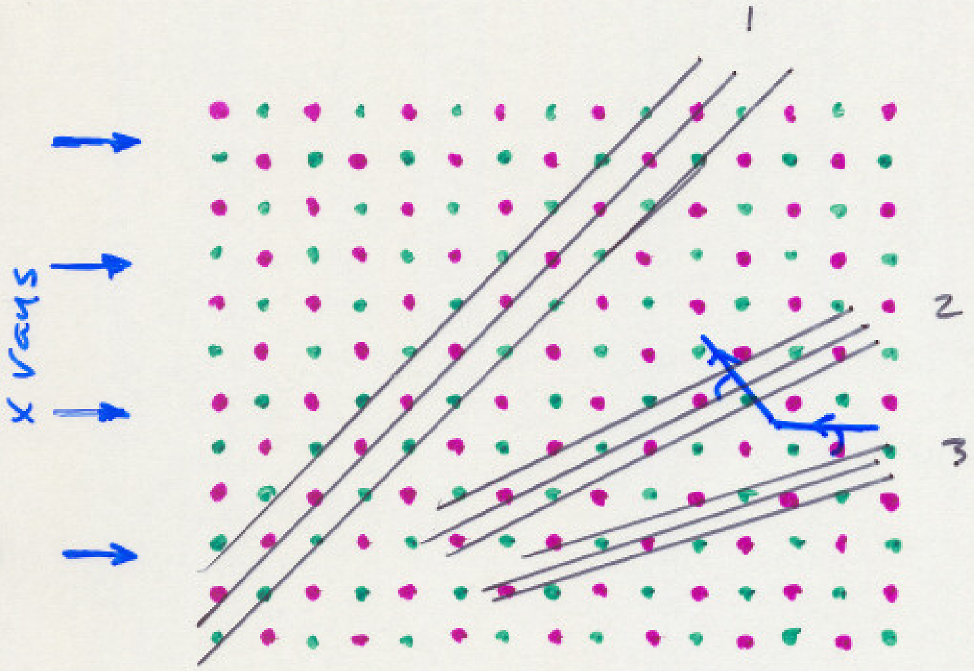
– depends on the material

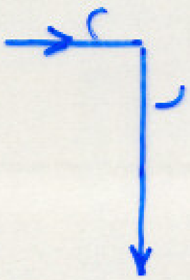
they did NaCl ... a cubic lattice

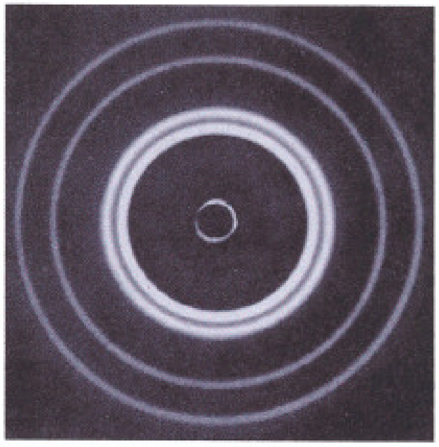
→ pick the angle, determine λ

→ measure the angle of maxima ...

determine the crystal structure


X rays
1
2
3





Xray diffraction from Al

Ni

has an inter-layer distance of $d = 2.15\,\text{Å}$

Mo primary xray wavelength ~ $0.63\,\text{Å}$

What angles will result?

$$n\lambda = 2d\sin\theta_n$$

$$\sin\theta_n = n\frac{\lambda}{2d} \approx n\left(\frac{0.63}{4.3}\right)$$

$$\sin\theta_n = n\,(0.15) = \frac{n}{6.7}$$

So, first angle is @ $n=1$ $\quad \sin\theta_1 = 0.15$

$$\theta_1 \doteq 8.6°$$

then $\quad n=2 \quad \sin\theta_2 = 2(0.15)$

$$\theta_2 = 17°$$

notice when $n \geq 7$, $\sin\theta > 1$ ...

so, there will only be 6 angles at which light will give "bright spots"

so... it's 1923 — what's known?

- h explains — Blackbody radiation
  Planck, Einstein
- h appears to explain — Hydrogen
  Bohr
- h suggests — light is indeed quantized...
  Einstein, Millikan, Compton

All together... light seems to have a dual nature:

either wave or particle...

or

both wave and particle

nobody understands yet!

In any case... something formerly a wave
↓
both.

the object formerly known as wave is $\underline{\underline{\gamma}}$

Into this comes a 16 page PhD thesis from Paris

Prince Louis de Broglie

"Because photons have wave properties and particle properties, perhaps all forms of matter have wave as well as particle characteristics."

yeah, right.

Remember, photons satisfied

$$p = \frac{h}{\lambda}$$

de Broglie suggested that this is generally true -- or, more (disturbing)

$$\lambda = \frac{h}{|\vec{p}|}$$ called the de Broglie wavelength

if an object is non-relativistic

$$\lambda = \frac{h}{mv}$$

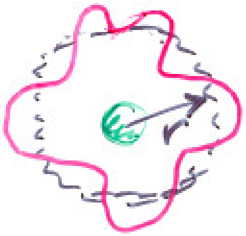
if relativistic

$$\lambda = \frac{h}{mv\gamma}$$

he noticed something intriguing:

presume electron is a <u>wave</u> trapped in an orbit... without changing phase

→ a standing wave of "electron"



so: circumference = an integral # wavelengths

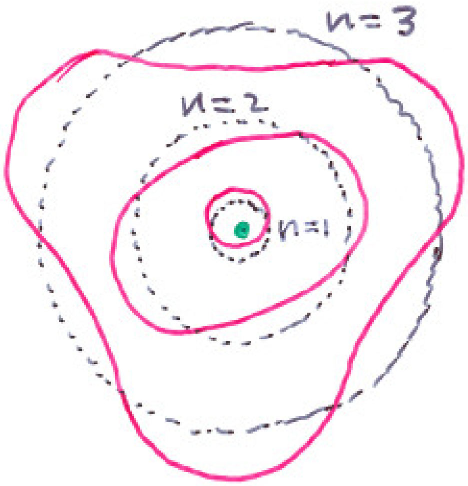
$$2\pi r = n\lambda$$

$$= n\frac{h}{p}$$

$$rp = L = \frac{nh}{2\pi} = n\hbar$$

Bohr's condition!

The de Broglie picture:



the nodes move as the electron "waves" within each Bohr orbit.

pick on the 3rd Bohr orbit...

λ_e ?

$$2\pi r = n\lambda$$

$$\lambda = \frac{2\pi r}{n}$$

$r = n^2 a_0$ $\nearrow$ $\qquad a_0 = 0.53 \times 10^{-10}$ m

$$\lambda = \frac{2\pi n^2 a_0}{n}$$

$$\lambda = 2\pi n a_0$$

$$\lambda = (2\pi)(3)(0.53 \times 10^{-10} \text{ m})$$

$$\lambda \simeq 10 \times 10^{-10} = 1 \text{ nm}$$

v_e ?

$$\lambda = \frac{h}{p} \qquad \text{try} \qquad p = m_e v$$

$$v = \frac{h}{m\lambda} = \frac{h}{m 2\pi r} \cdot n = \frac{n\hbar}{mr}$$

$$v = \frac{n\hbar}{mr}$$

$$= \frac{n\hbar}{mn^2a_0} \qquad = \frac{1}{n}\frac{\hbar}{ma_0}$$

$$v = \frac{1}{n}\frac{\hbar}{a_0 m}\left(\frac{c^2}{c^2}\right)$$

$$= \frac{1}{n}\frac{\hbar c^2}{a_0 mc^2}$$

$$v/c = \beta = \frac{1}{n}\frac{\hbar c}{a_0 mc^2} \qquad mc^2 = 0.511\ \text{MeV}$$

$$\beta = \frac{1}{n}\frac{197.3\ \text{eV}\cdot\text{nm}}{(0.053\ \text{nm})(0.511\times10^6\ \text{eV})}$$

$$\beta = \frac{1}{n}\ 0.00729$$

$$v = \frac{1}{n}\ 2.18\times10^6\ \text{m/s}$$

$$v_3 = 727{,}000\ \text{m/s}$$