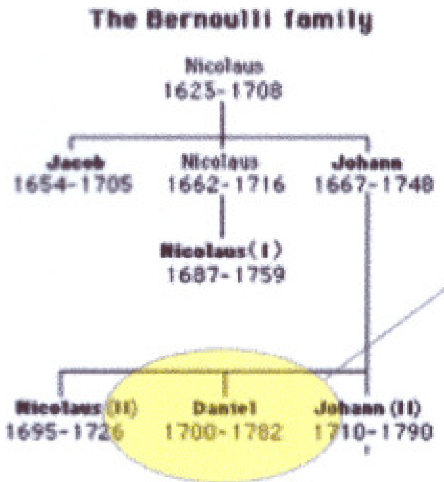
The applications of Newton's laws kept coming

Thomas Jefferson

- even designed a plow contour to maximize the overturn of earth for minimum force

Bernoulli's were a family of always amazing, sometimes emminent mathematicians

- but dysfunctional beyond belief

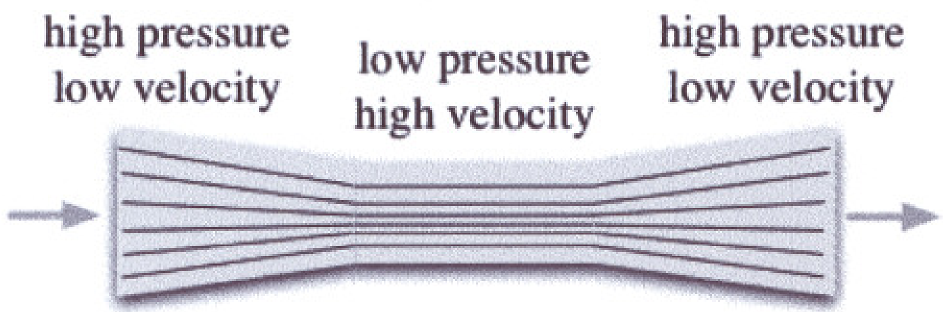


Daniel was the most accomplished, certainly as a practical developer of both Leibnitz and Newton's work

It was he who uncovered the use of Newton's laws, when combined with the conservation of vis viva to fully explain fluid dynamics

$$P + \rho v^2 = \text{constant}$$

Another conservation law, explaining the flow of fluids through varying sized pipes - motivated by Harvey's model of the circulation system



All published in *Hydrodynamica*, in 1738, 40, 42 along with an atomic explanation of Boyle's Law

THE DEVIL OF GASES IS

IN THE DETAILS...

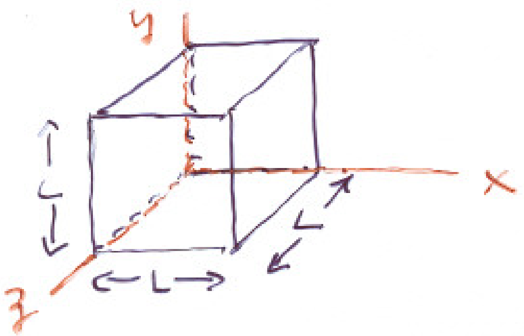
"KINETIC THEORY OF GASES"

very old, very cool calculation... soon after N.!

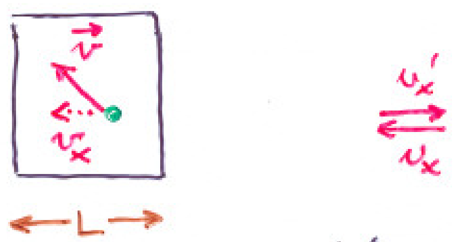
IMAGINE A VOLUME FILLED WITH MOLECULES:

- identical, mass m
- point-like ⇒ no size ⇒ no interactions
- numerous enough to ignore fluctuations $\sim \frac{1}{\sqrt{N}}$

• "IDEAL GAS" •



a cube of molecules



recoils elastically from walls

Δt = average time between $x = 0$ and $x = L$ collisions

$$\Delta t = \frac{2L}{|v_x|}$$

momentum is transferred to wall — elastic

so. $\Delta P_x = 2m\,|v_x|$

Rate at which momentum is transferred —

$$\frac{\Delta P_x}{\Delta t} = \frac{2m\,|v_x|}{2L} = \frac{m v_x^2}{L} = \langle F_x \rangle$$

by one molecule

Total force applied to $x=0$ or $x=L$ walls:

$$\langle F \rangle = \frac{N m v_x^2}{L}$$

The pressure —

$$P = \frac{\langle F \rangle}{A} = \frac{N m v_x^2}{L^2 \cdot L} = \frac{N m v_x^2}{V}$$

Assumes all molecules have the same speeds..

Use <u>AVERAGES</u>

$$v_x^2 \longrightarrow \langle v_x^2 \rangle$$

$$P = \frac{N m \langle v_x^2 \rangle}{V}$$

x isn't special -- y and z are, on average, the same.

$$\langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle \equiv \langle v^2 \rangle$$

so, $$\langle v_x^2 \rangle = \frac{1}{3} \langle v^2 \rangle$$

So,

$$P = \frac{Nm \langle v^2 \rangle}{3V}$$

or. $$PV = \frac{Nm \langle v^2 \rangle}{3}$$

BOYLE worked with the grumpy Robert Hooke and found that at a constant T:

$PV =$ constant — Boyle's Law

"springiness of air"

A century later Jacques Charles found

$V/T =$ constant -- at constant Pressure

Charles' Law

THESE GO TOGETHER...

$PV = nRT$ Ideal Gas Law

or $PV = NkT$ "Equation of State"

n = # moles

R = Universal Gas Constant = $8.314510 \times 10^{23}\ mol^{-1}$

k = Boltzmann's Constant = $1.38066 \times 10^{-23}\ J/K$

N = # molecules

HERE'S THE MAGIC:

$$PV = \frac{Nm\langle v^2\rangle}{3} = nkT$$

$$\Rightarrow \quad kT = \frac{m\langle v^2\rangle}{3}$$

$$kT = \frac{2}{3}\left(\frac{1}{2}m\langle v^2\rangle\right) = \frac{2}{3}\langle K\rangle$$

Gas Temperature can be thought of as

THE AVERAGE KINETIC ENERGY

OF THE MOLECULES OF A GAS

$\cancel{\Delta U = N(\Delta K}) \qquad U = NK$ total internal energy

So: $PV = \frac{2}{3}U$

$\langle K \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT$

$\langle v^2 \rangle = \frac{3kT}{m} \Rightarrow \langle v_x^2 \rangle = \frac{kT}{m}$

FLIP IT..

$$v_{RMS} \equiv \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3kT}{m}}$$

a measure of the average speed of a molecule from

IS Its temperature!

What's the average speed of air molecules at 20°C?

mass of N_2 is 28 amu.

~~28 x 10~~

$$m = 28 \times 1.66 \times 10^{-27}\ \text{kg}$$

$$= 4.648 \times 10^{-26}\ \text{kg}.$$

$$v_{RMS} = \sqrt{\frac{3kT}{m}}$$

$$= \sqrt{\frac{(3)(1.38 \times 10^{-23}\ \text{J/K})(293\ \text{K})}{4.65 \times 10^{-26}\ \text{kg}}}$$

$$v_{RMS} = 510\ \text{m/s}$$

Ideal.. what about my perfume?

SO.. MORE TO THE VELOCITIES OF GAS MOLECULES THAN JUST AVERAGES...

- they are distributed over wide ranges

MAXWELL FIGURED THIS OUT...

... introducing probability into physics..
(described in Chapter 9 – here's a sketch, we'll come back)

LET'S THINK ABOUT A

A. NUMBER DISTRIBUTION

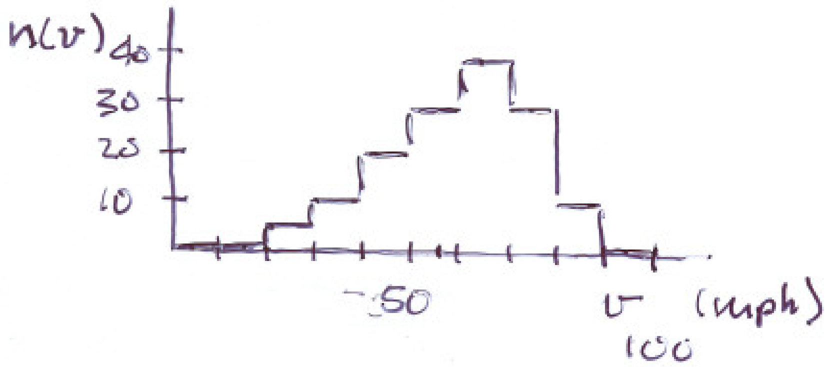
&

B. PROBABILITY DISTRIBUTION

consider I94 to Chicago...

what's the # distribution of cars at a given speed...

guess something like:



between, say B.C. and Kzoo watch for 30 minutes and find

the total # of cars is $N = 5 + 10 + 20 + 30 + 40 + 30 + 10$

$N = 145$

the probability that a car might be going at 55 mph...

$$f(55) = \frac{30}{145} = 0.21 \sim 21\%$$

if the measured distribution is representative of any time...

The number density is defined so that

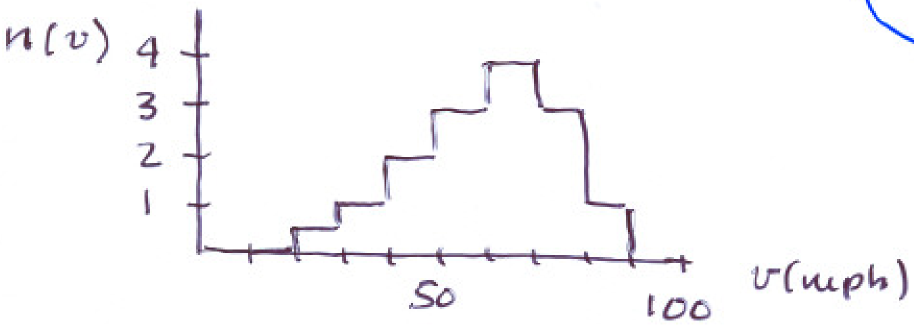
$$\int_0^{100} n(v)\,dv = N$$

so, since a histogram with finite Δx, the integral here is a sum...

$$\sum_{i=1}^{10} n_i \Delta v = N$$

the bin width = 10 mph

145 cars total



$$\sum_{i=1}^{10} n_i \Delta v = (0 + 0 + .5 + 1 + 2 + 3 + 4 + 3 + 1)\cdot 10 = 145$$

$n(v)\,dv$ is the number of cars between v and $v + dv$ — here $v + \Delta v$

$f(v)\,dv$ is the probability distribution of finding a car with velocity between v and $v + dv$

WHAT MAXWELL DID WAS CALCULATE THE NUMBER DENSITY (OR PROBABILITY) FOR MOLECULAR SPEEDS IN A GAS

using simple ideas...

Again, assume that v_x, v_y, v_z are independent...

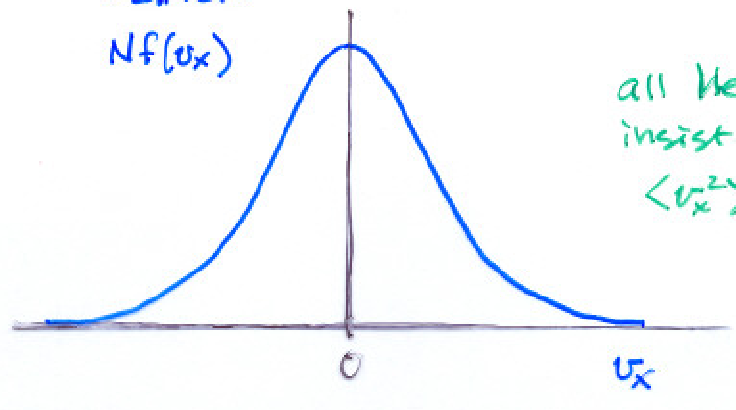
so probabilities multiply

(we'll do this later... here are the results)

The probability distribution for a single component of velocity, say v_x

$$N f(v_x)\, dv_x = N \left(\frac{m}{2\pi kT}\right)^{1/2} e^{-\frac{1}{2} m v_x^2/kT} dv_x$$

Gaussian



all he did was insist that $\langle v_x^2 \rangle = \frac{kT}{m}$

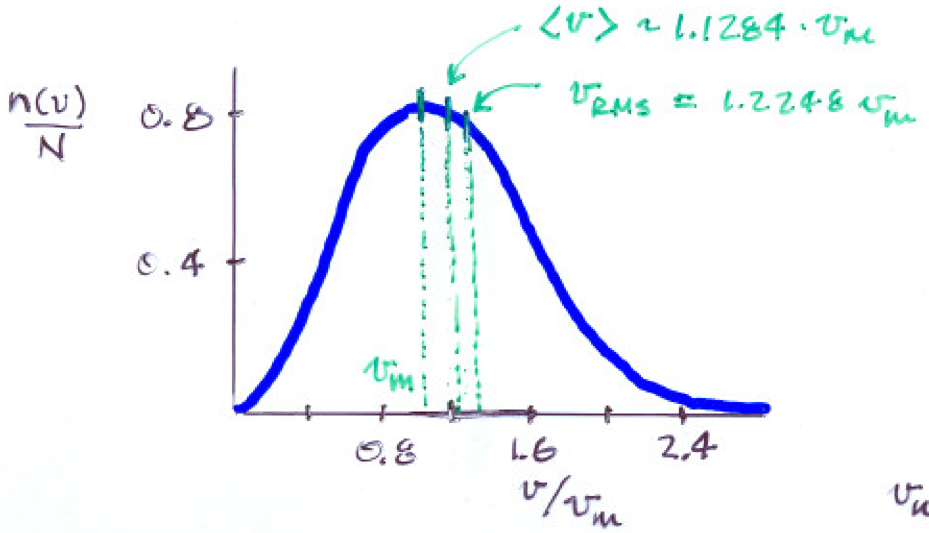
SPEEDS .. ANOTHER STORY

⋮ } stands for algebra.. which I'll do at chapter 9..

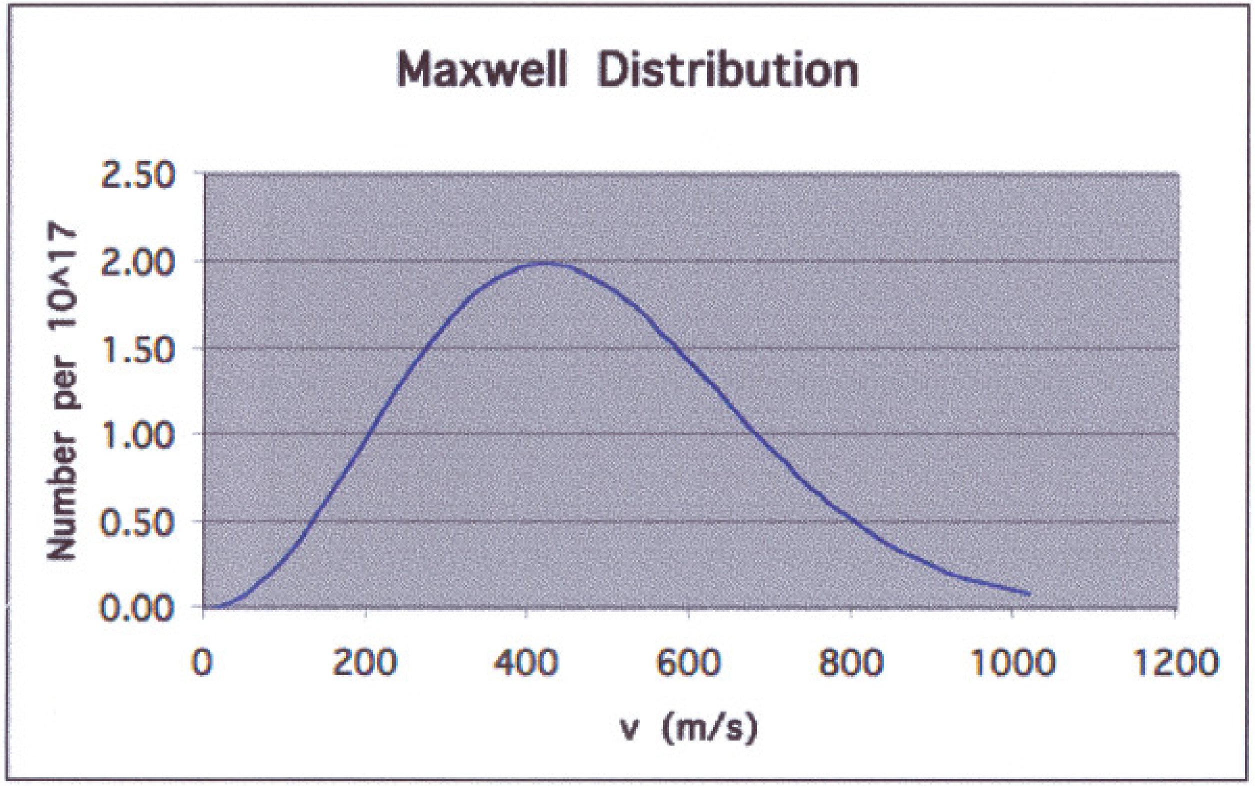
$$n(v)\,dv = 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{1}{2}mv^2/kT}$$

called the Maxwell Speed Distribution

- You give me the constituent (m)
- You give me the temperature (T)
- I'll tell you how many molecules there are at any speed (v)

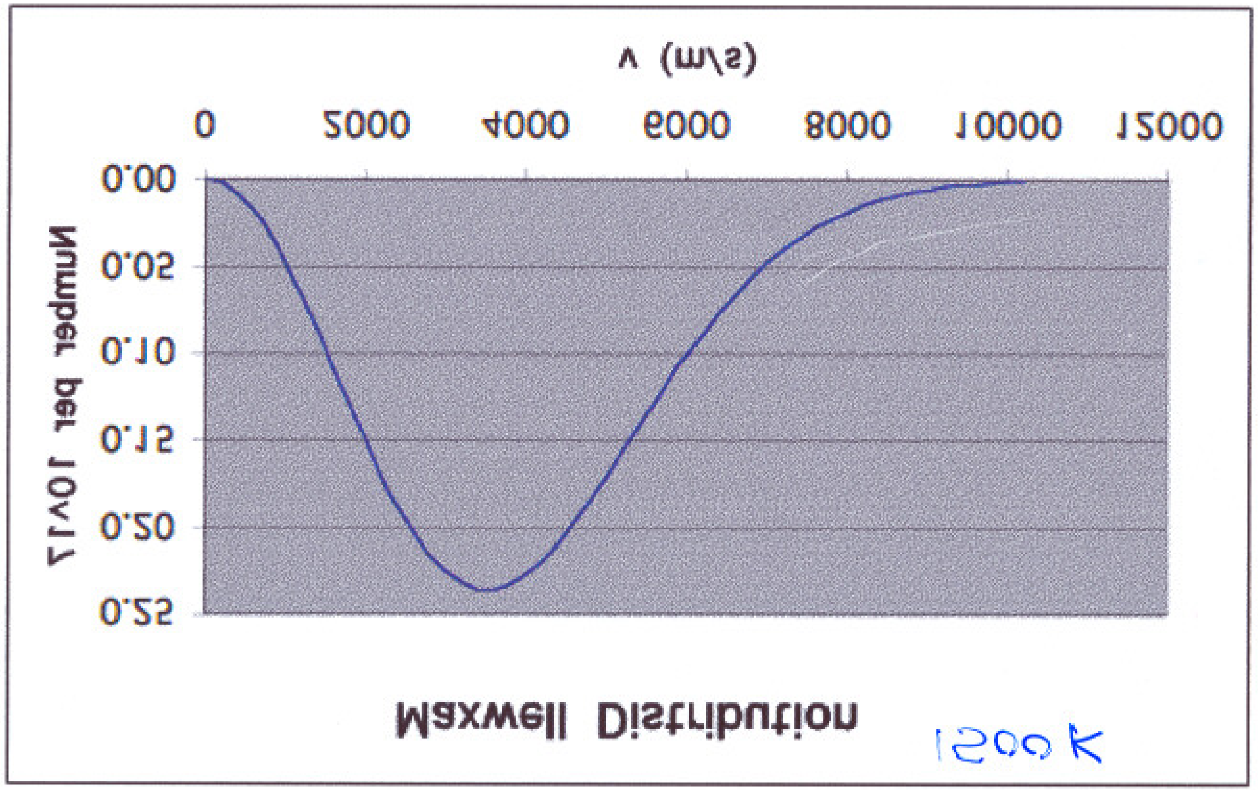


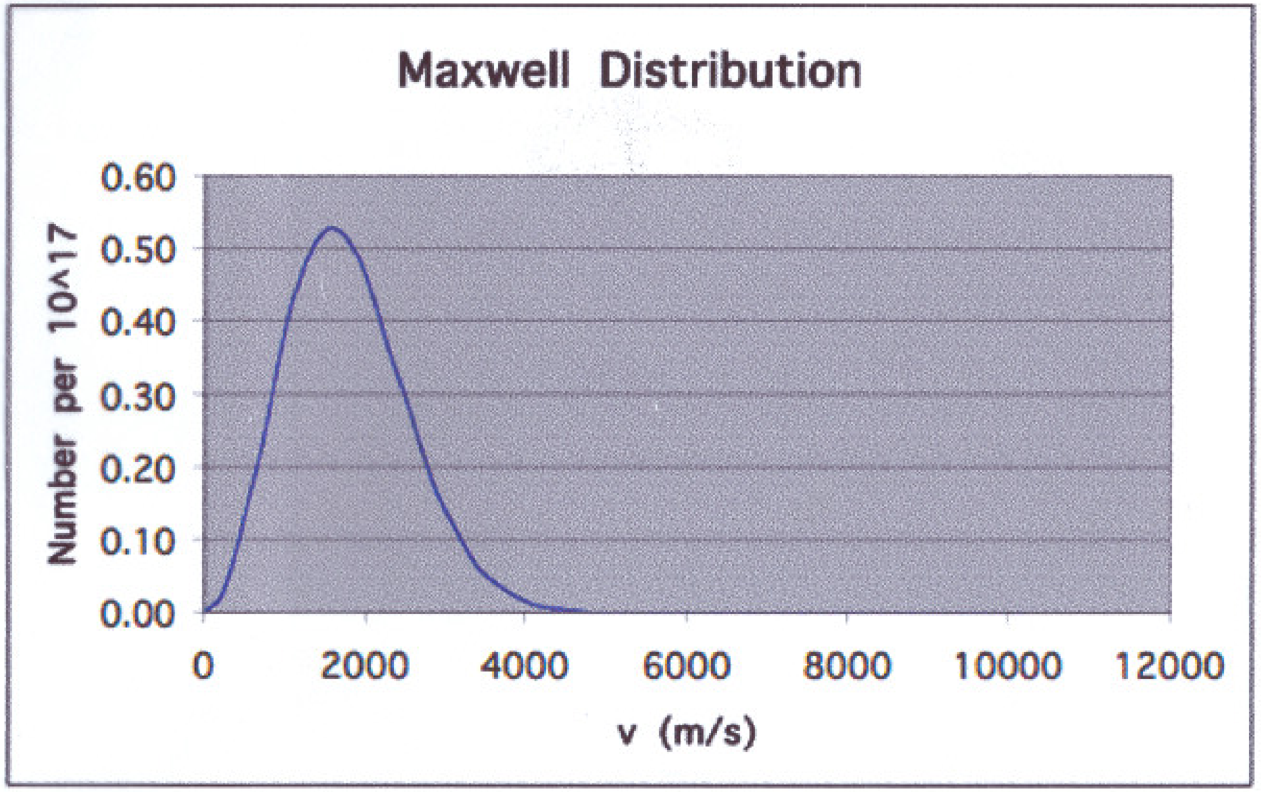
$$v_m = \sqrt{\frac{2kT}{m}}$$



Room temperature N_2 molecules

Here, the escape velocity is about 10km/s and that tail above that amount is enough, over 4.5 billion years, to have lost all of the original, primordial hydrogen in our atmosphere.





Room temperature H_2 molecules

notice the scale difference from before

IDEAL GAS...

A remarkably robust model for many kinds of gases:

- large N
- point objects → no size
- identical molecules / bits...
- no forces among particles — just the walls.

→ most gases approach this for small P!

The Ideal Gas Law is remarkably consistent with our simple Bernoulli-inspired mechanical model.

$$PV = nRT$$

or

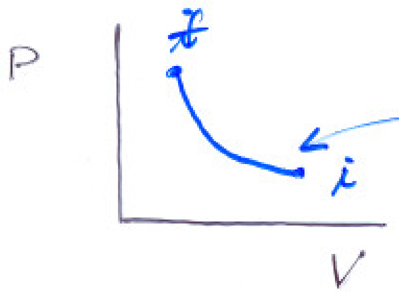
$$PV = NkT$$

GO BACK TO P-V DIAGRAMS, WORK et al.

for the specific Ideal gas model.

ISOTHERMAL EXPANSION...

using $PV = nRT$ = constant (constant T)

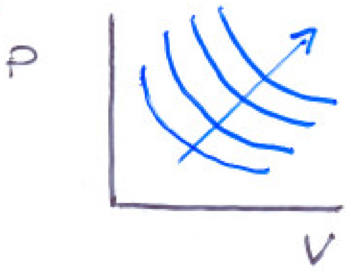


a particular shape in P–V for an "ISOTHERM"

same Temperature along the curve

so, $P = \frac{nRT}{V}$ is the curve

$P = \frac{\text{constant}}{V}$ hyperbole



each a different temperature
HIGH T out
Low T in

WHAT ABOUT WORK DONE BY AN IDEAL GAS SYSTEM?

$$\Delta W = \int_i^f P\,dV$$

$$= \int_{V_i}^{V_f} \frac{nRT}{V}\,dV = nRT \ln V_f/V_i$$

$V_f > V_i$ volume bigger → expansion ⇒ $\Delta W > 0$

$V_f < V_i$ volume smaller → compression ⇒ $\Delta W < 0$