

# THERE IS A BEST ENGINE

AN IDEALIZED CYCLE DUE IN SPIRIT  
TO THE FRENCH MILITARY ENGINEER

Nicolas Léonard Carnot ~ 1824

enunciated a version of  $2_1^{\text{nd}}$  -- which translates  
into

$2_3^{\text{nd}}$  Thermal energy will not, of its own accord,  
flow from a cooler object to a warmer object.

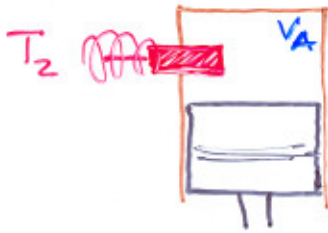
and found the maximum efficiency thermal  
engine ... will still believing in caloric

THE "CARNOT CYCLE" :

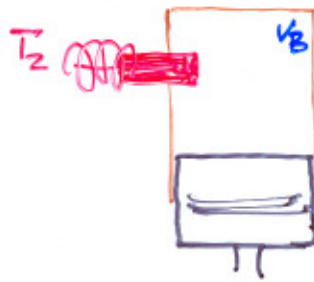
2 adiabatic transitions }  
2 isothermal transitions } single cycle

# CARNOT ENGINE

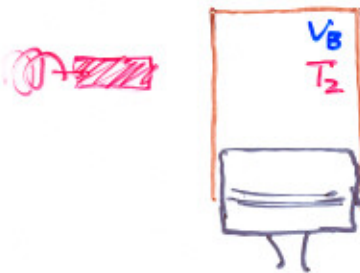
# CARNOT CYCLE



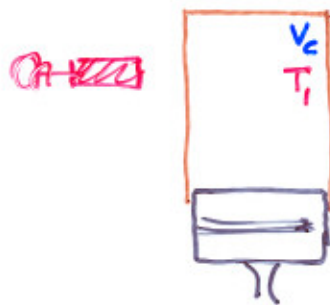
A



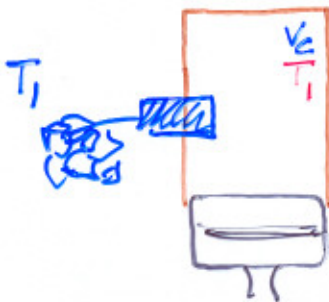
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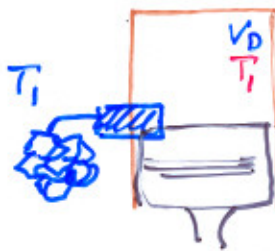
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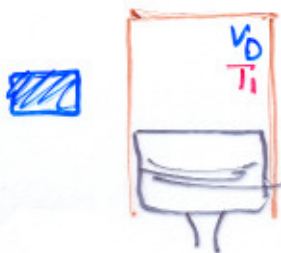
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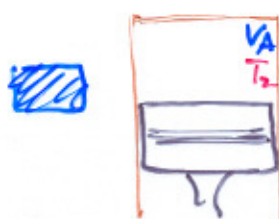
C



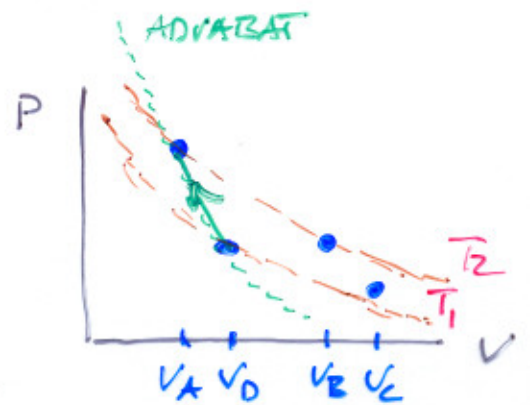
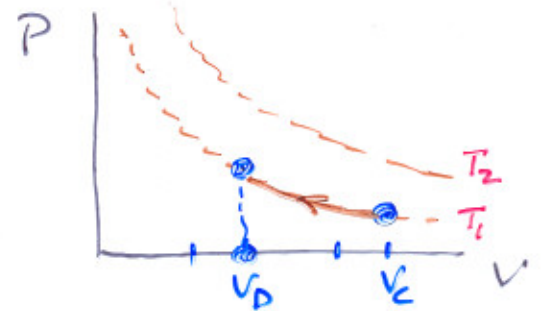
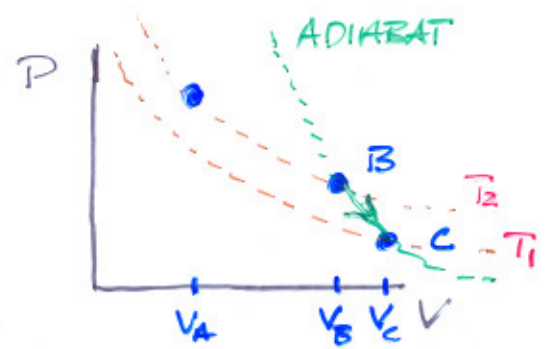
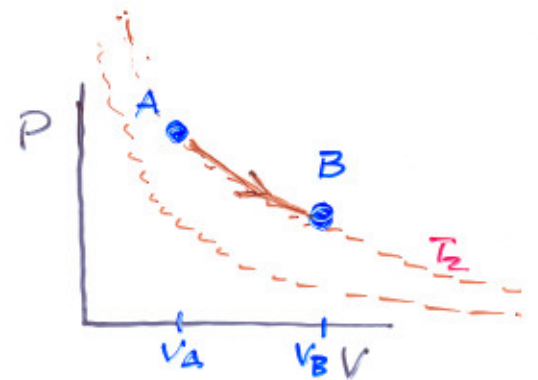
D

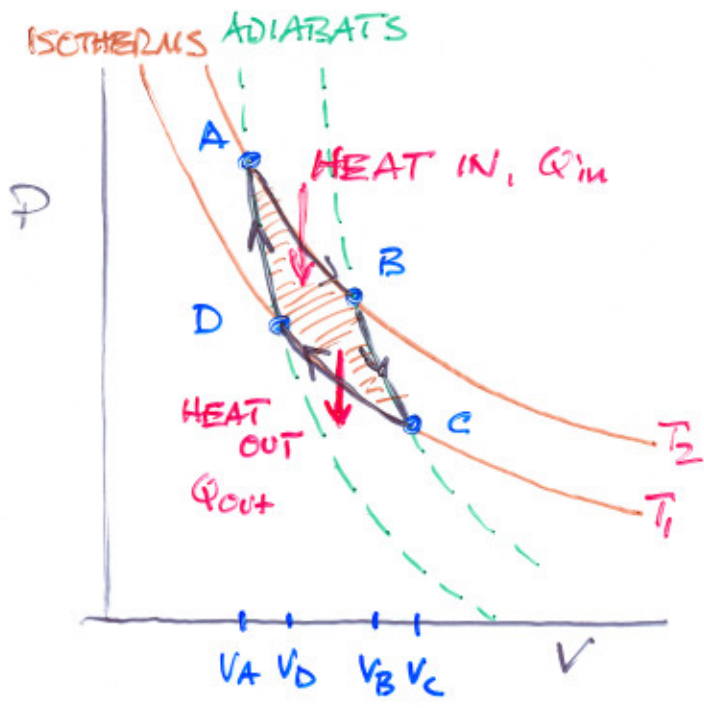


D



A





$$\epsilon = \frac{W}{Q_{in}} -$$

$$= \frac{Q_{out} - Q_{in}}{Q_{in}} -$$

$$\epsilon = 1 - \frac{Q_{out}}{Q_{in}}$$

Assume an ideal gas -

A → B  
 isothermal expansion  
 $P_A V_A = P_B V_B$

B → C  
 adiabatic expansion  
 $P_B V_B^\gamma = P_C V_C^\gamma$

C → D  
 isothermal compression  
 $P_C V_C = P_D V_D$

D → A  
 adiabatic compression  
 $P_D V_D^\gamma = P_A V_A^\gamma$

A → B isothermal expansion

$$\Delta U = \Delta Q - \Delta W = 0$$

$$W_{AB} = Q_{in}$$

$$W_{AB} = \int_A^B P dV = \int_A^B \frac{nRT_2}{V} dV = nRT_2 \ln\left(\frac{V_B}{V_A}\right) > 0$$

C → D isothermal contraction

$$W_{CD} = |Q_{out}|$$

$$= \int_C^D P dV = \int_C^D \frac{nRT_1}{V} dV = nRT_1 \ln\left(\frac{V_D}{V_C}\right) < 0$$

$$Q_{out} = nRT_1 \ln\left(\frac{V_C}{V_D}\right) = -W_{CD} = nRT_1 \ln\left(\frac{V_C}{V_D}\right) > 0$$

B → C adiabatic expansion

remember

$$W_{BC} = \frac{nR}{\gamma-1} (T_2 - T_1)$$

and

$$\Delta U = \underset{\substack{\uparrow \\ \text{gets cooler}}}{-n C_V} (T_2 - T_1) = n C_V (T_1 - T_2)$$

W (by gas)      Q (added)       $\Delta U$

$$nRT_2 \ln(V_B/V_A) \quad Q_{in} = nRT_2 \ln(V_B/V_A) \quad 0$$

$$\frac{nR}{\gamma-1} (T_2 - T_1) \quad 0 \quad nC_V(T_1 - T_2)$$

$$nRT_1 \ln(V_D/V_C) \quad Q_{out} = nRT_1 \ln(V_D/V_C) \quad 0$$

$$\frac{nR}{\gamma-1} (T_1 - T_2) \quad 0 \quad nC_V(T_2 - T_1)$$

net change:

$$nR(T_2 - T_1) \ln\left(\frac{V_B}{V_A}\right) - nR(T_2 - T_1) \ln\left(\frac{V_D}{V_C}\right) \quad 0$$

$D_3$  slow this line





from

$$\frac{Q_{out}}{Q_{in}} = \frac{T_1}{T_2} \frac{\ln(V_C/V_D)}{\ln(V_B/V_A)}$$

and

$$\frac{V_B}{V_A} = \frac{V_C}{V_D} \quad (\text{which you need to show for } D_3)$$

The "Carnot Efficiency" is

$$\epsilon_c = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{T_1}{T_2} = 1 - \frac{T_c}{T_H}$$

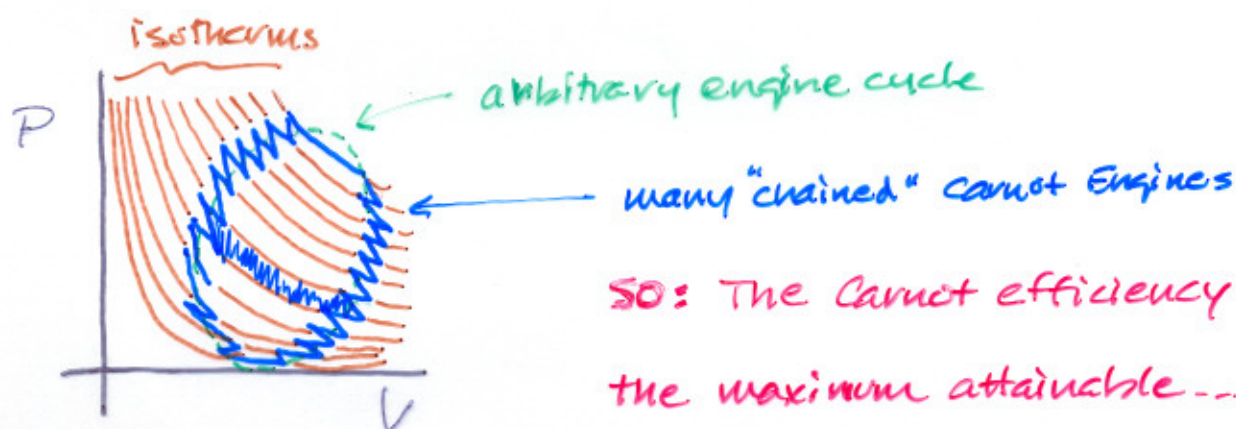
$$\epsilon_c = 1 - \frac{T_c}{T_H} \quad \text{depends only on temperatures}$$

Imagine a bunch of Carnot Engines:

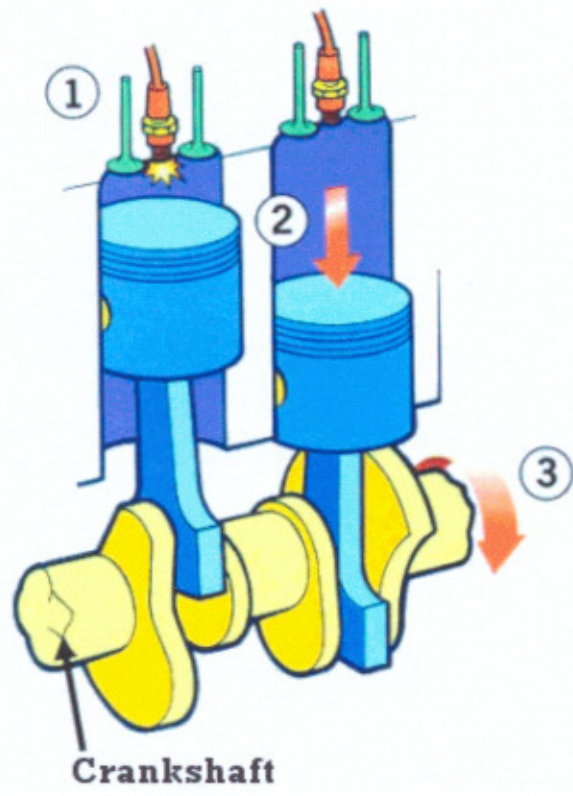
$Q_{in}$  for one =  $Q_{out}$  of the previous one...

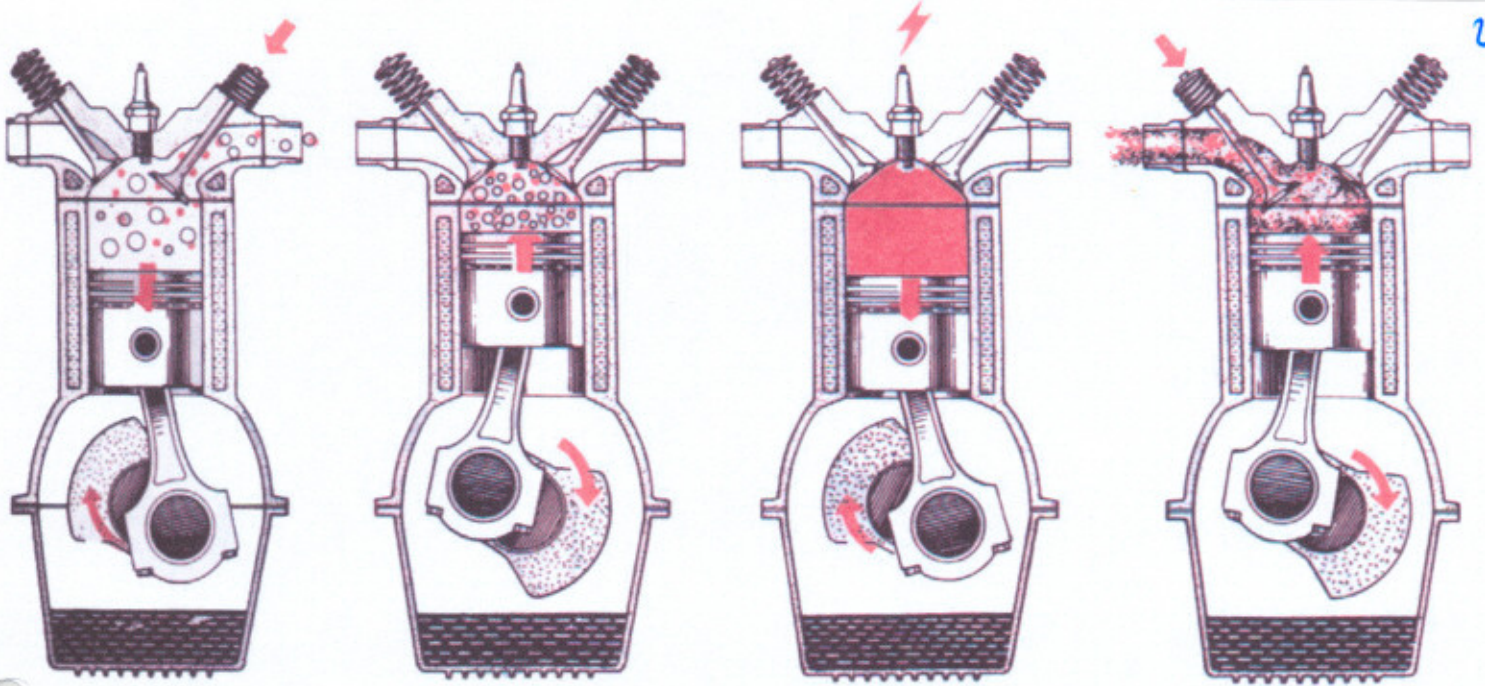
An arbitrary engine cycle can be thought of as

the limit of an infinite number of Carnot engines



SO: The Carnot efficiency is the maximum attainable... it is an idealization:  $\epsilon_{any} < \epsilon_c$





Air intake

Compression

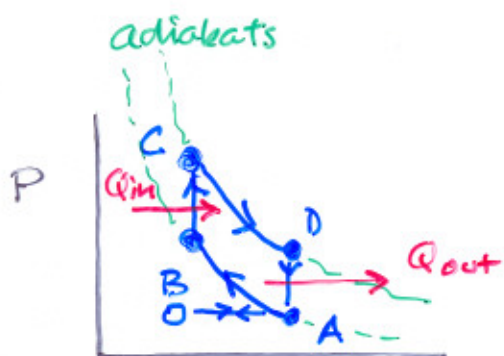
Combustion

Exhaust Emission

207



# REAL ENGINES



"Otto Cycle"

~ an automobile engine

OA - air in at atmospheric pressure

AB - adiabatic compression

BC -  $Q_{in}$  represents the heating from a spark plug

CD - adiabatic expansion - work done... turning the crank shaft

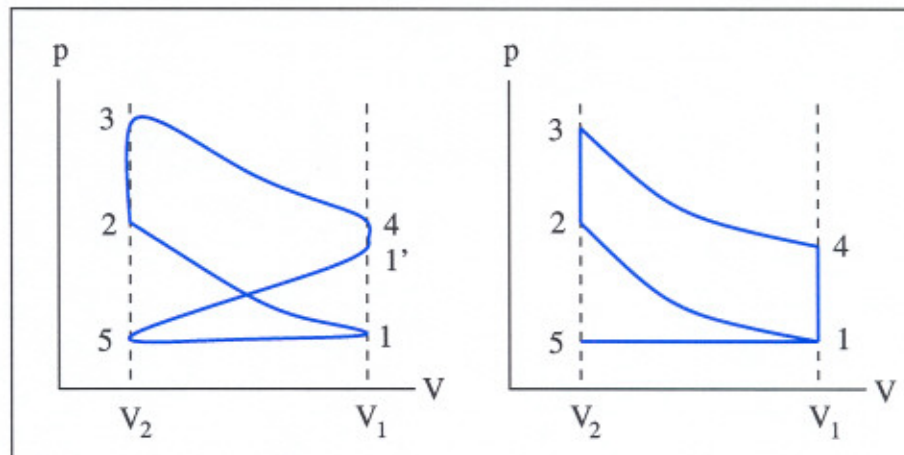
DA - cooling (exhaust stroke, almost)

AO - actual exhaust at atmospheric pressure

$$\epsilon_{otto} = 1 - \frac{1}{\left(\frac{V_A}{V_B}\right)^{\gamma-1}}$$

$$\epsilon_{otto} = 1 - \frac{T_A}{T_D} \quad \left( \epsilon_c = 1 - \frac{T_A}{T_C} \right)$$

## Gasoline engine (Otto cycle) [1m65]



### Four-stroke Otto cycle (left)

- 1-2: compression stroke
- 2-3-4: power stroke (spark plug ignites at 2)
- 4-1'-5: exhaust stroke (exhaust valve opens at 4)
- 5-1: intake stroke (intake valve opens at 5)

### Idealized Otto cycle (right)

- 1-2: adiabatic compression of air-fuel mixture ( $S = \text{const}$ )
- 2-3: explosion of air-fuel mixture ( $V = \text{const}$ )
- 3-4: adiabatic expansion of exhaust gas ( $S = \text{const}$ )
- 4-1: isochoric release of exhaust gas ( $V = \text{const}$ ).
- 1-5-1: intake stroke (thermodynamically ignored)

Parameter:  $K \doteq V_1/V_2$  (compression ratio).

The compression ratio  $K$  must not be chosen too large to prevent the air-fuel mixture from igniting spontaneously, i.e. prematurely.

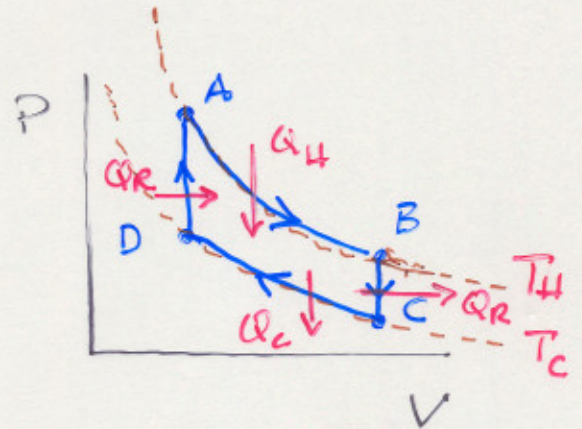
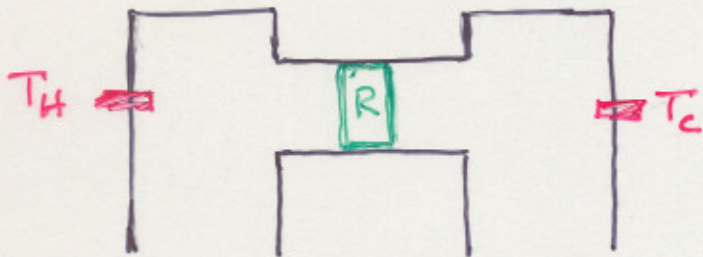
# STIRLING ENGINE

2/2



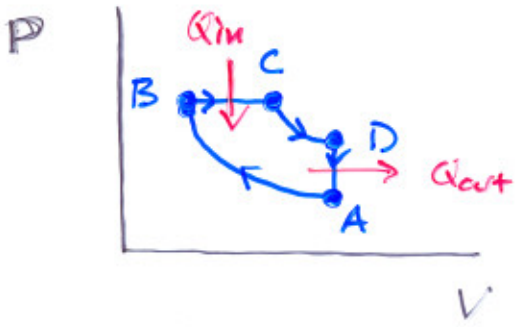
real-live, near-Carnot engine concept

Robert Stirling ~ 1816



- AB - isothermal expansion  
L - down  $\rightarrow$  power  
 $Q_H$  in
- BC - isochoric! up!  
R - ~~up~~ down L - down!  
 $Q_C$  - out
- CD - isothermal compress.  
R - up  
 $Q_C$  - out
- DA - isochoric  
L - down R - up  
 $Q_R$  - in





Diesel Cycle



# ENTROPY

Suppose we have two objects at different temperatures



- This configuration is useful
- could run an engine between them and get  $W$  out.
  - could warm anything  $T_A < T_H$

without transferring heat in or out — put them together.

Let them come to equilibrium —  $T_M$

$$T_M > T_C \quad \text{right?}$$

$$T_M < T_H$$

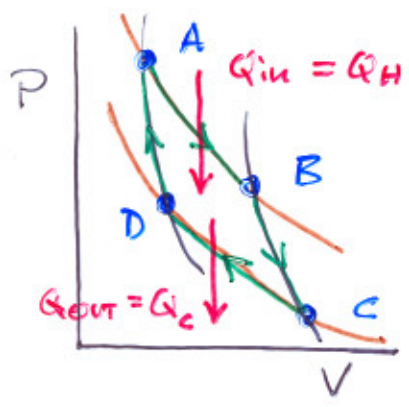


- total energy is the same
- no work was done on it

But, this configuration is somehow less useful — “lower quality” energy

- can't run an engine.
- can't warm anything  $T_A > T_M$

Remember the Carnot Cycle...



we found

$$\frac{Q_{out}}{Q_{in}} = \frac{T_c}{T_H} = \frac{Q_c}{Q_H}$$

rearrange

$$\frac{Q_c}{T_c} = \frac{Q_H}{T_H}$$

$$0 = \frac{Q_H}{T_H} - \frac{Q_c}{T_c} \quad \text{for the whole cycle}$$

Something called  $\frac{Q}{T}$  appears to not change in the cycle. This is called **ENTROPY**

$$\Delta S = \frac{\Delta Q}{T} \Rightarrow \Delta S = 0 \text{ for closed, complete cycle}$$

or... think lots of infinitesimal Carnot cycles to make one arbitrary cycle:

$$\Delta S = \oint \frac{dQ}{T} = 0 \quad \text{for an arbitrary, closed system}$$

Symbolic for whole cycle

What's an obvious characteristic of a cycle?

reversibility

$$\Delta S = \frac{\Delta Q}{T}$$

was for the isothermal stages  
of the Carnot cycle.

$$\& \quad \Delta S = \frac{\Delta Q}{T}$$

for any isothermal  
REVERSIBLE process

$$* \quad \oint \frac{dQ}{T} = 0$$

for any complete <sup>reversible</sup> cycle

The entropy of the universe is unaffected  
by any reversible process

Look closely at some arbitrary path from

$$i \rightarrow f \Rightarrow T_i V_i \rightarrow T_f V_f$$

reversible, ideal gas

$$\text{1st: } dQ = dU + dW$$

$$dW = PdV$$

$$dU = nC_V dT$$

$$P = \frac{nRT}{V}$$

so:

$$dQ = nC_V dT + nRT \frac{dV}{V}$$

$$\frac{dQ}{T} = nC_V \frac{dT}{T} + nR \frac{dV}{V} \quad i \rightarrow f \Rightarrow$$

$$\int_{T_i}^{T_f} \frac{dQ}{T} = nC_V \int_{T_i}^{T_f} \frac{dT}{T} + nR \int_{V_i}^{V_f} \frac{dV}{V}$$

$$\Delta S = nC_V \ln(T_f/T_i) + nR \ln(V_f/V_i)$$

nothing said about the path ...

$\Delta S$  is dependent only on initial state

and the final state  $\Rightarrow$  a State Function

State Functions:  $P, V, S$



# REMEMBER MY "ARROW OF TIME" DISCUSSION?

Let's consider an IRREVERSIBLE process...

can consider entropy changes now since we find that  $S$  is a state function

## WHAT'S AN IRREVERSIBLE PROCESS?

- in a reversible process, one can describe each step as having a definite thermodynamic state
- an irreversible process has a well defined thermodynamic state only at the beginning and the end

