Instead of 2 waves, add more and more and the groups become wave localized

\[ \Delta x \rightarrow \Delta x \rightarrow \text{flat} \]

in a limit

\[ \text{flat} \rightarrow \text{localized} \rightarrow \text{WAVE PACKET} \]

the "flat" regions \( \Rightarrow \) a phase relationship \( y = \pi \)
among all waves in the packet

\[
y = 2A \cos \left\{ \frac{\Delta k x - \Delta \omega t}{2} \right\} \cos \left\{ \frac{\hbar x - \hbar \omega t}{\hbar} \right\}
\]

the envelope

\[ \Downarrow \]

going flat at particular \( x \)'s

\[ \Delta x = x_2 - x_1 \text{ here are zero} \]

and \( \frac{\Delta k x_2}{\Delta \omega} - \frac{\Delta k x_1}{\Delta \omega} = \pi \) for more points.
\[ \frac{\Delta h}{2} (x_2 - x_1) = \pi \]
\[ \Delta h \Delta k = 2\pi \]

Likewise in TIME
\[ \Delta \omega \Delta t = 2\pi \]

So... to have fine localization
\[ \Delta x \text{ small} \]
\[ \Delta h \text{ large} \]

must have large range of wavelengths.

\[ \text{since } \Delta x = \frac{2\pi}{\Delta h} \]

In electronics
\[ \Delta t = \frac{2\pi}{\Delta \omega} \]

Sine pulse requires high frequency bandwidth.

\[ \Rightarrow \text{ all true in all kinds of waves.} \]
2 waves-one twice the other cos(x)  and cos(11\sqrt{2}\times x)

\[ \cos(x) + \cos(11\sqrt{2}\times x) + \cos(11\times x) \]
For many waves superimposed

\[ y(x,t) = \sum_{n} A_n \cos(h_n x - \omega_n t) \]

**Fourier Series**

which can be extrapolated to a continuous distribution

\[ y(x,t) = \int_{0}^{\infty} \tilde{y}(h) \cos(h x - \omega t) \, dh \]

In this case, we can again deal with an envelope.

For example

\[ y(x,0) = A e^{-\Delta h^2 x^2} \cos(h_0 x) \]

---

*traveling waves*

*Gaussian envelope*

*real localization*
SO... you want "pauticles"?

when all you've got are waves?

WAVE PACKETS are your thing!

For many, many waves together...:

\[ v_q = \frac{\Delta w}{\Delta k} \rightarrow \frac{dw}{dk} \bigg|_{k_0} \]

\[ v_q = \frac{dw}{dk} \bigg|_{k_0} \]

with \( w = h v_p \)

\[ v_q = v_p \bigg|_{k_0} + h \frac{d v_p}{dk} \bigg|_{k_0} \]

\( v_p \) can be a function of a material \( \lambda \)

can be \( v_p(\lambda) \) or therefore \( v_p(k) \)

such as dispersive media - the individual waves travel at different speeds

\[ \rightarrow v_q \]

\[ \text{in time} \]

\[ \text{The individual wave separating} \]
Figure 2. (a) & (b) Shockwave pulse train in a 106-m long length of CGW-B-10 fiber with nominally 90% reflecting mirrors across each end. Pulse spreading effects were made negligible by injecting 30-nsec wide laser pulses that were broad compared to the fiber dispersion. (b) Pulse 2014m, L = 954m, 514 Pulse 106m, L = 106m.
Back to particles... de Broglie particles.

+ couple of standard relations:

\[ E = hf \]

\[ E = \frac{\hbar \nu}{2\pi} = \frac{\hbar \omega}{2\pi} \]

\[ E = \hbar \omega. \]

\[ p = \frac{\hbar}{\lambda} = \frac{\hbar k}{2\pi} \]

\[ p = \hbar k. \]

Look at phase velocity:

\[ v_p = f\lambda = \frac{E}{p} \]

\[ E = \sqrt{p^2c^2 + m^2c^4} \]

\[ v_p = \sqrt{\frac{p^2c^2 + m^2c^4}{p^2}} = c \sqrt{1 + \left(\frac{mc^2}{p}\right)^2} \]

\[ v_p = c \sqrt{1 + \left(\frac{mc^2}{\hbar k}\right)^2} \]

\[ \uparrow \quad v_p = v_p(c) \quad \text{DISPERSE IN EMPTY SPACE} \]
Look at group velocity

\[ v_{gr} = \frac{d\omega}{dn} = \frac{d}{dP} \left( \frac{E}{\hbar} \right) = \frac{dE}{dp} \]

\[ E^2 = m^2 c^4 + p^2 c^2 \]
\[ 2E \, dE = 2p \, dp \, c^2 \]
\[ \frac{dE}{dp} = \frac{pc^2}{E} \]

\[ v_{gr} = \frac{pc^2}{E} \]

Not obvious — look non-relativistically, \( p = mu \).

And \( E = \frac{p^2}{2m} \)
\[ \frac{dE}{dp} = \frac{2p}{2m} = \frac{mu}{m} = v \] the "mechanical" velocity

\[ v_{gr} = \frac{dE}{dp} = v \]

Is the group velocity

Act H H A T! one can think of a "particle" with "mechanical" velocity \( v \) as a "wavepacket" with group velocity \( v_{gr} \)
WHAT DOES IT MEAN

to determine the location of something?

gotta "look" at it...

t not quiet with your eyes... but in principle

cut different.

But... suppose your probe and your target are

both particles and waves...

& suppose your target is tiny -- like an electron

& your probe is light.

This was thought about by Werner Heisenberg

--- formally --- derived from Quantum Mechanics

--- and abstractly --- using Thought Experiments
**What is a Shadow?**

- Light, small wavelength
- Big object
- or, a slit

If the slit is $a \lambda$:

- $a \sin \theta_0 = n \lambda$
- $\theta_0 = \frac{n \lambda}{a}$

- $\lambda$ - smaller $\lambda$ -- more localized to the image of the slit for $m=1$
KEEP THIS IN MIND!

What's the INTENSITY of light?

For any wave, its the square of the amplitude.

For light, $I \propto |E_{\text{rms}}|^2$

↑ the electric field
Here's the rub...

\[ p_y = \frac{h}{\lambda} \]

(smaller the \( \lambda \), higher the \( p \))

So, how about the location of an electron?

How well can you determine that an electron is here relative to here.

... using light.
Heisenberg "thought experiment"

\[ P_y = \frac{h}{\lambda} \]

the resolving power... the resolution... of a microscope.

\[ \Delta x = \frac{\lambda}{\sin \alpha} \]

best resolution, smallest \( \lambda \)

can't see any smaller than this for a given \( \lambda \)

... classically, reduce \( \lambda \)

(Ignore the change of \( \theta \) by scattered \( X \))
The proton can scatter anywhere
... but may be seen if it enters the lens.

biggest $\Delta p_{y'x}$ is for

$\Delta p_{y'x} = 2 p_{y'x} \sin \alpha$

But, by momentum conservation, this also equals $\Delta P_{y'x} = 2 P_{y'x} \sin \alpha$

$= \frac{2 h \sin \alpha}{\lambda}$

smallest disturbance $e$ -- largest $\lambda$

multiply:

$\Delta x \Delta P_{y'x} = \frac{\lambda}{\sin \alpha} \cdot \frac{2 h \sin \alpha}{\lambda}$

$\Delta x \Delta P_x = 2h$

This is an inescapable conclusion... since proton's wavelength cannot be arbitrarily reduced -- had to have at least $1\%$ to "illuminate" the electron.
Just bad measuring?

No. It's due to the impossibility of using less than a single photon.

How about electrons?

- An electron single slit
- An electron double slit

- Single slit - same as for light.

\[ \theta_s \sim \frac{\lambda}{a} \]  
@ first minimum.
Double slit

\[ a = \frac{\lambda}{d} \]

first one
then
the other

minima occur at

\[ \sin \theta_d = \frac{\lambda}{d} \]

\[ \theta_d \leq \theta_s \left( \frac{a}{2d} \right) \]

the interference minima much more frequent.
Okay... trick it.

Figure out which slits the electrons go through

The monitor emits photons which scatter from an electron... and into our eye

→ and identifies which slit.
TIGHTEN YOUR SEAT BELTS:

Remember, there is constructive interference for 2 slit diffraction at

\[ d \sin \theta = m \lambda \quad m = 0, 1, 2, \ldots \]

Want the separation on the screen between adjacent maxima:

\[ d \sin \theta_m = m \lambda \]

\[ d \sin \theta_{m+1} = (m+1) \lambda \]

\[ \sin \theta_{m+1} - \sin \theta_m = \frac{(m+1) \lambda - m \lambda}{d} \]

\[ = \frac{\lambda}{d} \]

Any vertical spot on the screen is at

\[ y_m = L \tan \theta_m \]

for small angles \( \tan \theta_m \approx \sin \theta_m \)

\[ y_m \approx L \sin \theta_m \]
So,

\[ y_{n+1} - y_n = L \sin \theta_{n+1} - L \sin \theta_n \]

\[ = L (\sin \theta_{n+1} - \sin \theta_n) \]

\[ y_{m+1} - y_m = \frac{L \lambda}{d} \]

Resolution - distinguishing
2 peaks - says
INTERFERENCE!!

Okay... remember the monitor?

Must distinguish one slit from the other

\[ \Delta y \approx \frac{d}{2} \text{ or so...} \]
So... the monitor photon KICKS the electron

\[ \Rightarrow \text{giving it some } \Delta p_y \]

Well, from our microscopic thought experiment...

\[ \Delta y \Delta p_y \approx 2\hbar \]

\[ \Delta p_y = \frac{2\hbar}{d/2} = 4\hbar/\bar{d} \]

\[ \Delta \theta = \frac{\Delta p_y}{p} = \frac{4\hbar}{\bar{p} \bar{d}} \]

\[ p = \frac{\hbar}{\lambda} \]

\[ \Delta \theta = \frac{4\lambda}{\bar{d}} \]

so... the ADDED vertical displacement due to the monitor is

\[ y = \Delta y \lambda = \frac{4\lambda \lambda}{\bar{d}} \]

The ORIGINAL distance between maxima was

\[ y_{max} - y_{min} = \frac{7\lambda}{\bar{d}} \]