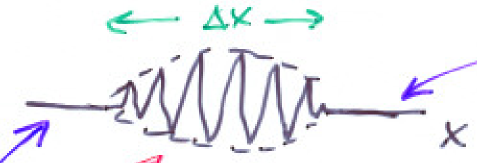
Instead of 2 waves, add more and more and the groups become more localized...

in a limit...



flat

flat

localized

WAVE PACKET

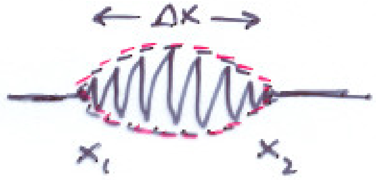
the "flat" regions ⇒ a phase relationship of π among all waves in the packet

for

$$y = 2A\cos\underbrace{\left\{\frac{\Delta k x}{2} - \frac{\Delta \omega t}{2}\right\}}_{\text{the envelope}}\cos\left\{\bar{k}x - \bar{\omega}t\right\}$$

$\downarrow$

going flat at particular x's



$\Delta x = x_2 - x_1$ here are zero

and $\frac{\Delta k x_2}{2} - \frac{\Delta k x_1}{2} = \pi$ for these points.

$$\frac{\Delta k}{2}(x_2 - x_1) = \pi$$

$$\Delta k \, \Delta x = 2\pi$$

likewise in TIME

$$\Delta \omega \, \Delta t = 2\pi$$

So... to have fine localization

Δx small

must have large range of wavelengths.

Δk large

since $\quad \Delta x = \dfrac{2\pi}{\Delta k}$

in electronics $\quad \Delta t = \dfrac{2\pi}{\Delta \omega}$

short pulse

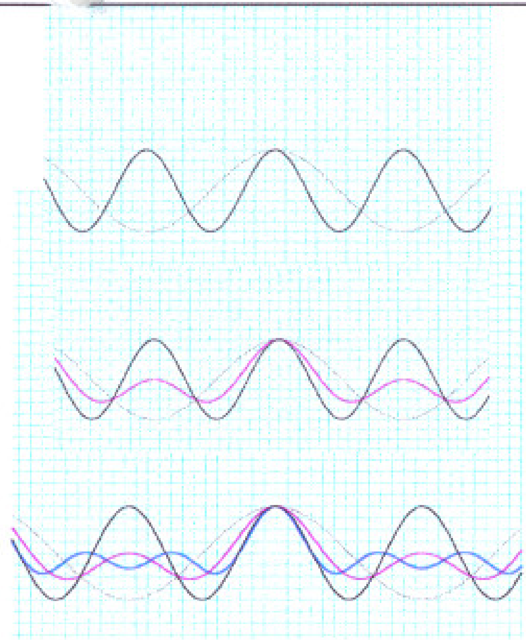
requires high frequency bandwidth.

$\Rightarrow$ all true for all kinds of waves.

2 wavelengths...one twice the other cos(x) and cos(1/2x)

add them:

add another cos(x) + cos(1/2x) + cos(1/3x)



For many waves superimposed

$$y(x,t) = \sum_n A_n \cos(k_n x - \omega_n t)$$

Fourier series

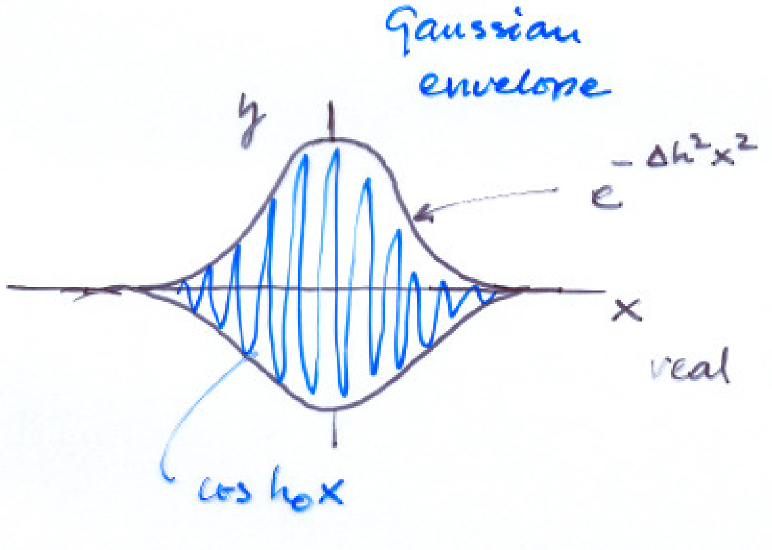
which can be extrapolated to a continuous distribution

$$y(x,t) = \int_0^\infty \tilde{y}(k)\cos(kx-\omega t)\,dk$$

In this case, we can again deal with an envelope.

For example

$$y(x,0) = \underbrace{A e^{-\Delta k^2 x^2}} \underbrace{\cos(k_0 x)}_{\text{traveling waves}}$$



real localization

SO... you want "particles"?

when all you've got are waves?

WAVE PACKETS are your thing!

For many, many waves together...

$$v_g = \frac{\Delta\omega}{\Delta k} \rightarrow \left.\frac{d\omega}{dk}\right|_{k_0}$$

the central wave number

$$v_g = \left.\frac{d\omega}{dk}\right|_{k_0}$$

with $\omega = k v_p$

$$v_g = \left. v_p \right|_{k_0} + k \left.\frac{dv_p}{dk}\right|_{k_0}$$

v_p can be a function of a material &

can be $v_p(\lambda)$ or therefore $v_p(k)$

Such are dispersive media – the individual waves travel at different speeds

in time

the individual waves separating.

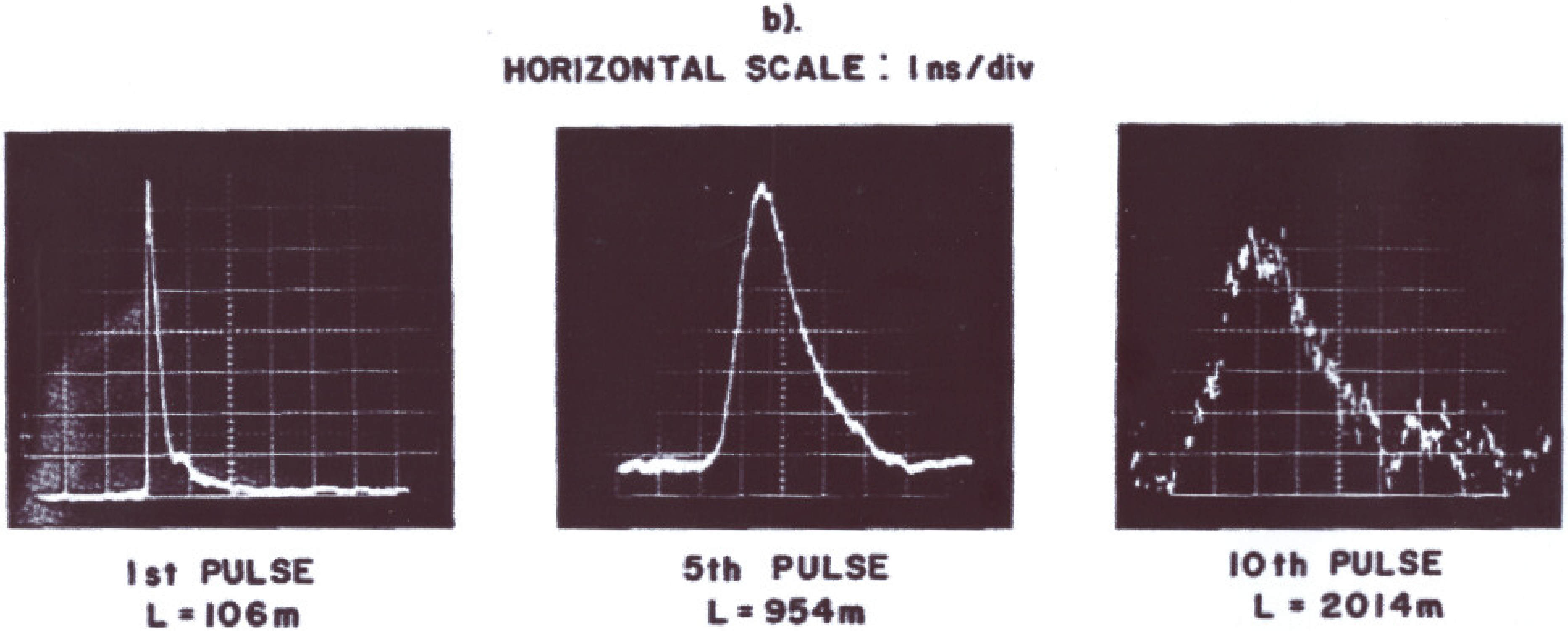


Fig. 3. (a) Shuttle pulse train in a 106-m length of CGW-Bell-10 fiber with nominally 90% reflecting mirrors against each end. Pulse spreading effects were made negligible by injecting 30-nsec wide laser pulses that were broad compared to the fiber dispersion. (b) Pulse spreading is observed when narrow impulses are injected into the fiber. The photographs are sampling scope displays of the first pulse received after $L = 106$ m, the fifth pulse after $L = 954$ m, and the tenth pulse after $L = 2014$ m.

L.G. Cohen
Applied Optics 14, 1353, 1975

Back to particles... de Broglie particles.

A couple of standard relations:

$E = hf$ from ~~$\omega = \frac{2\pi}{f}$~~ aack! $\omega = 2\pi f$

$E = h\frac{\omega}{2\pi} = \hbar\omega$

$E = \hbar\omega.$

$p = \frac{h}{\lambda} = h\frac{k}{2\pi}$ from $k = \frac{2\pi}{\lambda}$

$p = \hbar k.$

look at phase velocity...

$$v_p = f\lambda = \frac{E}{h}\frac{h}{p} = \frac{E}{p}$$

$$E = \sqrt{p^2c^2 + m^2c^4}$$

$$v_p = \sqrt{\frac{p^2c^2 + m^2c^4}{p^2}} = c\sqrt{1 + \left(\frac{mc}{p}\right)^2}$$

$$v_p = c\sqrt{1 + \left(\frac{mc}{\hbar k}\right)^2}$$

$v_p = v_p(k)$ -- DISPERSIVE IN EMPTY SPACE with at group velocity

$$v_{gr} = \frac{d\omega}{dk} = \frac{d(E/\hbar)}{d(p/\hbar)} = \frac{dE}{dp}$$

$$E^2 = p^2c^2 + m^2c^4$$

$$2E\,dE = 2p\,dp\,c^2$$

$$\frac{dE}{dp} = \frac{pc^2}{E}$$

$$v_{gr} = \frac{pc^2}{E}$$

not obvious... look non-relativistically. $p = mv$.

and $E = \frac{p^2}{2m}$

$$\frac{dE}{dp} = \frac{2p}{2m} = \frac{mv}{m} = v$$ the "mechanical" velocity

$$v_{gr} = \frac{dE}{dp} = v$$ is the group velocity

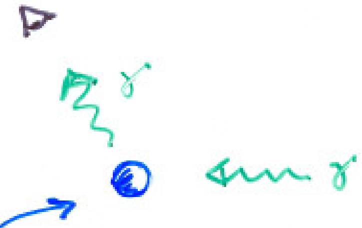
AH HAH! one can think of a "particle" with "mechanical" velocity v as a "wavepacket" with group velocity v_{gr}

WHAT DOES IT MEAN

to determine the location of something?

gotta "look" at it...

not just with your eyes... but in principle not different.



to "see" the ball

But... suppose your probe and your target are both particles and waves...

& suppose your target is tiny -- like an electron

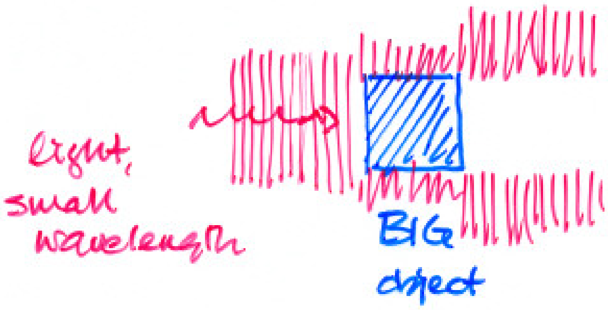
& your probe is <u>light</u>.

This was thought about by Werner Heisenberg

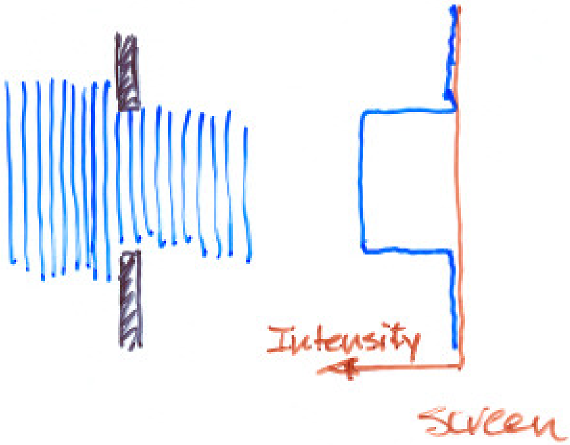
-- for<u>mally</u> -- derived from Quantum Mechanics

-- and abstractly -- using Thought Experiments

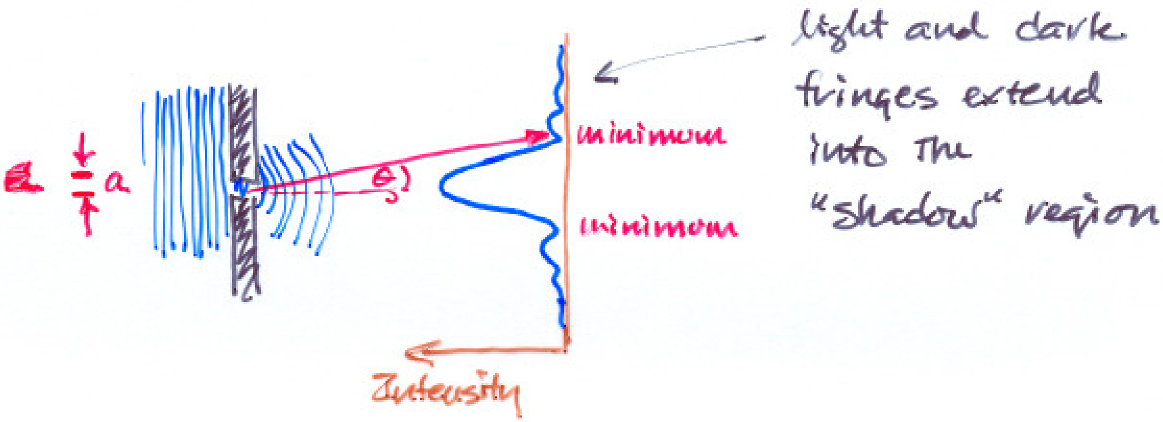
WHAT IS A SHADOW?



or, a slit



if the slit is $\sim \lambda$ --



$$a \sin \theta_s = m\lambda$$

$$\theta_s \sim \frac{m\lambda}{a}$$ ← smaller λ .. more localized to the image of the SLIT for $m=1$

KEEP THIS IN MIND:

What's the INTENSITY of light ?

for any wave,
it's the square
of the amplitude

For light $I \propto |E_{rms}|^2$

the electric field

HERE'S THE RUB...

light's also a particle.

$$p_\gamma = \frac{h}{\lambda}$$

smaller the λ, higher the p

SO, HOW ABOUT THE LOCATION OF

an electron

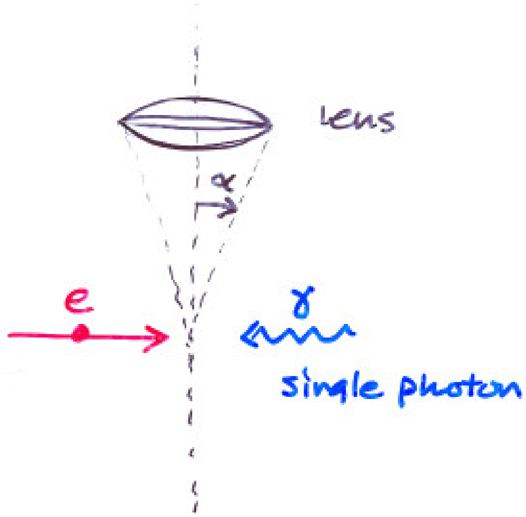
How well can you determine that an electron

is here

relative to here

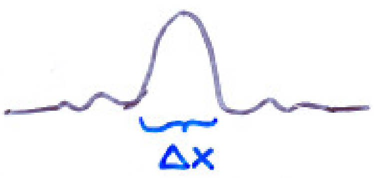
... using light

Heisenberg "thought experiment"



$$p_\gamma = \frac{h}{\lambda}$$

the resolving power -- the resolution -- of a microscope

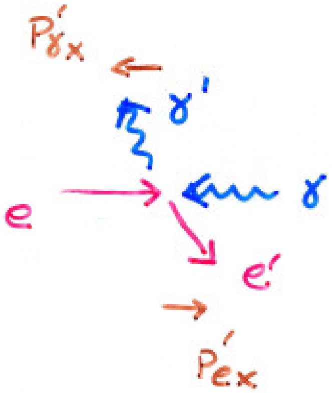


$$\Delta x \sim \frac{\lambda}{\sin\alpha}$$

best resolution, smallest λ

can't see any smaller than this for a given λ

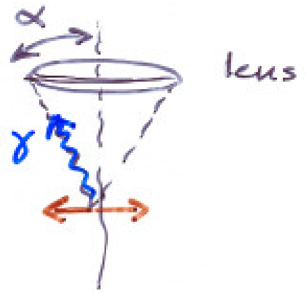
... classically, reduce λ



(Ignore the change of t by scattered γ)

The photon can scatter anywhere

... but only be <u>seen</u> if it enters the lens.



biggest $\Delta p'_{\gamma_x}$ is for

$$\Delta p'_{\gamma x} = 2 p_{\gamma x} \sin\alpha$$

But, by momentum conservation, this also equals

$$\Delta p'_{e_x} = 2 p_{\gamma_x} \sin\alpha$$
$$= 2 \frac{h}{\lambda} \sin\alpha$$

smallest disturbance of e ... largest λ

multiply:

$$\Delta x_e \, \Delta p_{e_x} = \frac{\lambda}{\sin\alpha} \cdot \frac{2h}{\lambda} \sin\alpha$$

$$\Delta x \, \Delta p_x = 2h$$

This is an inescapable conclusion ... since photon's wavelength cannot be arbitrarily reduced -- had to have at least 1 γ to "illuminate" the electron.

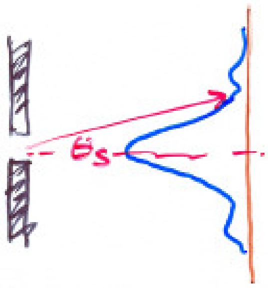
JUST BAD MEASURING?

no. It's due to the impossibility of using less than a single photon.

HOW ABOUT ELECTRONS?

an electron single slit
&
an electron double slit

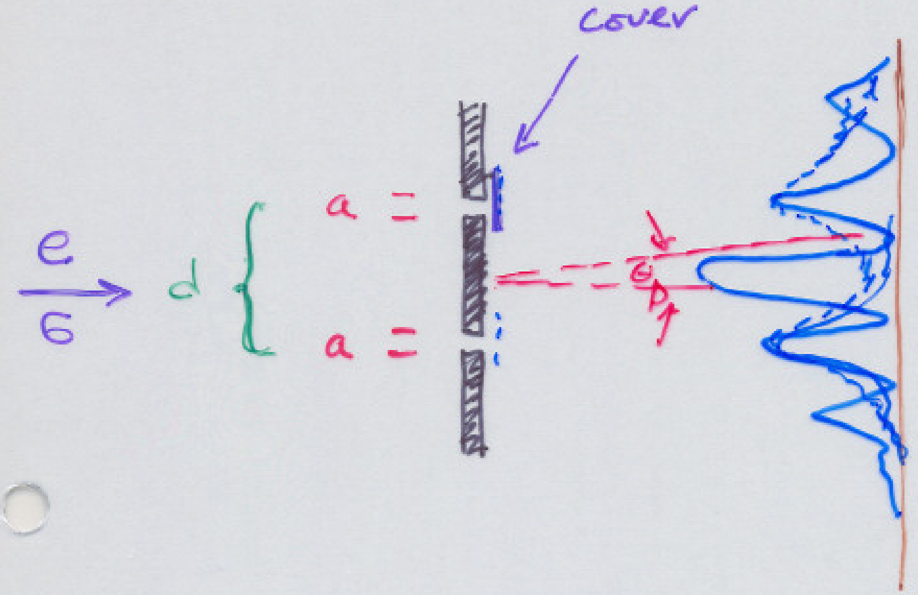
* single slit — same as for light.



} broad, reflecting the size of the slit.

$\theta_s \sim \frac{\lambda}{a}$ @ first minimum.

Double slit



first one

then

the other

minima occur at $\qquad \sin\theta_D = \dfrac{\lambda/2}{d}$

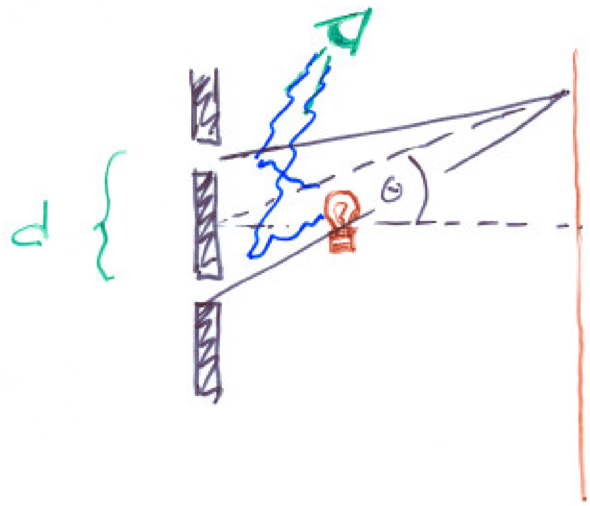
$$\theta_D \simeq \theta_s \left(\frac{a}{2d}\right)$$

the interference minima much more frequent.

OKAY... TRICK IT.

FIGURE OUT WHICH SLITS

THE ELECTRONS GO THROUGH



some device
which we watch

The monitor emits photons which scatter from
an electron -- and into our eye

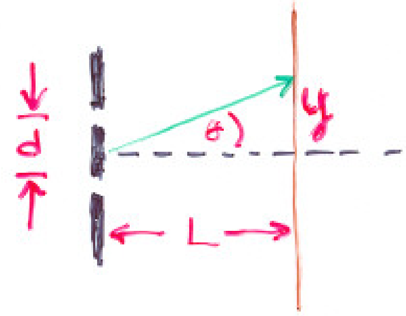
→ and identifies which slit.

TIGHTEN YOUR SEAT BELTS:

Remember, there is constructive interference for 2 slit diffraction at

$$d\sin\theta = m\lambda \qquad m = 0, 1, 2 \ldots$$

Want the separation on the screen between adjacent maxima →



$$d\sin\theta_m = m\lambda \qquad d\sin\theta_{m+1} = (m+1)\lambda$$

$$\sin\theta_{m+1} - \sin\theta_m = \frac{(m+1)\lambda}{d} - \frac{m\lambda}{d}$$

$$= \frac{\lambda}{d}$$

Any vertical spot on the screen is at

$$y_m = L\tan\theta_m$$

for small angles $\tan\theta_m \sim \sin\theta_m$

$$y_m \sim \cancel{L\tan\theta_m}\ L\sin\theta_m$$

so,

$$y_{m+1} - y_m = L\sin\theta_{m+1} - L\sin\theta_m$$

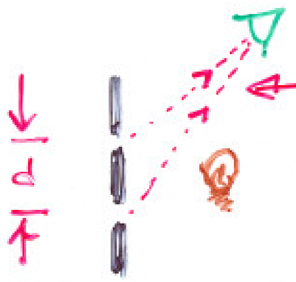
$$= L(\sin\theta_{m+1} - \sin\theta_m)$$

$$y_{m+1} - y_m = \frac{L\lambda}{d}$$

... sort of a resolution -- distinguishing 2 peaks ... says INTERFERENCE !!

Okay... remember the monitor?

Must distinguish one slit from the other



must distinguish

like measuring the y position @ slits by

$\Delta y \sim \frac{d}{2}$ or so...

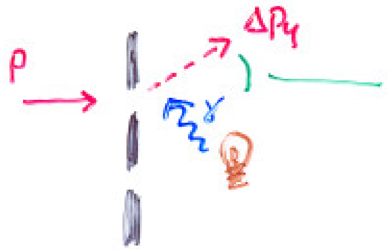
So... the monitor photon KICKS the electron

→ giving it some Δp_y

Well, from our microscope thought experiment...

$$\Delta y \, \Delta p_y \simeq 2h$$

$$\Delta p_y \simeq \frac{2h}{d/2} \sim \frac{4h}{d}$$



$\Delta\theta$ an additional angle

$$\Delta\theta = \frac{\Delta p_y}{p} = \frac{4h}{pd} \qquad p = \frac{h}{\lambda}$$

$$\Delta\theta = \frac{4\lambda}{d}$$

so... the ADDED vertical displacement due to the monitor is $y = \Delta\theta L = \frac{4L\lambda}{d}$

The ORIGINAL distance between maxima was

$$y_{m+1} - y_m = \frac{L\lambda}{d}$$